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MATH SCREENING—IDENTIFYING STUDENTS AT RISK: LITERATURE REVIEW

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Executive Summary

Historically, research about reading difficulties has taken the spotlight within the field of cognitive and educational psychology. Most of this research has focused on how to improve reading comprehension, fluency, and speed. Only recently has research into mathematics learning processes and difficulties moved to the forefront.

Many students struggle with mathematics at various points in their education. Finding ways to pinpoint those areas of weakness can provide educators with valuable information to provide effective instruction when and how it's needed. In the published research of the last decade, three tasks in particular have shown up consistently as reliable predictors of math deficiencies in students: number line estimation, computation and magnitude comparison. Number line estimation tasks involve the ability to accurately place a value on a number line without reference points on the line. Computation is the basic task of addition, subtraction, multiplication and division that are age appropriate. Magnitude comparison is being able to indicate which of two values is greater.

Careful screening and progress monitoring using these research-based measures can help to determine whether students have general Mathematical Difficulties (MD) or the more persistent Mathematical Learning Difficulties (MLD), which may require additional supports and strategies to overcome.

Mathematical difficulties (MD) represent a broad spectrum of disabilities present in children who struggle with math. Butterworth and Reigosa (2007) suggest that math difficulties stem from slower processing of numerical information, specifically in the areas of estimation and comparison. Difficulty recognizing, representing, and mentally manipulating small numerosities may be at the heart of this slower processing speed. From a more quantitative perspective, when a student receives a low standardized math achievement test score

"... early prediction will increase the likelihood that children's difficulties with mathematics will be diminished or avoided through the benefits of early intervention."
Professor Michele M. M. Mazzocco

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that falls below a cut-off point, usually the 35th percentile, then the student is considered to have problems learning math (Mazzocco, 2007). Identifying which students have MD can be accomplished through the following:

1. Early testing services to determine learning difficulties.
2. Frequent progress monitoring (or also called curriculum-based measurement or general outcome measurement). Progress monitoring as defined by Dr. Anne Foegen is "an empirically developed approach to formative evaluation that relies on frequent assessment using brief measures that serve as indicators of general proficiency in a content area" (Foegen, 2008).
3. Intervention for low-performing students. Throughout the intervention period a series of progress monitoring probes are administered to ensure that students are progressing appropriately.
4. Data-based decision-making. Based on progress of students selected for intervention, further testing may be required to determine whether a more serious learning disability is present.

"Using general outcome measures provides a way to use measurement for planning and evaluating instructional programs for students and for making peer-referenced comparisons."
(Fuchs & Dino, 1991)

Screening for Math Difficulties

Researchers have looked at various tasks and determined which ones are the best indicators of poor math achievement. Three tasks are most commonly found as key indicators of mathematical difficulties in students of all ages: computation (Fuchs, et. al, 2005; Griffin, 2007; Foegen, 2008b), comparison (Jordan, 2007; Holloway & Ansari, 2009; Markovits & Sowder, 1994; Wilson and Dehaene, 2007) and estimation (Siegler & Booth, 2004 and 2005; Schneider, Grabner, & Paetsch, 2009; Penner-Wilger, et. al., 2009).

Computation tasks are the most traditional way of measuring mathematical competence in students. Asking them to add, subtract, multiply and divide whole numbers, fractions and decimals is the standard method of determining proficiency in math. There are many national mathematics achievement tests that ask students to do basic arithmetic. In a study by Fuchs, et. al. (2005) they found that weekly tutoring of at-risk students helped them perform better on Curriculum-Based Measurement (CBM) Computation tests. These improved scores significantly exceeded the scores of their normally achieving peers. A study performed by Dr. Anne Foegen on progress monitoring in middle school found that computation, or basic facts measurement, was a good indicator (r= 0.92 for 6th graders and 0.95 for 7th graders) of math proficiency with strong criterion validity and acceptable levels of reliability (Foegen, 2008b).

Comparison is another measure that determines the ability of students to think on a larger scale and in terms of numerical magnitude. Number comparison requires recognition and judgment of magnitudes of numbers (Penner-Wilger, et. al., 2009). Comparison tasks come in both symbolic and non-symbolic representations of numbers, as appropriate for different age groups. For younger students, a non-symbolic representation might look like the comparison of a group of stars, but for the older students a symbolic representation would be comparing the numerals "4" and "6." The most common result from the numerical comparison task is the distance effect (Noël, Rousselle, & Mussolin, 2005; Holloway & Ansari, 2009; Penner-Wilger, et. al., 2009), which occurs when participants are faster to judge number pairs that have a larger difference (e.g. 2 vs. 8) than pairs with smaller differences (e.g. 3 vs. 4). This effect has been found in both adults and children, but for adults the effect is lessened (Noël, Rousselle, & Mussolin, 2005).

For 6-, 7- and 8-year-olds, Holloway and Ansari (2009) found that as calculation scores decreased, the size of the symbolic numerical distance effect increased. As the distance effect became more prominent in children, their performance on calculation problems dropped. This finding suggests that the distance effect observed in the comparison tasks is linked to performance on mathematical fluency and calculation abilities that indicate achievement in math. In the same study the authors also found that, when presented with two numbers, 7- to 8-year-olds were significantly faster at determining which number was larger compared to 6-year-olds.

A study by Wilson and Dehaene (2007) found that by using simple tasks we can easily detect problems with math achievement in older students. There is a clear impairment of number sense in most students with either acalculia or developmental dyscalculia, and simple tests using enumeration and number comparison have already been implemented in some schools based on Butterworth's 2005 research into diagnosing dyscalculic children. What Wilson and Dehaene found was a validation for number sense and number comparison as an appropriate tool to use to measure math difficulties in adults and older students.

Estimation is "a process of translating between alternate quantitative representations at least one of which is inexact" (Siegler & Booth, 2005). Consider a few examples of estimation in our daily lives. How many people attended the recital? How much will a Honda Accord cost? How long will you be on the phone? Estimation is an important part of mathematical cognition and everyday life. Extensive research has demonstrated that translating a written number to its placement on a number line provides good information about how a student represents numerical magnitude. There are three reasons outlined in Siegler and Booth's 2004 study that articulate the benefits of number line estimation tasks. First, the task does not require any prior knowledge of particular units or measurements, and as such is a pure measure of numerical estimation. Second, the task is applicable to activities learned in classrooms. Third, this task helps researchers test out different models of estimation and its mapping in the brain. With this task we gain insight into the way children view numerical concepts and number relationships.

In their 2004 study, Siegler and Booth found that within each grade they studied, kindergarten, first and second grade, "the smaller the child's percent absolute error of estimates, the higher was the child's achievement test score." This shows that number line estimation tasks are good predictors of math achievement in this age group. Research performed by Penner-Wilger et. al. (2009) found that performance on estimation tasks was a good indicator of math abilities in grades 1 and 2. This indicates that estimation is a task that is also useful in discovering who, in the elementary school classroom, is struggling with math abilities.

Schneider, Grabner and Paetsch (2009) found that number line estimation was also a good predictor of math achievement for fifth and sixth graders. After testing these students on basic number line estimation and on math achievement tests, performance on the estimation task significantly predicted performance on the math achievement tests.

Studies show that number sense is a key factor in identification of students who need intervention. While a student may understand that 8 is greater than 6, the speed of this recognition can point to a learning disability.

"The number-line task is a robust tool [that] taps into participants' mapping of spatial and numerical quantities across a wide range of values. The task has proven useful for characterizing subjects' representations across a wide range of ages."
John E. Opfer

Diagnosing Mathematical Learning Disability

Children who fail to show progress following early testing, intervention, and progress monitoring, can be considered for further diagnosis for Mathematical Learning Disability (MLD) Most broadly considered, MLD is a behaviorally defined disorder, which can only be diagnosed using observation of performance on common tasks, and basic measurable abilities on mathematics-related problems. In recent studies performed by David Geary, three different distinct types of MLD have been defined (Geary, 2005). The first type is when a child is unable to retrieve facts from their long-term memory storage. This results in a greater number of counting errors and immature or inappropriate usage of strategies to solve problems. The second type of MLD is described as a difficulty to generate correct answers because of an inability to block out irrelevant associations. An example of this would be when a child answers 4 + 8 as 5 or 9 because those answers are close to 4 and 8, the addends. This is irrelevant information that is interfering with the retrieval process to arrive at the appropriate answer. The third type of MLD is associated with impairments of visuospatial representations. This prevents students from accurately representing concepts, numbers, and geometry in their minds prior to arriving at the right answer.

> **"Approximately 6 to 10% of individuals in the general population have a persistent mathematical learning disability (MLD) or dyscalculia. Children with a learning disability may need additional time or assistance, or will need modified instruction or activities, to help them acquire the skills and concepts that other children attain with relative ease."**
>
> **Professor Michele M. M. Mazzocco**

Common Math Errors made by Children with MLD. The biggest trends that have been found among children with MLD is in the errors they make, which include: poor counting knowledge, poor place value knowledge, poor estimation abilities, inappropriate strategy use for solving basic math problems, basic number processing difficulties, retrieval error (saying 6 + 3 = 7 or 4 since those values are close to one of the addends), and taking longer to complete simple tasks (Mazzocco, 2007; Geary, Hoard, Nugent, & Byrd-Craven, 2007).

At a more serious level, there is another disorder called dyscalculia (sometimes referred to as acalculia). It is a type of MLD that is so severe it may affect very basic everyday mathematical tasks, such as determining which of two numbers is larger. Developmental dyscalculia is found in approximately 3.6% of children (Butterworth and Reigosa, 2007). Dyscalculia is also specifically selective only for math. Children with this disorder may have relatively normal performance in other subjects in school (Butterworth and Reigosa, 2007).

Assessments by Grade Level

Effective test measures have been studied at each grade level, from kindergarten to 12th grade. A summary of those findings are presented below.

Kindergarten

Counting

The Task. In standard counting tasks children are asked to enumerate a number of items.

Developmentally. It has been shown that by kindergarten children understand the elementary features of counting (Geary & Hoard, 2005). Gelman and Gallistel led the way in determining the "basic features" of counting in the late 1970s. The first feature is called "one-one correspondence" this means that kindergarteners understand that the word "one" corresponds to only one number "1" and the word "two" corresponds to only one value "2." The second basic feature is called stable order and this is the understanding that numbers follow a correct and sequential order. Cardinality is the third basic feature and it means that when counting objects the last number articulated indicates the number of items that are present. The fourth basic feature is abstraction this means counting can be applied to any set of objects. The last basic feature of counting is called order irrelevance, this means that when counting objects one can choose to count them in any order he or she wishes and still arrive at the same number of items. It is at this age that children start using their knowledge of counting to make “quantity estimates” (Griffin, 2007).

Comparison

The Task. In a standard comparison task, children are asked to indicate which of two sets contains more (or fewer) items.

Developmentally. In a study performed by Jordan (2007) she studied the predictive factors in kindergarten that determined the mathematical abilities of first graders. She found that magnitude comparison was one of the best predictive factors of first grade mathematics achievement (Jordan, 2007). This shows that the ability of a kindergartner to compare quantities of items is a good indicator of future math performance once they get to first grade.

Grades 1&2

Computation

The Task. In a standard computation task students are asked to add and subtract values.

Developmentally. By first grade children understand the concept of counting and they are beginning to add and subtract basic numbers and items. In a study performed by Fuchs, et. al. (2005), testing computational skills was a good indicator of math achievement in first graders.

Comparison

The Task. In a standard comparison task, children are asked to indicate which of two numbers is larger (or smaller).

Developmentally. At this developmental age comparing numbers should be simple to perform with three digit numbers. It has been shown that a basic understanding of numerical magnitude early on in educational development is crucial to understanding higher level processing (Holloway and Ansari, 2009).

Estimation

The Task. In a standard estimation task students are given a numeral and asked to place the number where it goes on a number line with only the end points labeled.

Developmentally. Research performed by Siegler and Booth tells us that number line estimation steadily improves during elementary school (2005 and 2004). Percent error on number line estimation tasks has been shown to decrease from kindergarten to second grade, from 24% to 10% respectively (Siegler & Booth, 2005).

Grades 3-5

Computation

The Task. In a standard computation task for this age group students are asked to add, subtract, multiply and divide.

Developmentally. It has been shown that 9-10 year-olds have developed a conceptual understanding of computation that allows them to perform three-digit computation problems (Griffin, 2007). Since this is where 9-10 year olds should be in their math development, testing them on this level of computation is therefore an appropriate task to use in this program.

Study after study has confirmed that rapid recall of basic facts, for students at all levels, is a significant predictor of math abilities.

Comparison

The Task. In a standard comparison task for this age group students are asked to indicate which of two values is more (or less).

Developmentally. At around this age student's development centers around a bidirectional conceptual understanding (Griffin, 2007). This means that these students can understand quantities along two dimensions, i.e. dollars and cents or hours and minutes. This is valuable in the study of comparison because this means students can understand how numbers and values relate to each other. Student's number knowledge at this age is crucial to the later development of good math skills that they can then apply to more difficult problems.

Estimation

The Task. In a standard computation task for students of this age they are typically asked to multiply and divide greater whole numbers, fractions, decimals, integers, or all of the above.

Developmentally. On larger number lines, from 0 to 1,000, Siegler and Opfer (2003) report a percent absolute error improvement from 21% in second grade to 14% in fourth grade. This improvement shows us where students should be in terms of their math achievement at specific developmental stages. Siegler and Booth (2005) report that by third grade children should be able to estimate midpoints on number lines and use these midpoints as landmarks for estimating numerical magnitudes. By third grade they also report children should understand the idea of proportionality which manifests in the increased use of rulers for measures of magnitude and spatial understanding. This ability helps the performance on this estimation task and shows us where third graders are supposed to be in their mathematical concepts development.

Grades 6-8

Computation

The Task. In a standard computation task students will be asked to multiply and divide more complicated numbers.

Developmentally. At this age it is expected that students can perform more complicated mathematical problems. Also, strategy use is an important part of this developmental stage (National Council of Teachers of Mathematics, 2011). Students in middle school should be learning how to use different methods to arrive at the same answer to a particular problem. Strategy choice may lead the student to choose to use or not use a calculator. Increased fluency in computations helps ease the strategy choice.

Comparison

The Task. In a standard comparison task, students are asked to indicate which of two numbers is more (or less) in numerical value.

Developmentally. In a study performed by Markovits and Sowder (1994) they found that an intervention using comparison tasks on seventh grade students improved their number sense and mathematical achievement. Using an initial test that focused on basic abilities (such as comparing two numbers) they were able to diagnose the strategies used by children in seventh grade and use those strategies to build on their intervention tactics and shape how they approached their tutoring sessions with the children.

Estimation

The Task. In a standard estimation task students are given a value and asked to place in on a number line with only the end points labeled.

Developmentally. Throughout the elementary school years it has been shown that number line estimation improves steadily (Siegler & Booth, 2005). On the 1-1000 number line Siegler and Opfer (2003) found that percent absolute error improved from 21% in second grade to 14% in fourth grade, 7% in sixth grade and 1% in adults (Siegler and Opfer, 2003).

Grades 9-12

Computation

The Task. In a standard computation task for students of this age they are typically asked to multiply and divide greater whole numbers, fractions, decimals, integers, or all of the above.

Developmentally. The ability to perform simple calculation problems at this developmental stage in the math learning process is a clear indicator of math abilities. This is one of the measurements that is so basic to the foundation of math that without this skill, math classes would be difficult to pass.

Comparison

The Task. In a standard comparison task for students of this age group they will be asked to compare values and identify which is more (or less). But these values will be more difficult in that they will not just simply be whole numbers; some numbers may be fractions, some decimals and some expressions.

Developmentally. Number sense is a crucial developmental skill that matures within the first year of life and is the "core aspect of adult numerical cognition" (Wilson and Dehaene, 2007). This shows that under a normal developmental trajectory adults should be able to compare numerosities.

Estimation

The Task. In a standard estimation task students will be given a number line and asked to place values on that number line.

Developmentally. Students of this age should be able to answer and understand numbers and problem in different ways; one of these ways is estimation (National Council of Teachers of Mathematics, 2011). At this age students should understand and have a good mental representation of how numbers relate to one another on a number line.

Additional Research-based Assessments

In general, strategy use is a common measurement of mathematical achievement. The strategy chosen by the student reveals how he or she conceptualizes the problem. Some researchers, as noted by Geary et. al. (2007), have studied circumstances when younger children use their fingers to count as a strategy in solving a math problem. The persistence of this strategy throughout elementary school reveals the state of mathematical conceptual understanding. As students mature, they should grow out of using their fingers to count and add and subtract. Therefore, observing strategies used by students gives us insight into their mental representations of the math. Adults use multiple procedures when answering basic number combinations (LeFevre, et. al., 2003). LeFevre et. al. suggest that individual variability in performance on arithmetic problems may be better explained by procedure/strategy selection instead of simply performance of the individual problems.

There have been many other approaches to understanding how to measure mathematical abilities in students. Researchers have studied the effects of subitizing, or the ability to "quickly enumerate small sets without counting" (Penner-Wilger et. al., 2009). This is exemplified in how adults don't need to count three items to know that there are three present. The ability to subitize

"...use of standardized probing questions may be the most informative way to gauge who is struggling with concepts that others seem to grasp quite readily."

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has been shown to be related to mathematical understanding in some students.

Another type of estimation task has been studied with older students and adults (Dr. Dowker, 2005). This task tests the ability to estimate answers to arithmetic problems. It has been shown that this type of task can help reveal math difficulties in terms of one's ability to choose an appropriate strategy and the effectiveness and accuracy in using that particular strategy. There is another variation of this test that examines one's ability to judge another person's estimate to problems. This variation causes the participant to reflect on the cognitive procedure of the problem, rather than trying to figure it out on his or her own. These different methods of tackling the same problem shows the multitude approaches to measuring math achievement in students.

Conclusion

Reliable predictors of math performance can help educators identify students who are struggling or at risk of math failure. While mathematics cognition research lags several decades behind reading, much is now being done to study this important area. Research in the last decade has revealed three main areas that can be used to diagnose and discover learning deficiencies in math: estimation, computation and comparison. Tests based on these tasks can be used by educators to quickly pinpoint students who are at risk and intervene with appropriate instructional strategies to prevent or eliminate the learning gap.

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