Research Paper:
Building Calculation Fluency

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A child's mathematical development is of great importance to his or her early educational years (Smith-Chant, 2010). Exposure to arithmetic begins formally in kindergarten classrooms with the introduction of numbers and counting. By first and second grade, students are expected to apply their math knowledge to different arithmetic problems. This paper will examine the role of calculation fluency in transitioning from basic math performance to advanced arithmetic and will explore methods for teaching arithmetic facts and concepts more effectively.

What is Calculation Fluency?

Calculation fluency, sometimes called computational fluency, is defined as the ability to quickly and accurately perform arithmetic problems (Kilpatrick, 2001).

What does it mean to be fluent in calculation? Someone who is fluent in calculation has memorized basic arithmetic facts and is able to effectively use basic math skills. Basic arithmetic facts usually refer to single-digit, positive-integer addition and multiplication problems that involve only two terms, along with the corresponding subtraction and division problems (e.g., 3 + 7, 10 – 7, 3 x 7, 21 ÷ 7). Developing fluency in basic addition, subtraction, multiplication and division problems is important. Calculation fluency is an essential foundation for children's advanced arithmetic performance. Beginning with addition, children are encouraged to memorize facts from 0 + 0 through 9 + 9 (Hamann, 1985). Children also memorize basic subtraction facts by solving problems with single-digit whole numbers that yield positive-integer answers. By second grade, children are expected to solve addition and subtraction problems that involve numbers up to 100 (Blöte, 2001). Research suggests that fluency in division is attained by transforming a division problem, such as 18 ÷ 3 = ?, into a corresponding multiplication problem 3 x ? = 18 that is stored in memory (De Brauwer, 2009).

As children get older, fluency includes memorizing the multiplication table that ranges from 0 x 0 to 9 x 9 (Hamann, 1985).

Children depend on fluency most when math tasks involve more than two terms. Research supports the idea that solving math problems with three or more terms requires several processing stages (Geary, 1992). Depending on its complexity, a problem can be broken up into a few or many simpler parts to solve. For example, someone who is fluent in calculation can easily solve (5 x 5) + 6 by first retrieving what 5 x 5 is from memory and then adding 6. Someone who is not fluent in calculation would most likely break the problem down into countable parts. Solving 5 + 5 + 5 + 5 + 5 + 6 reduces both the speed and the accuracy with which one solves a problem. Experts on the topic of mathematical performance suggest that a memory-retrieval model, which depends on the retrieval of stored arithmetic facts from long-term memory, is the most effective way to solve simple arithmetic problems (Geary, 1992). Fluency in arithmetic is often identified by a child's use of memory-retrieval processes instead of counting.

Component Skills of Calculation Fluency

Calculation fluency is indicated by the following skills:

- Mental math
- Complex computation

Mental math: Mental math refers to the shift from children's dependence on algorithmic strategies to their dependence on retrieval strategies. Algorithmic strategies include finger and spoken counting to solve problems, whereas retrieval strategies are completely mental. Research suggests that children are able to perform mental math once they have learned and rehearsed answers to problems that are initially solved algorithmically (Barrouillet, 2005). Simple addition, subtraction, multiplication and division problems are used to determine whether children depend on retrieving arithmetic facts from memory in order to solve problems or whether they rely on various counting strategies. Someone who is fluent in arithmetic would be able to retrieve math facts from memory and therefore be able to solve single-digit problems quickly. Someone who is not fluent in arithmetic would either not know a substantial number of arithmetic facts and/or may not depend on these facts as a solution strategy (Geary, 1992). Repeated practice of simple arithmetic problems reinforces arithmetic facts that over time become stored in long-term memory. Therefore, the path to computational fluency does involve algorithmic strategies in its outset (Barrouillet, 2005).

Complex computation: Complex computation is defined as an addition, subtraction, multiplication, or division task with two or three digit whole numbers. Complex arithmetic problems require children to perform the same mathematical procedure repeatedly, thereby testing and strengthening their algorithmic strategies. Performing complex computational tasks depends on a child's knowledge of arithmetic facts as well a child's ability to do mental math. Complex computation is an indicator of calculation fluency because it relies on the efficient retrieval of arithmetic facts for an accurate solution (Fuchs, 2006).

Importance of Calculation Fluency

Numbers and early arithmetic skills are formally introduced to all children during preschool and kindergarten. However by first and second grades, many children begin to struggle with calculation fluency that is essential for the foundation of future and more advanced arithmetic skills (Arnold, 2002). Calculation fluency is important because it can be used to predict future math ability. Measures of calculation fluency are therefore helpful in identifying children who may be at risk for math deficiencies and allowing for early intervention (Jordan, 2009).

Fluency in calculation also enables children to successfully solve more advanced math problems. Fewer demands are made on fluent children's working memory in terms of basic skills since the arithmetic facts that they retrieve are stored in long-term memory. This allows their working memory to be efficiently used for advanced skills that require more effort (Smith-Chant, 2010). Children who are not fluent in arithmetic may be able to solve advanced arithmetic problems, but it would take them longer. Aside from the fact that this is inefficient, the slow processing speed of disfluent children usually causes the meaning of the problem to become lost. However, because fluent children are quick to solve the small components of a complex problem, the meaning of the entire problem is preserved (Baroody, 2009). Calculation fluency is important for measuring future math ability, enabling performance on advanced arithmetic, and reducing processing time in order to maintain meaningfulness in the problem.

Difficulty with Calculation Fluency

Research on mathematical performance and development suggests two sources for the difficulty that children have with calculation fluency. One source of calculation difficulty is a child's limited exposure to numbers and arithmetic facts during early math development (Arnold, 2002). Calculation difficulty also emerges because of the incongruence between how arithmetic concepts are initially taught and how they are later tested. This gap in arithmetic instruction requires children to transfer mathematical knowledge to different situations and it is often the context of the situation that proves to be difficult (Fuchs, 2008).

Building Fluency

Children are able to learn and remember an abundance of arithmetic facts that can be retrieved later on. While successful arithmetic fact retrieval establishes that basic mathematical knowledge was memorized, rote memorization is not the ultimate goal. Memorizing arithmetic facts is essential to calculation fluency, but it is not the only component of fluency. Understanding the reasons and concepts behind the memorized arithmetic facts makes them meaningful sources of information instead of remote facts. Learning math concepts and practicing with algorithmic strategies are necessary precursors to memorizing arithmetic facts (Baroody, 2003).

Students develop two types of knowledge of basic mathematical facts. The first type of mathematical knowledge is called declarative knowledge and it is a network of learned and memorized problems and their solutions. An example of an arithmetic fact that is considered declarative knowledge is 2 x 3 = 6. Procedural knowledge, the second type of mathematical knowledge, is used when a child does not have the answer to a problem stored as a fact in long-term memory. Procedural knowledge includes knowing when and how to use different strategies or algorithms to solve a problem. Both declarative and procedural knowledge are fundamental to calculation fluency (Rittle-Johnson, 2007). Declarative knowledge is necessary for arithmetic fact retrieval, and procedural knowledge is relied on when transitioning into using mental math and complex computation. Over time, problems that may have initially required procedural knowledge may become declarative knowledge when they are stored in long-term memory.

Though students may have a strong grasp of the arithmetic being taught, the shift from nonsymbolic to symbolic arithmetic may contribute to decreased performance (Fuchs, 2008). Symbolic and nonsymbolic problems are the two main types of arithmetic questions that grade school students typically encounter. A symbolic math problem is a question that uses conventional arithmetic symbols for addition, subtraction, multiplication and division equations (Sherman, 2009). A nonsymbolic or word problem uses words and or visuals to test the same arithmetic knowledge that is needed for conventional equations. Recent research suggests that the way in which mathematical knowledge is assessed contributes to students' understanding of and success with arithmetic (Fuchs, 2008).

Encouraging children to explain the concepts and processes underlying their arithmetic as they practice in the classroom is an important way to build calculation fluency (Baroody, 2003). Having grasped key mathematical concepts, children will be able to conceptualize complex computation and begin performing mental math. Memorizing arithmetic facts is best taught through repeated practice. It is important to note that practice and quizzes are not only useful for testing knowledge of arithmetic facts, but they are helpful in strengthening memorized arithmetic facts as well (Kilpatrick, 2001).

Approaches to Improving Fluency

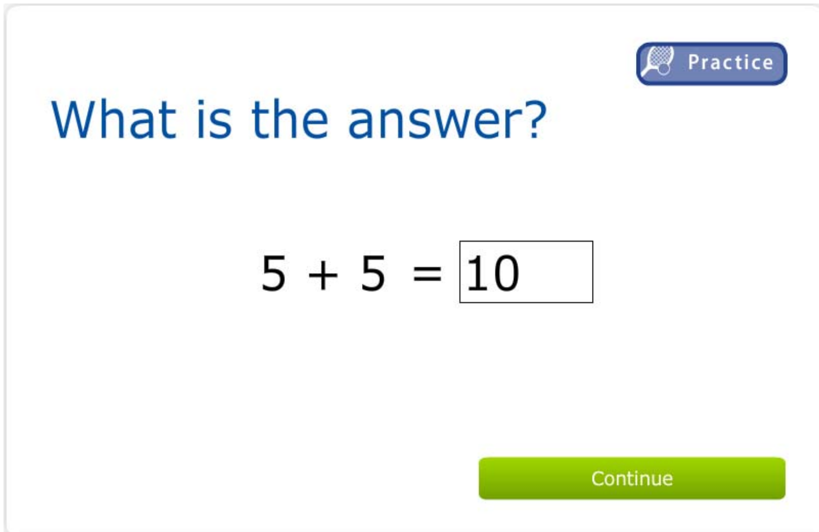
The challenge for parents and teachers alike is to create activities that will introduce key arithmetic facts to children in a way that they can understand. In order to do this, educators should be sensitive to the fact that children learn arithmetic most effectively when they are using tangible or visual objects, called manipulatives. Arnold et al. (2002) experimented with a classroom intervention to develop math skills among preschool children. The intervention was designed to incorporate math activities into the regular classroom routine to improve counting, number recognition and quantity comparison (Arnold, 2002). This experiment was successful in improving children's math development because it implemented creative and age-appropriate activities that exposed the children to math concepts and numbers. Consistent exposure to arithmetic contributes to an increased level of interest and performance. Over time, these arithmetic facts become stored in long-term memory and are quickly retrievable.

One effective method for introducing math concepts is to make them entertaining for young children. Children's mathematical development can be encouraged through stories, games and conversations involving quantities and numbers. It is most important to foster involvement with numeracy and to develop an awareness of math. Just as new vocabulary is actively introduced to children during the course of natural conversation, arithmetic facts and concepts should also be embedded in the daily activities of a child. Having a child count specific objects in a room or name the numbers on street signs are practical examples of incorporating mathematics into a child's vision of the world (Arnold, 2002).

Another effective method for introducing math concepts is to use manipulatives. Not only are manipulatives easier to use than symbols, but they also prove to be better conceptual tools (Sherman, 2009). When children first learn arithmetic, it is important that they have real situations for the context of the problem. Research suggests that children's ability to solve problems with manipulatives develops before their ability to solve symbolic problems. By using manipulatives, educators can introduce arithmetic at an earlier age and the knowledge of nonsymbolic arithmetic actually improves children's ability to perform on symbolic problems (Sherman, 2009). This approach strengthens the reliance on algorithmic strategies and simultaneously reinforces arithmetic facts to prepare for mental math. The math concepts that children develop using manipulatives can then be transferred to symbolic problems. Although the contexts of nonsymbolic and symbolic arithmetic problems are different, research supports the idea that children can transfer their understanding from one context to the other. Specifically the order of this transfer, from nonsymbolic to symbolic, is particularly effective for teaching the arithmetic facts that are part of symbolic or conventional arithmetic. Introducing arithmetic facts at a young age prepares children for mental math that they will use as a part of calculation fluency.

How to Assess Calculation Fluency

Calculation fluency can be assessed through timed mathematical tests on addition, subtraction, multiplication and division. Both accuracy and speed contribute to calculation fluency and should therefore be tested simultaneously. The time that a child spends on a math problem indicates whether he or she is using arithmetic fact retrieval strategy or an algorithmic strategy. Although the time it takes to complete a problem is dependent on its complexity, if a problem is not solved with relative speed it can be assumed that the child is using an algorithmic strategy such as counting instead of mental math.



High scores of speed and accuracy on timed tests on addition, subtraction, multiplication and division indicate that the child has successfully learned, stored and retrieved the arithmetic facts that are responsible for calculation fluency (Hamann, 1985).

Conclusion

Calculation fluency is the ability to quickly and accurately solve mathematical problems (Kilpatrick, 2001). Mental math and complex computation are two important skills for calculation fluency. Calculation fluency is critical for advanced arithmetic ability (Smith-Chant, 2010). Algorithmic strategies and basic arithmetic facts should be taught and rehearsed so that arithmetic facts can be memorized in meaningful ways. Calculation fluency can be assessed through mathematical tests that measure both the speed and accuracy with which a child solves a problem.

- Someone who is **fluent in calculation** is able to retrieve arithmetic facts from memory and is therefore able to solve basic math problems quickly.
- **Teaching arithmetic facts** is best done through repeated practice. Over time, these arithmetic facts become stored in a child's long-term memory and are quickly retrievable.
- **Understanding the concepts** and reasons behind the memorized arithmetic facts makes them meaningful sources of information instead of remote facts.

- **Implications for Educators:**
  - Consistent exposure to arithmetic contributes to an increased level of interest and performance.
  - Encouraging children to explain the concepts and processes underlying their arithmetic as they practice in the classroom is an important way to build calculation fluency.
  - Having grasped key mathematical concepts, children will be able to conceptualize complex computation and perform mental math.

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