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**Answer Key** ........................................ 149
Getting the Idea

An expression is a combination of numbers and operation signs such as +, −, × and ÷. To write a numerical expression from words look for the relationship between the words and the numbers in the situation.

<table>
<thead>
<tr>
<th>Operation</th>
<th>Problem</th>
<th>Numerical Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>addition</td>
<td>3 more than 10</td>
<td>$3 + 10$</td>
</tr>
<tr>
<td></td>
<td>the sum of 10 and 3</td>
<td>$3 + 10$</td>
</tr>
<tr>
<td></td>
<td>3 increased by 10</td>
<td>$3 + 10$</td>
</tr>
<tr>
<td></td>
<td>the total of 10 and 3</td>
<td>$3 + 10$</td>
</tr>
<tr>
<td></td>
<td>3 combined with 10</td>
<td>$3 + 10$</td>
</tr>
<tr>
<td>subtraction</td>
<td>20 minus 8</td>
<td>$20 - 8$</td>
</tr>
<tr>
<td></td>
<td>8 fewer than 20</td>
<td>$20 - 8$</td>
</tr>
<tr>
<td></td>
<td>8 subtracted from 20</td>
<td>$20 - 8$</td>
</tr>
<tr>
<td></td>
<td>20 decreased by 8</td>
<td>$20 - 8$</td>
</tr>
<tr>
<td></td>
<td>the difference of 8 from 20</td>
<td>$20 - 8$</td>
</tr>
<tr>
<td>multiplication</td>
<td>5 times 12</td>
<td>$5 \times 12$</td>
</tr>
<tr>
<td></td>
<td>5 multiplied by 12</td>
<td>$5 \times 12$</td>
</tr>
<tr>
<td></td>
<td>the product of 5 and 12</td>
<td>$5 \times 12$</td>
</tr>
<tr>
<td>division</td>
<td>25 partitioned into 5 equal groups</td>
<td>$25 \div 5$</td>
</tr>
<tr>
<td></td>
<td>25 split into 5 equal groups</td>
<td>$25 \div 5$</td>
</tr>
<tr>
<td></td>
<td>25 shared by 5 equally</td>
<td>$25 \div 5$</td>
</tr>
</tbody>
</table>
Example 1
Write a numerical expression to represent the problem below.

A box of 36 character cards is shared equally among 3 friends.

Strategy Use a strip diagram to represent the problem.

Step 1 Represent the problem using a strip diagram.

<table>
<thead>
<tr>
<th>36</th>
<th></th>
<th></th>
</tr>
</thead>
</table>

Step 2 Write the expression.

36 ÷ 3

Solution A numerical expression that represents the problem is 36 ÷ 3.

Sometimes you may need to write a numerical expression using more than one operation. For example, “the sum of 45 and 35, multiplied by 3” can be written as (45 + 35) × 3.

Example 2
Write a numerical expression to represent the problem below.

Subtract 3 from 12, then multiply by 4.

Strategy Separate the problem into two parts. Write a numerical expression for each part.

Step 1 The comma separates the problem into two parts. Write a numerical expression for the first part “subtract 3 from 12.”

(12 − 3)

Step 2 Write a numerical expression for the second part “then multiply by 4.”

× 4

Step 3 Combine the parts.

(12 − 3) × 4

Solution The expression is (12 − 3) × 4.
Example 3
How does the expression $4 \times (27 - 12)$ compare to $27 - 12$?

Strategy Look how the expressions are the same and different.

Step 1 Look to see how the expressions are the same.
Both expressions have $27 - 12$.

Step 2 Look to see how the expressions are different.
The expression $4 \times (27 - 12)$ shows the common part multiplied by 4.
So, $4 \times (27 - 12)$ is 4 times as many as $27 - 12$.

Solution The expression $4 \times (27 - 12)$ is 4 times as many as $27 - 12$.

Coached Example
Write a numerical expression to represent the problem below.

Divide 30 by 5, then add 12.

Write a numerical expression for “divide 30 by 5.”

(_______ ___ _________)

Write a numerical expression for “then add 12.”

____ _________

Combine the parts.

(_______ ___ _________) ___ _________

The expression is (_______ ___ _________) ___ _________.
Order of Operations

Getting the Idea

When evaluating an expression with more than one operation, use the **order of operations**. The order of operations is a set of rules used for evaluating an expression with more than one operation.

<table>
<thead>
<tr>
<th>Order of Operations</th>
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<tbody>
<tr>
<td>1. Operate inside the grouping symbols.</td>
</tr>
<tr>
<td>2. Multiply and divide from left to right.</td>
</tr>
<tr>
<td>3. Add and subtract from left to right.</td>
</tr>
</tbody>
</table>

Example 1

Evaluate this expression: $14 - 6 \div 3$

**Strategy**  
Use the order of operations.

**Step 1**  
There are no grouping symbols, so multiply and divide from left to right.  
$14 - 6 \div 3$  
$14 - 2$  

**Step 2**  
Add and subtract from left to right.  
$14 - 2$  
$12$

**Solution**  
$14 - 6 \div 3 = 12$
Example 2
Evaluate this expression: $2 + 3 \times 8 \div 2$

Strategy **Use the order of operations.**

**Step 1** There are no grouping symbols, so multiply and divide from left to right.
- Multiply.
  - $2 + 3 \times 8 \div 2$
  - $2 + 24 \div 2$
- Divide.
  - $2 + 24 \div 2$
  - $2 + 12$

**Step 2** Add and subtract from left to right.
- Add.
  - $2 + 12$
  - $14$

Solution $2 + 3 \times 8 \div 2 = 14$

Example 3
Evaluate this expression: $64 \div 8 + 15 \times 3 - 16$

Strategy **Use the order of operations.**

**Step 1** There are no grouping symbols, so multiply and divide from left to right.
- Divide.
  - $64 \div 8 + 15 \times 3 - 16$
  - $8 + 15 \times 3 - 16$
- Multiply.
  - $8 + 15 \times 3 - 16$
  - $8 + 45 - 16$
Step 2  Add and subtract from left to right.

Add.
\[ 8 + 45 - 16 \]
\[ 53 - 16 \]

Subtract.
\[ 53 - 16 \]
\[ 37 \]

Solution  \[ 64 \div 8 + 15 \times 3 - 16 = 37 \]

Coached Example

What is the value of the expression shown below?

\[ 100 - 60 \div 5 \times 8 + 17 \]

Use the order of operations.

Divide, then multiply.
\[ 100 - \underline{12} \times 8 + 17 \]
\[ 100 - \underline{96} + 17 \]

Subtract, then add.
\[ \underline{4} + 17 \]

\[ 100 - 60 \div 5 \times 8 + 17 = \underline{94} \]
Getting the Idea

You can use the order of operations to evaluate an expression with grouping symbols. First operate in parentheses ( ), then brackets [ ], then braces { }. For example:

\[
8 \times \{3 + [5 - (3 - 2)]\} = 8 \times \{3 + [5 - 1]\} = 8 \times \{3 + 4\} = 8 \times 7 = 56
\]

Example 1

Evaluate this expression: \([(2 + 3) \times 8] \div 2\)

**Strategy** Use the order of operations. Work inside the grouping symbols first.

**Step 1** Operate within the parentheses.

\[
[(2 + 3) \times 8] \div 2 = [5 \times 8] \div 2
\]

**Step 2** Operate within the brackets.

\[
[5 \times 8] \div 2 = 40 \div 2 = 20
\]

**Step 3** Multiply and divide from left to right.

\[
40 \div 2 = 20
\]

**Solution** \([(2 + 3) \times 8] \div 2 = 20\)
Example 2
Evaluate this expression: \(87 \div 3 - [15 - (4 \times 3)] + 2\)

Strategy Use the order of operations. Work inside the grouping symbols first.

Step 1 Operate inside the parentheses.
\[
87 \div 3 - [15 - (4 \times 3)] + 2
\]
\[
87 \div 3 - [15 - 12] + 2
\]

Step 2 Operate inside the brackets.
\[
87 \div 3 - [15 - 12] + 2
\]
\[
87 \div 3 - 3 + 2
\]

Step 3 Multiply and divide from left to right.
\[
87 \div 3 - 3 + 2 = 29 - 3 + 2
\]

Step 4 Add and subtract from left to right.
\[
29 - 3 + 2 = 26 + 2 = 28
\]

Solution \(87 \div 3 - [15 - (4 \times 3)] + 2 = 28\)

Example 3 below shows the evaluation of the expression from Example 2 without grouping symbols. Notice that the answer is not the same as in Example 2.

Example 3
Evaluate this expression: \(87 \div 3 - 15 - 4 \times 3 + 2\)

Strategy Use the order of operations.

Step 1 Multiply and divide from left to right.
\[
87 \div 3 - 15 - 4 \times 3 + 2
\]
\[
29 - 15 - 4 \times 3 + 2 = 29 - 15 - 12 + 2
\]

Step 2 Add and subtract from left to right.
\[
29 - 15 - 12 + 2
\]
\[
14 - 12 + 2 = 2 + 2 = 4
\]

Solution \(87 \div 3 - 15 - 4 \times 3 + 2 = 4\)
Example 4
Evaluate this expression: \( 9 \times [(4 + 2) \div 3] \)

Strategy  Use the order of operations.

Step 1  Operate inside the parentheses.
\[
9 \times [(4 + 2) \div 3] \\
9 \times [6 \div 3]
\]

Step 2  Operate inside the brackets.
\[
9 \times [6 \div 3] = 9 \times 2
\]

Step 3  Multiply.
\[
9 \times 2 = 18
\]

Solution  \( 9 \times [(4 + 2) \div 3] = 18 \)

Coached Example

Evaluate this expression: \( [(4 + 3) \times 2 - 8] \div 3 \)

Use the order of operations.

Operate inside grouping symbols.
\[
[(4 + 3) \times 2 - 8] \div 3 \\
[_______ \times 2 - 8] \div 3 \\
[_______ - 8] \div 3 \\
_______ \div 3
\]

Divide.
\[
_______ \div 3 \\
_______
\]

\( [(4 + 3) \times 2 - 8] \div 3 = _________ \)
Patterns

Getting the Idea

A pattern is a series of numbers or figures that follows a rule. The rule of the pattern tells you how to get from each number in the pattern to the next number. The rule can also help you find a missing number in a pattern.

The pattern below is an increasing pattern. The rule is add 3.

\[ 5, 8, 11, 14, 17, \ldots \]

The pattern below is a decreasing pattern. The rule is subtract 5.

\[ 100, 95, 90, 85, \ldots \]

You can create a new pattern using a rule and a starting number. Each number in a pattern is called a term.

Example 1

Write a new pattern that starts with 3, has 6 terms, and uses the rule add 4.

Strategy Use the rule to write a new pattern.

Step 1 Start with the first term.

The first term is 3.

Step 2 Use the rule to extend the pattern.

Add 4 to the first term to find the second term.

\[ 3 + 4 = 7 \]

The second term is 7.

Step 3 Continue to extend the pattern until there are 6 terms.

Add 4 to each sum.

\[ 7 + 4 = 11 \]
\[ 11 + 4 = 15 \]
\[ 15 + 4 = 19 \]
\[ 19 + 4 = 23 \]
Step 4  Write the terms in the pattern.

3  7  11  15  19  23

Solution  The new pattern is 3, 7, 11, 15, 19, 23.

Example 2

Write a new pattern starting with 3 that has 6 terms and uses the rule multiply by 4.

Strategy  Use the rule to write a new pattern.

Step 1  Start with the first term.

The first term is 3.

Step 2  Use the rule to extend the pattern.

Multiply the first term by 4 to find the second term.

\[3 \times 4 = 12\]

The second term is 12.

Step 3  Continue to extend the pattern until there are 6 terms.

Multiply each product by 4.

\[12 \times 4 = 48\]
\[48 \times 4 = 192\]
\[192 \times 4 = 768\]
\[768 \times 4 = 3,072\]

Step 4  Write the terms in the pattern.

3  12  48  192  768  3,072

Solution  The new pattern is 3; 12; 48; 192; 768; 3,072.
Look back at Examples 1 and 2. Even though they start with the same term, the multiplication pattern increases much faster than the addition pattern.

You can create two patterns to form a relationship between the corresponding terms.

**Example 3**
Create two patterns with 5 terms that both start with 0. Pattern A uses the rule add 2. Pattern B uses the rule add 8. Make a table to show the corresponding terms. How are the terms in the two patterns related?

**Strategy**  
Use the rules to write two new patterns.

**Step 1**  
Write the terms for Pattern A.  
Use the rule add 2. Start with 0.  
0, 2, 4, 6, 8

**Step 2**  
Write the terms for Pattern B.  
Use the rule add 8. Start with 0.  
0, 8, 16, 24, 32

**Step 3**  
Make a table to show the corresponding terms.

<table>
<thead>
<tr>
<th></th>
<th>Pattern A</th>
<th>Pattern B</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td>8</td>
</tr>
<tr>
<td>4</td>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>6</td>
<td>6</td>
<td>24</td>
</tr>
<tr>
<td>8</td>
<td>8</td>
<td>32</td>
</tr>
</tbody>
</table>

**Step 4**  
Compare the terms.  
8 ÷ 2 = 4  
16 ÷ 4 = 4  
24 ÷ 6 = 4  
32 ÷ 8 = 4  
Each term in Pattern B is 4 times the value of the corresponding terms in Pattern A.

**Solution**  
The terms in Pattern B are always 4 times the value of the corresponding terms in Pattern A.
Create two patterns each with 5 terms that both start with 0. Pattern A uses the rule add 4 and Pattern B uses the rule add 12. What is the relationship between the patterns?

Create the pattern with the rule add 4.

0, 0 + 4 = ________, ________, ________, ________.
The first 5 terms of Pattern A are 0, ________, ________, ________, ________.

Create the pattern with the rule add 12.

0, 0 + 12 = ________, ________, ________, ________.
The first 5 terms of Pattern B are 0, ________, ________, ________, ________.

Complete the table.

<table>
<thead>
<tr>
<th>Pattern A</th>
<th>0</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pattern B</td>
<td>0</td>
</tr>
</tbody>
</table>

Find the relationship between the patterns.

Divide each term in Pattern B by the corresponding term in Pattern A.

_______ ÷ _________ = _________
_______ ÷ _________ = _________
_______ ÷ _________ = _________
_______ ÷ _________ = _________

The relationship is that each term in Pattern B is ________ the value as each corresponding term in Pattern A.
Domain 1 • Lesson 5

Graph Patterns

Getting the Idea

You can show the relationship between two values in a graph. You can graph the values as ordered pairs on a coordinate plane.

(Note: Look ahead to Lessons 30 and 31 for more information on coordinate planes.)

An ordered pair \((x, y)\) is a pair of numbers used to locate a point on a coordinate plane. The first number in an ordered pair is called the \(x\)-coordinate. The second number in an ordered pair is called the \(y\)-coordinate. For example, in \((3, 5)\), the \(x\)-coordinate is 3 and the \(y\)-coordinate is 5.

Example 1

Nina ran 6 miles each hour she ran in a long-distance race. She ran for 4 hours. Make a graph that shows the pattern.

Strategy  Translate the pattern into a graph.

Step 1  Write the relationship between hours and miles run.

She ran 6 miles each hour.

1 hour = 6 miles
2 hours = 12 miles
3 hours = 18 miles
4 hours = 24 miles
Step 2  Make a table of values. List the ordered pairs.

<table>
<thead>
<tr>
<th>Number of Hours (x)</th>
<th>Number of Miles (y)</th>
<th>Ordered pairs (x, y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>6</td>
<td>(1, 6)</td>
</tr>
<tr>
<td>2</td>
<td>12</td>
<td>(2, 12)</td>
</tr>
<tr>
<td>3</td>
<td>18</td>
<td>(3, 18)</td>
</tr>
<tr>
<td>4</td>
<td>24</td>
<td>(4, 24)</td>
</tr>
</tbody>
</table>

Step 3  Graph the ordered pairs on a coordinate plane.

Draw a straight line that connects the points.

Solution  The graph in Step 3 shows the pattern.

You can make ordered pairs of corresponding terms from two patterns and then graph the ordered pairs.

Example 2
Create two patterns with 5 terms that both start with 0. Use two different rules: add 3 and add 6. Form ordered pairs of corresponding terms from the two patterns and graph them. How do the terms in the two patterns seem to be related?

Strategy  Use the rules to write two new patterns. Write ordered pairs of corresponding terms, then graph.

Step 1  Write the pattern for the x terms.

Use the rule add 3. Start with 0.

0, 3, 6, 9, 12
Step 2 Write the pattern for the $y$ terms.
Use the rule add 6. Start with 0.
0, 6, 12, 18, 24

Step 3 Write $(x, y)$ ordered pairs using the corresponding terms from each pattern.
(0, 0)
(3, 6)
(6, 12)
(9, 18)
(12, 24)

Step 4 Graph the ordered pairs on a coordinate plane.

Step 5 Compare the terms to see how they are related.
$y = x \times 2$
$6 = 3 \times 2$
$12 = 6 \times 2$
$18 = 9 \times 2$
$24 = 12 \times 2$

The value of the $y$-coordinate is always 2 times the value of the $x$-coordinate.

Solution The graph is shown in Step 4. The value of the $y$-coordinate is 2 times the value of the $x$-coordinate.
Create two patterns with 5 terms that both start at 0. Use two different rules: add 1 and add 4. Form ordered pairs of corresponding terms from the two patterns and graph them. How do the terms in the two patterns seem to be related?

Write the first 5 terms for the \( x \)-coordinate.

0, _____, ____, _____, _____

Write the first 5 terms for the \( y \)-coordinate.

0, _____, ____, _____, _____

Write ordered pairs of corresponding terms.

(0, 0)
(1, ________)
(2, ________)
(______, ________)
(______, ________)

Graph the ordered pairs on the coordinate plane below.

Compare the terms to see how they are related.

\( y = x \times 4 \)

4 = 1 \times 4

8 = 2 \times ________

12 = 3 \times ________

16 = ________ \times ________

The value of the \( y \)-coordinate is ________ times the value of the \( x \)-coordinate.
Multiply Whole Numbers

Getting the Idea

In a multiplication problem, the numbers you multiply are called factors, and the result is called the product. When multiplying two- or three-digit numbers, multiply by the ones and then the tens to find the partial products. Then add the partial products to find the product.

Example 1
Find the product.

\[ 523 \times 18 = \]

**Strategy** Multiply by the ones and then the tens. Add the partial products.

**Step 1** Rewrite the problem vertically. Multiply 523 by the 8 ones in 18.

\[
\begin{array}{c}
12 \\
523 \\
\times 18 \\
\hline
4184 \\
\end{array}
\]

\[ \leftarrow 8 \times 523 \]

**Step 2** Multiply 523 by the 1 ten in 18.

Write a 0 in the ones place before multiplying.

\[
\begin{array}{c}
523 \\
\times 18 \\
\hline
4184 \\
\end{array}
\]

\[ 5230 \quad \leftarrow 10 \times 523 \]

**Step 3** Add the partial products.

\[
\begin{array}{c}
523 \\
\times 18 \\
\hline
4184 \\
+ 5230 \\
\hline
9414 \\
\end{array}
\]

**Solution** \[ 523 \times 18 = 9414 \]
You can write an **equation** to solve a real-world problem. Use a **variable** to represent the unknown value.

**Example 2**

Mrs. Robinson is the principal of a school with 465 students. The librarian told Mrs. Robinson that there are 16 times as many books in the library as there are students in the school. How many books are in the library?

**Strategy**  **Write an equation for the problem, then solve.**

**Step 1**  Write an equation for the problem.

Let $b$ represent the total number of books in the library.

$465 \times 16 = b$

**Step 2**  Rewrite the problem. Multiply 465 by the ones digit in 16.

$$
\begin{array}{c}
33 \\
465 \\
\times 16 \\
\hline
2790 \\
\end{array} 
\quad \text{← 6 \times 465}
$$

**Step 3**  Multiply 465 by the tens digit in 16.

Use a 0 as a placeholder in the partial product.

$$
\begin{array}{c}
465 \\
\times 16 \\
\hline
2790 \\
4650 \\
\end{array} 
\quad \text{← 10 \times 465}
$$

**Step 4**  Add the partial products.

$$
\begin{array}{c}
465 \\
\times 16 \\
\hline
2790 \\
+ 4650 \\
\hline
7440
\end{array}
$$

**Solution**  There are 7,440 books in the library.
Example 3
The new A5 computer sells for $1,499. Yesterday, Electronic World sold 23 of the A5 computers. How much money did Electronic World make from the sale of the A5 computers yesterday?

Strategy  Write an equation for the problem, then solve.

Step 1  Write an equation for the problem.
Let \( m \) represent the total amount of money earned.
\[
$1,499 \times 23 = m$
\]

Step 2  Rewrite the problem vertically and multiply.

\[
\begin{array}{c}
11 \\
122 \\
1,499 \\
\times 23 \\
\hline
4497 \\
29980 \\
\hline
34,477 \\
\end{array}
\]

Solution  Electronic World made $34,477 from the sale of the A5 computers yesterday.

You can use the distributive property to multiply numbers. To use the distributive property, rewrite one of the factors as a sum of two or more numbers. Then multiply each of the addends by the other factor and add the products.

For example, this array model shows how to multiply 12 \( \times \) 28.

\[
12 \times 28 = 336
\]
\[
12 \times (20 + 8)
\]
\[
(12 \times 20) + (12 \times 8)
\]
\[
240 + 96 = 336
\]
Example 4
Use the distributive property to find $65 \times 128$.

**Strategy**  Use the distributive property.

**Step 1** Write the second factor as a sum of each place value.

\[ 128 = 100 + 20 + 8 \]

**Step 2** Multiply each addend by 65.

\[ 65 \times 128 \]

\[ 65 \times (100 + 20 + 8) = (65 \times 100) + (65 \times 20) + (65 \times 8) \]

\[ = 6,500 + 1,300 + 520 \]

**Step 3** Add the products.

\[ 6,500 + 1,300 + 520 = 8,320 \]

**Solution**  $65 \times 128 = 8,320$

Example 5
A rug buyer bought 15 rugs that each cost $462. How much did the rugs cost in all?

**Strategy**  Write an equation for the problem. Use the distributive property.

**Step 1** Write an equation for the problem.

Let $c$ represent the total cost of the rugs.

\[ 15 \times $462 = c \]

**Step 2** Write the second factor as the sum of each place value.

\[ 462 = 400 + 60 + 2 \]

**Step 3** Multiply each addend by 15.

\[ 15 \times 462 \]

\[ 15 \times (400 + 60 + 2) = (15 \times 400) + (15 \times 60) + (15 \times 2) \]

\[ = 6,000 + 900 + 30 \]

**Step 4** Add the products.

\[ 6,000 + 900 + 30 = 6,930 \]

**Solution**  The rugs cost $6,930 in all.
A theater sold 329 tickets to an afternoon performance for $26 each. How much money did the theater take in for this performance?

Write an equation for the problem.

Let $m$ represent the total amount of money.

$\text{_________ } \times \text{_________ } = m$

Rewrite the problem.

Multiply 329 by the ones digit in 26.

What is the partial product? _________________

Use a ________ as a placeholder in the ones place of the second partial product.

Multiply 329 by the tens digit in 26.

What is the partial product? _________________

Add the _______________ _______________ to find the product.

What is the product? _________________

The theater took in _______________ for this performance.
Divide Whole Numbers

Getting the Idea

In a division problem, the number that is being divided is the **dividend**. The number that divides the dividend is the **divisor**. The answer to a division problem is the **quotient**. If there is a number left over after the division is complete, then the quotient has a **remainder**.

Example 1

There are 851 seats in an auditorium. Each of the 23 rows in the auditorium has the same number of seats. How many seats are in each row?

**Strategy**  
Write an equation for the problem. Then divide.

**Step 1**  
Write an equation for the problem.

Let \( s \) represent the number of seats in each row.

\[
851 \div 23 = s
\]

**Step 2**  
Set up the division problem.

\[
23 \overline{)851}
\]

**Step 3**  
Decide where to place the first digit in the quotient.

The first digit of the quotient will be in the tens place.

**Step 4**  
Divide 85 tens.

\[
\begin{array}{c}
23 \overline{)851} \\
- 69 & \leftarrow 3 \times 23 = 69 \\
\hline
16 & \leftarrow 85 - 69 = 16
\end{array}
\]

**Step 5**  
Bring down the 1 one. Divide 161 ones.

\[
\begin{array}{c}
23 \overline{)851} \\
- 69 \\
\hline
161 \\
- 161 & \leftarrow 7 \times 23 = 161 \\
\hline
0 & \leftarrow 161 - 161 = 0
\end{array}
\]

**Solution**  
There are 37 seats in each row.
Since multiplication and division are inverse operations, you can check division by using multiplication. Multiply the quotient by the divisor. If the product equals the dividend, the quotient is correct.

\[
\begin{array}{c}
37 \\
\times 23 \\
\hline
111 \\
+ 740 \\
\hline
851
\end{array}
\]

The product equals the dividend, so the quotient is correct.

When solving a division word problem with a remainder, you need to interpret the remainder. You may ignore the remainder, add 1 to the quotient, or the remainder may be the answer.

Example 2
Tina has 426 stickers. She divides them equally among 15 friends. How many stickers will each friend get?

Strategy  Write an equation for the problem. Then divide.

Step 1  Write an equation for the problem.
Let \( s \) represent the number of stickers each friend will get.

\[ 426 \div 15 = s \]

Step 2  Set up the problem. The first digit of the quotient will be in the tens place.
Divide 42 tens.

\[
\begin{array}{c}
2 \\
15)426 \\
- 30 \, \Rightarrow \, 2 \times 15 = 30 \\
\hline
12 \, \Rightarrow \, 42 - 30 = 12
\end{array}
\]

Step 3  Bring down the 6 ones. Divide 126 ones.

\[
\begin{array}{c}
28 \, \text{R6} \\
15)426 \\
- 30 \downarrow \\
\hline
126 \\
- 120 \, \Rightarrow \, 8 \times 15 = 120 \\
\hline
6 \, \Rightarrow \, 126 - 120 = 6
\end{array}
\]
Step 4  Interpret the remainder.

There are 6 stickers left over. There is no way to divide 6 stickers among 15 friends, so drop the remainder.

Solution  Each friend will get 28 stickers.

You can also check a quotient with a remainder. Multiply the quotient by the divisor and add the remainder to the product.

\[28 \times 15 = 420 \quad 420 + 6 = 426\]  \(\rightarrow\) The sum equals the dividend.

Example 3
Spencer wants to put his 2,188 stamps in a binder. Each page in the binder holds 24 stamps. How many stamps will be on the last page in the binder?

Strategy  Divide each place from left to right.

Step 1  Set up the division problem.

\[
24 \overline{)2,188}
\]

Step 2  Divide each place from left to right.

\[
\begin{align*}
91 \text{ R} 4 \\
24 \overline{)2188} \\
- 216 \\
28 \\
- 24 \\
4
\end{align*}
\]

Step 3  Interpret the remainder.

The quotient is 91. That means 91 pages are full with 24 stickers on each page.

The remainder is 4. That means there are 4 stickers left over.

The question asks how many stamps will be on the last page of the binder, so the remainder is the answer.

Solution  There will be 4 stamps on the last page in the binder.
Katie has 568 oranges to put into bags. Each bag can hold 12 oranges. How many bags does Katie need for all the oranges?

Write the problem below that you can use to help answer the question. Then solve it.

The quotient is ________.
The remainder is ________.
The quotient means that ________ bags can be filled with 12 oranges.
The remainder means that there will be ________ oranges left over.

Interpret the remainder. The question asks how many bags Katie needs for all the oranges, so ________________________________________________________________.

You can check your answer by multiplying ________ times ________ and adding ________.

Katie needs ________ bags for all the oranges.
In Lesson 7, you learned that to check a division problem, you multiply the quotient by the divisor and add the remainder to the product. If the result is equal to the dividend, the quotient is correct. You can use this idea to write an equation.

\[ \text{dividend} = \text{quotient} \times \text{divisor} + \text{remainder} \]

Example 1
Divide: \(785 \div 25\). Write the result as an equation.

Strategy
Divide. Identify the dividend, quotient, divisor, and remainder to write an equation.

Step 1
Divide.

\[
\begin{array}{c}
25 \overline{)785} \\
-75 \downarrow \\
35 \\
-25 \\
10
\end{array}
\]

Step 2
Write an equation.

\[
\text{dividend} = \text{quotient} \times \text{divisor} + \text{remainder}
\]

\[
785 = 31 \times 25 + 10
\]

Solution
The equation is \(785 = 31 \times 25 + 10\).
When the result of division has a remainder, the result can be written as a mixed number. A **mixed number** is a number that has a whole-number part and a fraction part.

\[
\begin{align*}
15 \text{ R}2 & \quad \text{quotient} + \quad \frac{\text{remainder}}{\text{divisor}} = \quad \text{mixed number} \\
9 \overline{)137} & \quad 15 + \quad \frac{2}{9} = \quad 15\frac{2}{9}
\end{align*}
\]

**Example 2**

What is the result of \(1,190 \div 13\) written as a mixed number?

**Strategy**  
Divide. Then write the mixed number.

**Step 1**  
Divide.

\[
\begin{align*}
91 \text{ R}7 & \quad \text{quotient} + \quad \frac{\text{remainder}}{\text{divisor}} = \quad \text{mixed number} \\
13 \overline{)1190} & \quad 91 + \quad \frac{7}{13} = \quad 91\frac{7}{13}
\end{align*}
\]

**Solution**  
The mixed number is \(91\frac{7}{13}\).
Divide: 963 ÷ 34. Write the result as an equation and as a mixed number.

Write the problem below. Then solve it.

What is the dividend? __________
What is the quotient? _________
What is the divisor? __________
What is the remainder? _________

Write the equation as dividend = quotient × divisor + remainder.

963 = __________ × __________ + __________

Write the mixed number as quotient + \( \frac{\text{remainder}}{\text{divisor}} \) = mixed number.

___________ + __________ = __________

The equation for 963 ÷ 34 is _____ = ____________________ and the mixed number is __________.
A decimal is a number with a decimal point. A decimal point (.) separates the ones place from the tenths place.

The grids below represent one tenth, one hundredth, and one thousandth.

0.1
one tenth

0.01
one hundredth

0.001
one thousandth

To read or write a decimal number less than one, read the number to the right of the decimal point. Then read the least place value. For example, 0.7 is seven tenths, and 0.36 is thirty-six hundredths.

To read or write a decimal number greater than 1, use the word and to separate the whole-number part from the decimal part. For example, 2.003 is two and three thousandths.

There are different ways to read and write decimals.

<table>
<thead>
<tr>
<th>Hundreds</th>
<th>Tens</th>
<th>Ones</th>
<th>.</th>
<th>Tenths</th>
<th>Hundredths</th>
<th>Thousandths</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>9</td>
<td>6</td>
<td>.</td>
<td>7</td>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>

base-ten numeral: 196.748
number name: one hundred ninety-six and seven hundred forty-eight thousandths
expanded form: \(100 + 90 + 6 + 0.7 + 0.04 + 0.008\)

Each place in a decimal has a value that is 10 times the value of the place to its right. For example, in 6.666, the 6 in the hundredths place has a value of 0.06. That is 10 times the value of the 6 in the thousandths place.

\[0.006 \times 10 = 0.06\]
Each place in a decimal has a value that is $\frac{1}{10}$ the value of the place to its left. For example, in 6.666, the 6 in the thousandths place has a value of 0.006. That is $\frac{1}{10}$ the value of the 6 in the hundredths place.

**Example 1**

A lab sample has a mass of 0.222 gram. What is the value of the 2 in the thousandths place in relation to the 2 in the hundredths place?

**Strategy**  Use a place-value chart.

**Step 1** Write each digit of the number in a chart.

<table>
<thead>
<tr>
<th>Ones</th>
<th>.</th>
<th>Tenths</th>
<th>Hundredths</th>
<th>Thousandths</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>.</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
</tbody>
</table>

**Step 2** Find the value of the 2 in the thousandths place: 0.002. The digit to its left is in the hundredths place: 0.02.

$0.002 \div 0.02 = \frac{1}{10}$

The value of the 2 in the thousandths place is $\frac{1}{10}$ the value of the 2 in the hundredths place.

**Solution** The value of the 2 in the thousandths place is $\frac{1}{10}$ the value of the 2 in the hundredths place.

**Example 2**

What decimal describes the shaded part of the grids?

**Strategy** Count the number of small shaded squares in each grid.

**Step 1** There are 100 small squares in the grid on the left and all are shaded. Each small square is one hundredth, or 0.01. So the entire grid is equal to $100 \times 0.01$ or 1.
Step 2  There are 100 small squares in the grid on the right and 64 are shaded.
   Each small square is one hundredth, or 0.01.
   So the shaded squares are equal to $64 \times 0.01$ or 0.64.

Step 3  Write the decimal for each grid and combine them.
   $1 + 0.64 = 1.64$

Solution  The decimal 1.64, or one and sixty-four hundredths, describes the shaded part of the grids.

Example 3
The winning speed in a car race was 125.044 miles per hour. How do you write that speed in expanded form?

Strategy  Make a place-value chart to find the value of each digit.

Step 1  Write the decimal in a place-value chart.

<table>
<thead>
<tr>
<th>Hundreds</th>
<th>Tens</th>
<th>Ones</th>
<th>.</th>
<th>Tenths</th>
<th>Hundredths</th>
<th>Thousandths</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>5</td>
<td>.</td>
<td>0</td>
<td>4</td>
<td>4</td>
</tr>
</tbody>
</table>

Step 2  Find the value of each digit.
   1 hundred $= 1 \times 100 = 100$
   2 tens $= 2 \times 10 = 20$
   5 ones $= 5 \times 1 = 5$
   4 hundredths $= 4 \times 0.01 = 0.04$
   4 thousandths $= 4 \times 0.001 = 0.004$

Step 3  Write the expanded form of the number.
   $125.044 = 100 + 20 + 5 + 0.04 + 0.004$

Solution  In expanded form, 125.044 is written as $100 + 20 + 5 + 0.04 + 0.004$.

Another way to write expanded form is with multiplication.
For example, write 347.392 in expanded form.

$347.392 = 300 + 40 + 7 + 0.3 + 0.09 + 0.002$

Then write 347.392 in expanded form with multiplication.
Multiply each digit in the number by the value its place represents. You can use fractions or decimals to write a number in expanded form. The fraction $\frac{1}{10}$ is equivalent to the decimal 0.1.

$$347.392 = 3 \times 100 + 4 \times 10 + 7 \times 1 + 3 \times \frac{1}{10} + 9 \times \frac{1}{100} + 2 \times \frac{1}{1000}$$

**Example 4**

Write the decimal 468.721 in expanded form with multiplication.

**Strategy** Use a place-value chart.

**Step 1** Write the decimal in a place value chart.

<table>
<thead>
<tr>
<th>Hundreds</th>
<th>Tens</th>
<th>Ones</th>
<th>.</th>
<th>Tenths</th>
<th>Hundredths</th>
<th>Thousandths</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>6</td>
<td>8</td>
<td>7</td>
<td>2</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

**Step 2** Show each digit as a multiplication expression.

- 4 hundreds $\rightarrow 4 \times 100$
- 6 tens $\rightarrow 6 \times 10$
- 8 ones $\rightarrow 8 \times 1$
- 7 tenths $\rightarrow 7 \times \frac{1}{10}$
- 2 hundredths $\rightarrow 2 \times \frac{1}{100}$
- 1 thousandth $\rightarrow 1 \times \frac{1}{1000}$

**Step 3** Write the expanded form with multiplication.

$$4 \times 100 + 6 \times 10 + 8 \times 1 + 7 \times \frac{1}{10} + 2 \times \frac{1}{100} + 1 \times \frac{1}{1000}$$

**Solution** $468.721 = 4 \times 100 + 6 \times 10 + 8 \times 1 + 7 \times \frac{1}{10} + 2 \times \frac{1}{100} + 1 \times \frac{1}{1000}$
The currency of China is the yuan. When Alana went to China, $1 was worth about 6.837 yuan. What is the number name and the expanded form with multiplication for 6.837?

To write the number name, first write the decimal in a place-value chart.

<table>
<thead>
<tr>
<th>Ones</th>
<th>.</th>
<th>Tenths</th>
<th>Hundredths</th>
<th>Thousandths</th>
</tr>
</thead>
</table>

Separate the decimal into two parts: the whole-number part and the decimal part.
Write the number name for 6. _________________
Write the word that separates the whole-number part from the decimal part. ______
Write the decimal part as you would a whole number. __________________________
What is the least place value of the decimal part? _________________
The number name for 6.837 is _____________________________________________.

Write the expanded form with multiplication.
Find the value of each digit.

6 ones = ____________
8 tenths = ____________
3 hundredths = ____________
7 thousandths = ____________

Write the expanded form. 6.837 = __________________________________________________________________________
Write the expanded form with multiplication.

6.837 = _________________________________________________________________________________________
Comparing decimals is similar to comparing whole numbers. When comparing decimals, start by comparing the numbers in the greatest place. If they are the same, compare the digits in the next place to the right. Do this until you can determine which digit is greater.

Decimals can be compared using the following symbols.
- $=$ means is equal to.
- $<$ means is less than.
- $>$ means is greater than.

**Example 1**
Which symbol makes this number sentence true? Use $>$, $<$, or $=$.

$32.135 \bigcirc 32.035$

**Strategy** Line up the numbers on the decimal point. Compare the digits, starting with the greatest place value.

**Step 1** Line up the digits on the decimal point.
- $32.135$
- $32.035$

**Step 2** Look for the greatest place where the digits are different.
- The digits in the tenths place are different.

**Step 3** Compare the digits in the tenths place.
- $1 > 0$, so $32.135 > 32.035$

**Solution** $32.135 \bigg(>\bigg) 32.035$
Example 2
Which symbol makes this number sentence true? Use >, <, or =.

\[ 47.085 \bigcirc 47.09 \]

**Strategy**  
Line up the numbers on the decimal point. Compare the digits starting with the greatest place value.

**Step 1**  
Line up the digits on the decimal point.

\[
\begin{align*}
47.085 \\
47.09
\end{align*}
\]

**Step 2**  
Look for the greatest place where the digits are different.

The digits in the hundredths place are different.

**Step 3**  
Compare the digits in the hundredths place.

\[ 8 < 9, \text{ so } 47.085 < 47.09 \]

**Solution**  
\[ 47.085 \bigcirc 47.09 \]

Example 3
Which of these numbers is the least? Which is the greatest?

\[ 7.35 \quad 6.989 \quad 7.038 \]

**Strategy**  
Compare the decimals.

**Step 1**  
Compare the whole-number parts.

Since \( 6 < 7 \), the least number is 6.989.

The other two numbers have 7 as the whole-number part.

**Step 2**  
Compare the tenths for the other two numbers.

The tenths place of 7.35 is 3.

The tenths place of 7.038 is 0.

Since \( 3 > 0 \), \( 7.35 > 7.038 \).

**Solution**  
The least number is 6.989. The greatest number is 7.35.
Example 4
Emma is mailing some packages. The weights of the packages, in pounds, are shown below. Order the weights from least to greatest.

9.42  3.201  4.083  11.2

Strategy  Compare the decimals.

Step 1  Compare the whole-number parts.
They are all different.

Step 2  Order the whole-number parts.
3, 4, 9, 11

Step 3  Since the whole-number parts are all different, order the decimals the same way.
3.201, 4.083, 9.42, 11.2

Solution  The weights of the packages in order from least to greatest are 3.201 pounds, 4.083 pounds, 9.42 pounds, and 11.2 pounds.

Coached Example

Which represents the lesser distance: 4.295 kilometers or 4.3 kilometers?

Compare the _______________-number parts first.

The whole-number parts are ______________.

Next compare the digits in the ______________ place.

Use >, <, or = to compare.

________ 〇  _______, so 4.295 〇 4.3.

The lesser distance is ______________ kilometers.
Rounding decimals is similar to rounding whole numbers. To round a decimal, look at the digit to the right of the place you are rounding to:

- If the digit is 5 or greater, round up.
- If the digit is 4 or less, round down.

Example 1
What is 32.86 rounded to the nearest tenth?

Strategy  Use rounding rules to round to the nearest tenth.

Step 1  Look at the digit to the right of the place you are rounding to.
        The digit to the right of the tenths place is in the hundredths place.
        32.86
        The digit is 6.

Step 2  Use the rounding rules to decide if you should round up or down.
        Since 6 > 5, round up.
        32.86 rounds to 32.9.

Solution  Rounded to the nearest tenth, 32.86 is 32.9.
Example 2

In her physical education class, Jenny ran 1 mile in 7.38 minutes. What is Jenny’s time rounded to the nearest minute?

Strategy Use a number line to round.

Step 1 Locate 7.38 between 7 and 8 on a number line.

Step 2 Decide if 7.38 is closer to 7 or 8 on the number line.

7.38 is closer to 7 than to 8.
So 7.38 rounded to the nearest minute is 7.

Solution Jenny’s time rounded to the nearest minute is 7 minutes.

Example 3

Sean has 23.69 meters of string. How many meters of string, rounded to the nearest tenth of a meter, does Sean have?

Strategy Use rounding rules.

Step 1 Look at the digit to the right of the place that you are rounding to.

The digit to the right of the tenths place is 9.

Step 2 Use rounding rules to decide if you should round up or down.

9 > 5, so round up.
So 23.69 rounded to the nearest tenth is 23.7.

Solution Sean has 23.7 meters of string, rounded to the nearest tenth of a meter.
Example 4
Dwayne ran 11.374 kilometers. Rounded to the nearest hundredth, how many kilometers did he run?

**Strategy** Use rounding rules.

**Step 1** Look at the digit to the right of the place that you are rounding to.
   The digit to the right of the hundredths place is 4.
   11.374

**Step 2** Use rounding rules to decide if you should round up or down.
   4 < 5, so round down.
   11.374 rounded to the nearest hundredth is 11.37.

**Solution** Rounded to the nearest hundredth, Dwayne ran 11.37 kilometers.

Coached Example

In his swim class, Alan swam 1 lap in 18.27 seconds. What is Alan’s time, rounded to the nearest whole second?

Round 18.27 to the nearest _____________________________.

Look at the digit to the __________ of the place you are rounding to.

The digit in that place is ________, which means you round ________________.

Rounded to the nearest whole second, 18.27 is ________.
To multiply a whole number by a **power of 10**, add on zeros at the end of the whole number.

To multiply a whole number by 10, put one zero at the end of the number.

For example, $12 \times 10 = 120$

To multiply a whole number by 100, put two zeros at the end of the number.

For example, $12 \times 100 = 1,200$

To multiply a whole number by 1,000, put three zeros at the end of the number.

For example, $12 \times 1,000 = 12,000$

Each place in a number has a value that is 10 times the value of the place to its right. For example, in 3.37 the 3 before the decimal point has a value that is 10 times the 3 after the decimal point.

Likewise, each place in a number has a value that is $\frac{1}{10}$ the value of the place to its left. For example, in 3.37 the 3 after the decimal point has a value that is $\frac{1}{10}$ the value of the 3 before the decimal point.

**Example 1**

What is the product?

$$53 \times 10 = \square$$

**Strategy** Use mental math.

Any whole number multiplied by 10 is the number with one zero at the end of the number.

$$53 \times 10 = 530$$

**Solution** $53 \times 10 \times 530$
Example 2
Jackie rides her bicycle for 13 miles a day. If she does this for 100 days, how many miles will she ride in all?

Strategy  Use mental math.

Step 1  Write an equation for the problem.
Let $m$ represent the number of miles she will ride in all.
$13 \times 100 = m$

Step 2  Multiply.
Any whole number multiplied by 100 is the number with two zeros at the end.
$13 \times 100 = m$
$13 \times 100 = 1,300$

Solution  Jackie will ride 1,300 miles in all.

You can write a power of 10 with an exponent. The exponent tells how many times 10 is used as a factor.

For example, 100 is a power of 10 since $10 \times 10 = 100$. The number 10 is used as a factor 2 times. This is written as $10^2$ and read as ten to the second power or ten squared.

Example 3
What is the value of $10^6$? When it is written using base-ten numerals, how many zeros does $10^6$ have?

Strategy  Use a pattern.

Step 1  Use a pattern.
$10^1 = 10$
$10^2 = 10 \times 10 = 100$
$10^3 = 100 \times 10 = 1,000$
$10^4 = 1,000 \times 10 = 10,000$
$10^5 = 10,000 \times 10 = 100,000$
$10^6 = 100,000 \times 10 = 1,000,000$
Step 2 Identify the pattern.

- $10^1$ evaluates to an answer with 1 zero.
- $10^2$ evaluates to an answer with 2 zeros.
- $10^3$ evaluates to an answer with 3 zeros.
- $10^4$ evaluates to an answer with 4 zeros.
- $10^5$ evaluates to an answer with 5 zeros.
- $10^6$ will have an answer with 6 zeros.

Solution The value of $10^6$ is $1,000,000$. Using base-ten numerals, $10^6$ has 6 zeros.

When multiplying or dividing a decimal by a power of 10, use the exponent to decide how many places to move the decimal point.

When you multiply by a power of 10, use the exponent to decide how many places to move the decimal point to the right.

For example,

- $6.32 \times 10^1 = 63.2 \quad \leftarrow$ move the decimal point one place to the right
- $6.32 \times 10^2 = 632 \quad \leftarrow$ move the decimal point two places to the right
- $6.32 \times 10^3 = 6,320 \quad \leftarrow$ move the decimal point three places to the right

When you divide a decimal by a power of 10, use the exponent to decide how many places to move the decimal point to the left.

For example,

- $6.32 \div 10^1 = 0.632 \quad \leftarrow$ move the decimal point one place to the left
- $6.32 \div 10^2 = 0.0632 \quad \leftarrow$ move the decimal point two places to the left
- $6.32 \div 10^3 = 0.00632 \quad \leftarrow$ move the decimal point three places to the left
Example 4
What is the product?

\[ 0.3 \times 10^2 = \square \]

**Strategy** Multiply by a power of 10.

**Step 1** Decide in which direction to move the decimal point.
You are multiplying by a power of 10, so move the decimal point to the right.

**Step 2** Find the number of places to move the decimal point in the product.
The exponent tells how many places to move to the right.
The exponent is 2, so move the decimal point 2 places.

**Step 3** Write the product.
Move the decimal point in 0.3 two places to the right.
Fill the empty places with zeros.

\[ 0.30 \]

**Solution** \[ 0.3 \times 10^2 = 30 \]

Example 5
What is the quotient?

\[ 627.4 \div 10^3 = \square \]

**Strategy** Divide by a power of 10.

**Step 1** Decide in which direction to move the decimal point.
You are dividing by a power of 10, so move the decimal point to the left.

**Step 2** Find the number of places to move the decimal point in the quotient.
The exponent tells how many places to move to the left.
The exponent is 3, so move the decimal point 3 places.
Lesson 12: Multiply and Divide by Powers of Ten

Step 3

Write the quotient.

Move the decimal point in 627.4 three places to the left.
Write a leading zero.

\[ 627.4 \div 10^3 = 0.6274 \]

Solution

Coached Example

What is the quotient?

\[ 0.9 \div 10^3 = \square \]

Will you multiply or divide 0.9 by a power of 10? _________________

When you divide by a power of 10, do you move the decimal point to the right or to the left? _________________

The _________________ tells how many places to move the decimal point.

What is the exponent, or the power of 10? ________

Move the decimal point in 0.9 _________________ places to the _________________ to find the quotient.

Fill the empty places with _________________.

\[ 0.9 \div 10^3 = \square \]

\[ 0.9 \div 10^3 = \square \]
You can add decimals the same way you add whole numbers. Just align the numbers on the decimal points and write a decimal point in the sum. Remember, when the sum of a column is 10 or greater, you will have to regroup 10 of that unit as 1 of the next greater unit. For example, 12 hundredths can be regrouped as 1 tenth and 2 hundredths.

\[
\begin{align*}
1.53 \\
+ 2.09 \\
\hline
3.62
\end{align*}
\]

**Example 1**
Find the sum: \(5.6 + 0.1\) = \_

**Strategy** Use mental math.
Think: What is 6 tenths plus 1 tenth? 7 tenths
\[
5.6 + 0.1 = 5.7
\]

**Solution** \(5.6 + 0.1 = 5.7\)

**Example 2**
Find the sum: \(1.26 + 0.65\) = \_

**Strategy** Use models.

**Step 1** Model the greater decimal using grids.
Use two grids and shade the first one completely. 1.26 is one and twenty-six hundredths. So, shade 26 squares in the second grid.
Step 2  Use the same model to add 0.65.

0.65 is sixty-five hundredths, so shade 65 more squares in the second grid.

![Grid with shaded squares]

Step 3  Write the total number of shaded squares as a decimal.

One grid is completely shaded, so it represents 1.
The other grid has 91 squares shaded, so it represents 0.91.
Together, the grids show the decimal 1.91.

Solution  \[1.26 + 0.65 = 1.91\]

Example 3
Amir recorded the snowfall during the first week of February. On Monday he recorded 12.78 inches, and on Thursday he recorded another 13.65 inches. How much snow did Amir record for the first week of February?

Strategy  Write an equation for the problem. Then add each place from right to left.

Step 1  Write an equation for the problem.

Let \(s\) represent the number of inches of snow for the first week of February.

\[12.78 + 13.65 = s\]

Step 2  Rewrite the problem vertically.

Align the numbers on the decimal point.
Write the decimal point in the sum.

\[
\begin{array}{c}
12.78 \\
+ 13.65 \\
\hline
\end{array}
\]
Step 3  Add the hundredths: \(8 + 5 = 13\) hundredths.
Regroup 13 hundredths as 1 tenth 3 hundredths.

\[
\begin{align*}
&1 \\
&12.78 \\
&+ 13.65 \\
&\underline{.3}
\end{align*}
\]

Step 4  Add the tenths: \(1 + 7 + 6 = 14\) tenths.
Regroup 14 tenths as 1 one 4 tenths.

\[
\begin{align*}
&11 \\
&12.78 \\
&+ 13.65 \\
&\underline{.43}
\end{align*}
\]

Step 5  Add the ones: \(1 + 2 + 3 = 6\) ones.

\[
\begin{align*}
&11 \\
&12.78 \\
&+ 13.65 \\
&\underline{6.43}
\end{align*}
\]

Step 6  Add the tens: \(1 + 1 = 2\) tens.

\[
\begin{align*}
&11 \\
&12.78 \\
&+ 13.65 \\
&\underline{26.43}
\end{align*}
\]

Solution  Amir recorded 26.43 inches of snow for the first week in February.

Sometimes it may be necessary to write an equivalent decimal before computing. Inserting a 0 at the right end of a decimal does not change its value.

Example 4
Find the sum: \(2.45 + 6.7 = \underline{\text{}}\).

Strategy  Add each place from right to left.

Step 1  Align the numbers on the decimal point.
Insert a 0 to the right of 6.7.
Now both addends have the same number of places.

\[
\begin{align*}
&2.45 \\
&+ 6.70
\end{align*}
\]

Solution  Amir recorded 26.43 inches of snow for the first week in February.
Lesson 13: Add Decimals

Step 2 Write the decimal point in the sum. Add from right to left.

\[ 5 + 0 = 5 \text{ hundredths} \]
\[ 4 + 7 = 11 \text{ tenths} \]
Regroup 11 tenths as 1 one 1 tenth.
\[ 1 + 2 + 6 = 9 \text{ ones} \]
\[ 1 \]
\[ 2.45 \]
\[ + 6.70 \]
\[ 9.15 \]

Solution \[ 2.45 + 6.7 = 9.15 \]

You can use the properties of operations to make computation easier.

<table>
<thead>
<tr>
<th>Additive identity property of 0</th>
<th>Addend</th>
<th>8.7 + 0 = 0 + 8.7 = 8.7</th>
</tr>
</thead>
<tbody>
<tr>
<td>The sum of any number and 0 is that number.</td>
<td>( a + 0 = 0 + a = a )</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Commutative property of addition</th>
<th>Addends</th>
<th>4.2 + 3.6 = 3.6 + 4.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>The order of addends can be changed. The sum does not change.</td>
<td>( a + b = b + a )</td>
<td>( 7.8 = 7.8 )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Associative property of addition</th>
<th>Addends</th>
<th>3.4 + (2.6 + 6.5) =</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addends can be grouped in different ways. The sum will be the same.</td>
<td>( (a + b) + c = a + (b + c) )</td>
<td>( (3.4 + 2.6) + 6.5 )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.4 + (2.6 + 6.5) =</td>
</tr>
<tr>
<td></td>
<td></td>
<td>3.4 + 9.1 = 12.5</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(3.4 + 2.6) + 6.5 =</td>
</tr>
<tr>
<td></td>
<td></td>
<td>6 + 6.5 = 12.5</td>
</tr>
</tbody>
</table>
Example 5
What number is missing from the equation below?

\[5.39 + \_ = 2.47 + 5.39\]

**Strategy** Use the commutative property of addition.

The commutative property of addition states that changing the order of the addends does not change the sum.

\[5.39 + 2.47 = 2.47 + 5.39\]

**Solution** The missing number is 2.47.

You can use an estimate to check if answers are reasonable. If an estimate is not close to the actual answer, an error was made in finding the answer. You can estimate by rounding to the nearest whole number or nearest dollar.

Example 6
Robert bought a sandwich for $2.29, a drink for $0.99, and a cookie for $1.59. How much did Robert spend in all?

**Strategy** Estimate the amount spent. Then find the actual cost.

**Step 1** Round each amount to the nearest dollar. Then add.

- $2.29 rounds down to $2.00
- $0.99 rounds up to $1.00
- $1.59 rounds up to $2.00

\[\$2.00 + \$1.00 + \$2.00 = \$5.00\]

The total should be about $5.00.

**Step 2** Add the costs of the items.

Align the numbers on the decimal point.

\[
\begin{array}{c}
\hline
& 1 & 2 \\
\$2.29 & 0.99 \\
+ 1.59 & \hline \\
\$4.87 & \\
\hline
\end{array}
\]
Step 3  Compare the actual answer to the estimate.

$4.87$ is close to $5.00$.

$4.87$ is a reasonable answer.

Solution  Robert spent $4.87$.

Coached Example

Harrison weighed three samples during science class. The samples had masses of $5.64$ grams, $9.5$ grams, and $2.07$ grams. Estimate the total mass of the three samples. Then find the actual total mass of the samples.

Find the estimated total mass. Round each mass to the nearest whole number.

5.64 rounds __________ to __________.

9.5 rounds __________ to __________.

2.07 rounds __________ to __________.

Add the rounded numbers: __________ + __________ + __________ = __________

The estimated mass of the three samples is __________ grams.

Find the actual total mass. Write and solve the addition problem.

Do all the digits have the same number of places to the right of the decimal point? __________

To write the problem, you need to insert a 0 to the right of __________.

Make sure the decimal points are __________.

Find the actual mass.

The actual mass of the 3 samples is __________ grams.

Is the actual answer close to the estimate? __________

The total mass of the samples is __________ grams.
Subtract Decimals

Getting the Idea
You can subtract decimals the same way you subtract whole numbers. Just align the numbers on the decimal points and write a decimal point in the difference. Remember, when there are not enough units to subtract from, you will have to regroup 1 of the next greater unit as 10 of the lesser unit. For example, you can regroup 7 tenths 3 hundredths as 6 tenths 13 hundredths.

\[
\begin{array}{c}
6.13 \\
1.73 \\
- 0.29 \\
\hline
1.44
\end{array}
\]

Example 1
Find the difference.

\[4.29 - 0.01 = \_\_\_\_\_
\]

Strategy Use mental math.
Think: What is 9 hundredths minus 1 hundredth?

\[4.29 - 0.01 = 4.28\]

Solution \[4.29 - 0.01 = 4.28\]
Example 2
Find the difference.

\[ 1.7 - 0.93 = \square \]

**Strategy** Use models.

**Step 1** Model the greater decimal using grids.

1.7 is one and seven tenths, or one and seventy hundredths.

Use two grids and shade the first one completely.

Shade 70 squares in the second grid.

**Step 2** Cross out squares to represent the number being subtracted.

0.93 is ninety-three hundredths, so cross out 93 of the shaded squares.

Cross out 70 shaded squares in the second grid.

Cross out 23 more in the first grid.

**Step 3** Count the number of shaded squares that are not crossed out.

77 squares are shaded and not crossed out.

77 hundredths = 0.77

**Solution** \[ 1.7 - 0.93 = 0.77 \]
Example 3
In 2010, Ms. Clark earned $528.56 per week. In 2000, she earned $390.73 per week. How much more did Ms. Clark earn per week in 2010 than in 2000?

Strategy Write an equation for the problem. Then solve.

Step 1 Write an equation for the problem.
Let $n$ represent how much more was earned per week in 2010.
$$528.56 - 390.73 = n$$

Step 2 Rewrite the problem.
Align the numbers on the decimal point.
Write the decimal point in the difference.
Subtract the hundredths.
6 hundredths $-$ 3 hundredths $= 3$ hundredths
$$\begin{align*}
528.56 \\
- 390.73 \\
\hline
3.3
\end{align*}$$

Step 3 Subtract the tenths.
Because 7 is greater than 5, regroup from the ones.
15 tenths $-$ 7 tenths $= 8$ tenths
$$\begin{align*}
7.15 \\
528.56 \\
- 390.73 \\
\hline
8.3
\end{align*}$$

Step 4 Subtract the ones.
7 ones $-$ 0 ones $= 7$ ones
$$\begin{align*}
7.15 \\
528.56 \\
- 390.73 \\
\hline
7.83
\end{align*}$$
Step 5  Subtract the tens.

Because 9 is greater than 2, regroup from the hundreds.

\[
\begin{align*}
12 \text{ tens} & - 9 \text{ tens} = 3 \text{ tens} \\
412715 \\
\$528.56 \\
- 390.73 \\
\hline
37.83
\end{align*}
\]

Step 6  Subtract the hundreds.

\[
\begin{align*}
4 \text{ hundreds} & - 3 \text{ hundreds} = 1 \text{ hundred} \\
Write \text{ the dollar sign in the difference.}
\end{align*}
\]

\[
\begin{align*}
412715 \\
\$528.56 \\
- 390.73 \\
\hline
\$137.83
\end{align*}
\]

Solution  Ms. Clark earned $137.83 more per week in 2010 than in 2000.

You can check the answer to a subtraction problem by using addition.

Since $137.83 + $390.73 = $528.56, the answer is correct.
Remember, you can estimate to check if answers are reasonable. If an estimate is not close to the actual answer, an error was made in finding the answer. You can estimate by rounding to the nearest whole number or nearest dollar.

Example 4
A bike trail is 36.25 miles long. Andrew stopped to rest after he had biked 13.8 miles of the trail. How many more miles must he ride to finish the trail?

**Strategy**  
Estimate the distance. Then find the actual distance left.

**Step 1**  
Round each number to the nearest whole number. Then subtract.  
36.25 rounds down to 36.  
13.8 rounds up to 14.  
36 miles − 14 miles = 22 miles  
The difference should be about 22 miles.

**Step 2**  
Find the actual distance.  
Align the numbers on the decimal point. Insert a 0 to the right of 13.8 so that both decimals have the same number of places.  
\[
\begin{array}{c}
512 \\
36.25 \\
- 13.80 \\
\hline
22.45
\end{array}
\]

**Step 3**  
Compare the actual answer to the estimate.  
22.45 is close to 22.  
22.45 is a reasonable answer.

**Solution**  
Andrew must ride 22.45 miles to finish the trail.
Lesson 14: Subtract Decimals

In all, Kobe ran 15.5 miles on Friday, Saturday, and Sunday. He ran 3.75 miles on Friday and 5.6 miles on Saturday. How many miles did Kobe run on Sunday?

Do all the digits have the same number of places to the right of the decimal point? __________

To write the problem, you need to insert a 0 to the right of ______ and ______.

First, ____________________ to find the total number of miles Kobe ran on Friday and Saturday.

Compute.

Kobe ran ___________ miles on Friday and Saturday.

Next, ____________________ the sum of those two days from the number of miles that Kobe ran in all.

Compute.

What is the result? __________

Kobe ran _________ miles on Sunday.
Multiplying decimals is similar to multiplying whole numbers. When you multiply decimals, remember to write the decimal point in the product. Where you place the decimal point depends on the decimal points in the factors.

**Example 1**
Find the product.

\[ 0.7 \times 0.4 = \square \]

**Strategy** Use a model.

**Step 1** Use a 10-by-10 grid.
Shade 0.7 of the columns of squares.

**Step 2** Shade 0.4 of the rows of squares.
The part that overlaps is the product.

There are 28 out of 100 squares, or 0.28, in the overlap.

**Solution**
\[ 0.7 \times 0.4 = 0.28 \]
When you multiply a decimal by a decimal, the product will have the same number of decimal places as the sum of the decimal places in the factors.

**Example 2**

Find the product.

\[0.28 \times 0.4 = \square\]

**Strategy**  
Multiply as you would with whole numbers.  
Write the decimal point in the product.

**Step 1**
Rewrite the problem. Multiply.

\[
\begin{array}{c}
3 \\
0.28 \\
\times 0.4 \\
\hline
112
\end{array}
\]

**Step 2**
Write the decimal point in the product.

There are 3 decimal places in the factors, so there will be 3 decimal places in the product.

\[
\begin{array}{c}
3 \\
0.28 \quad \leftarrow 2 \text{ decimal places} \\
\times 0.4 \quad \leftarrow 1 \text{ decimal place} \\
0.112 \quad \leftarrow 3 \text{ decimal places}
\end{array}
\]

**Solution**  
\[0.28 \times 0.4 = 0.112\]

**Example 3**

A jeweler bought 4.8 ounces of silver at $17.35 per ounce. How much did the jeweler pay for the silver?

**Strategy**  
Write an equation for the problem. Then solve.

**Step 1**
Write an equation for the problem.

Let \(n\) represent how much the jeweler paid for the silver.

\[17.35 \times 4.8 = n\]
Step 2  Rewrite the problem. Multiply the tenths.

\[
\begin{array}{c}
5.24 \\
\times 4.8 \\
\hline
13880
\end{array}
\]

Step 3  Multiply the ones. Write a 0 in the ones place of the second partial product.

\[
\begin{array}{c}
212 \\
\times 4.8 \\
\hline
13880 \\
69400
\end{array}
\]

Step 4  Add the partial products. Write the $ sign and the decimal point in the product.

\[
\begin{array}{c}
17.35 \quad \leftarrow 2 \text{ decimal places} \\
\times 4.8 \quad \leftarrow 1 \text{ decimal place} \\
13880 \\
69400 \\
\hline
83280 \quad \leftarrow 3 \text{ decimal places}
\end{array}
\]

Solution  The jeweler paid $83.28 for the silver.

Sometimes when you multiply with decimals, you will need to put zeros in the product.

Example 4
Find the product.

\[0.09 \times 0.7 = \square\]

Strategy  Multiply as you would with whole numbers. Write the decimal point in the product.

Step 1  Multiply.

\[
\begin{array}{c}
6 \\
0.09 \\
\times 0.7 \\
\hline
63
\end{array}
\]
Lesson 15: Multiply Decimals

Step 2

Write the decimal point in the product.

\[
\begin{align*}
&6 \\
&0.09 \quad \text{← 2 decimal places} \\
&\times 0.7 \quad \text{← 1 decimal place} \\
&0.063 \quad \text{← 3 decimal places}
\end{align*}
\]

Since 3 decimal places are needed in the product, write a zero in the tenths place.

Solution \[0.09 \times 0.7 = 0.063\]

You can use the properties of operations to make computation easier.

<table>
<thead>
<tr>
<th>Multiplicative identity property of 1</th>
<th>[a \times 1 = 1 \times a = a]</th>
<th>[9.3 \times 1 = 1 \times 9.3 = 9.3]</th>
</tr>
</thead>
<tbody>
<tr>
<td>The product of any number and 1 is that number.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Commutative property of multiplication</th>
<th>[a \times b = b \times a]</th>
<th>[2.8 \times 1.7 = 1.7 \times 2.8]</th>
</tr>
</thead>
<tbody>
<tr>
<td>The order of factors can be changed. The product does not change.</td>
<td>[4.76 = 4.76]</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Associative property of multiplication</th>
<th>[(a \times b) \times c = a \times (b \times c)]</th>
<th>[3.2 \times (4.5 \times 8.1) =]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factors can be grouped in different ways. The product will be the same.</td>
<td>[(3.2 \times 4.5) \times 8.1]</td>
<td>[3.2 \times (4.5 \times 8.1) =]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[3.2 \times 36.45 = 116.64]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[(3.2 \times 4.5) \times 8.1 =]</td>
</tr>
<tr>
<td></td>
<td></td>
<td>[14.4 \times 8.1 = 116.64]</td>
</tr>
</tbody>
</table>
Example 5
What number is missing from the equation below?
\[ 5.3 \times (7.9 \times 6.2) = (5.3 \times \underline{\quad}) \times 6.2 \]

**Strategy** Use the associative property of multiplication.
Changing the grouping of the factors does not change the product.
\[ 5.3 \times (7.9 \times 6.2) = (5.3 \times 7.9) \times 6.2 \]

**Solution** The missing number is 7.9.

Coached Example

Mr. Starr's class is taking a field trip to a museum. Tickets cost $13.95 each. If Mr. Starr buys 27 tickets, what is the total cost of the tickets?

Write the problem in vertical form.

\[
\begin{array}{c}
13.95 \\
\times 27
\end{array}
\]

Multiply by the ones: ______________
Write the first partial product.
Multiply by the tens: ______________
Write the second partial product.
Add the partial products: ______________ + ______________ = ______________
There are _____ decimal places in the factors, so the product will have _____
decimal places.
Write the decimal point and the $ in the product: ______________
The total cost of the tickets is ______________.
You can use a model to divide decimals.

**Example 1**
Find the quotient: \( 0.2 \div 5 = \square \).

**Strategy** Use a model.

**Step 1** Use a 10-by-10 grid.
Shade 0.2 of the model.

**Step 2** Use circles to separate the shaded area into 5 equal groups.

**Step 3** Count the number of squares in each group.
There are 4 squares in each group.
Four squares represents 4 hundredths, or 0.04.

**Solution** \( 0.2 \div 5 = 0.04 \)

When using paper and pencil, write the decimal point in the quotient above the decimal point of the dividend. Then divide just as you would with whole numbers.
Example 2
Mrs. Collins bought 8 towels for $66.24. Each towel was the same price. What was the price of one towel?

Strategy  Write an equation for the problem. Then divide from left to right.

Step 1  Write an equation for the problem.
Let $p$ represent the price of one towel.

\[
66.24 \div 8 = p
\]

Step 2  Write the decimal point in the quotient. Divide 66 ones.

\[
\begin{array}{c}
8. \\
8 \overline{)66.24} \\
-64 \quad \leftarrow 8 \times 8 = 64 \\
\hline
2 \quad \leftarrow 66 - 64 = 2
\end{array}
\]

Step 3  Bring down the 2 tenths. Divide the tenths.

\[
\begin{array}{c}
8.2 \\
8 \overline{)66.24} \\
-64 \downarrow \\
\hline
22 \\
-16 \quad \leftarrow 8 \times 2 = 16 \\
\hline
6 \quad \leftarrow 22 - 16 = 6
\end{array}
\]

Step 4  Bring down the 4 hundredths. Divide the hundredths.

\[
\begin{array}{c}
8.28 \\
8 \overline{)66.24} \\
-64 \\
\hline
22 \\
-16 \\
\hline
64 \\
-64 \quad \leftarrow 8 \times 8 = 64 \\
\hline
0 \quad \leftarrow 64 - 64 = 0
\end{array}
\]

Step 5  Write the dollar sign in the quotient.

\[
$8.28 = p$
\]

Solution  The price of one towel is $8.28.
Sometimes you may need to insert zeros in the quotient as placeholders.

**Example 3**
Find the quotient: \(0.42 \div 6 = \) \_

**Strategy** Divide each place, going from left to right.

**Step 1** Rewrite the problem.
Write the decimal point in the quotient. Since the dividend is less than 1, write a 0 in the ones place.

\[
\begin{array}{c}
0.6 \\
\hline
0.42 \\
\end{array}
\]

**Step 2** Divide the tenths.

\[
\begin{array}{c|c}
6 & 0.42 \\
\hline
\hline
-0 & 6 \times 0 = 0 \\
\hline
4 & 4 - 0 = 4 \\
\end{array}
\]

**Step 3** Bring down the 2 hundredths. Divide the hundredths.

\[
\begin{array}{c|c}
0.07 \\
\hline
6 & 0.42 \\
\hline
\hline
-0 & 6 \times 7 = 42 \\
\hline
42 & 42 - 42 = 0 \\
\end{array}
\]

**Solution** \(0.42 \div 6 = 0.07\)

When dividing a decimal by a decimal divisor, you can multiply the divisor by a power of 10 to form a whole number. Use the same power of 10 to multiply the dividend. Then use the new divisor and dividend to divide.

For example, divide \(1.5 \div 0.3\). Multiply the divisor and dividend by 10:

\[(1.5 \times 10) \div (0.3 \times 10) = \]

\[
\begin{align*}
15 & \div 3 = 5 \\
\end{align*}
\]

To find a decimal remainder, you can annex zeroes at the end of the dividend. Annexing a 0 to the right of a decimal does not change its value.
Example 4
Find the quotient: $82.6 \div 0.4 = \underline{\hspace{2cm}}$.

**Strategy**

Multiply by 10 to create a whole-number divisor. Then divide.

**Step 1**
There is one decimal place in the divisor, so multiply the divisor and the dividend by 10.

$$0.4 \times 10 = 4$$
$$82.6 \times 10 = 826$$

**Step 2**
Write the problem with the new divisor and dividend. Divide each place from left to right.

```
206
4)826
  - 8
  -- 02
  -- 0
  -- 26
  -- 24
    2
```

**Step 3**
Annex a 0 and continue dividing.

```
206.5
4)826.0
  - 8
  -- 02
  -- 0
  -- 26
  -- 24
    20
    -- 20
      0
```

**Solution**

$82.6 \div 0.4 = 206.5$

Remember to check the answer by multiplying the quotient by the divisor.

Since $206.3 \times 0.4 = 82.6$, the answer is correct.
Lesson 16: Divide Decimals

Coached Example

Madison paid $28.12 for 9.5 gallons of gas. What was the price of each gallon of gas?

Write an equation for the problem. _________________________________

What is the dividend? _____________

What is the divisor? _____________

How many decimal places are after the decimal point in the divisor? ______

By what number should you multiply both the divisor and dividend? _______

______ × 28.12 = _____________

______ × 9.5 = _____________

Write the problem with the new dividend and divisor. Write the decimal point in the quotient. Then divide each place.

Madison paid _____________ for each gallon of gas.
A fraction names part of a whole or a group. The number above the fraction bar is called the **numerator**. It shows the number of parts being considered. The number below the fraction bar is called the **denominator**. It shows the total number of equal parts in the whole or in the group. The fraction bar means divided by. So if $a$ and $b$ are whole numbers, $\frac{a}{b}$ means $a \div b$.

### Example 1

What fraction of the rectangle is shaded?

**Strategy** Find the denominator. Then find the numerator.

**Step 1** Count the number of equal parts in the rectangle.

There are 8 parts. This is the denominator.

**Step 2** Count the number of parts that are shaded.

There are 6 parts shaded. This is the numerator.

**Step 3** Write the fraction.

$$\frac{\text{numerator}}{\text{denominator}} = \frac{6}{8}$$

**Solution** $\frac{6}{8}$ of the rectangle is shaded.
Two fractions are **equivalent fractions** if they represent the same part of a whole. For example, the model below shows that $\frac{1}{2}$ is equivalent to $\frac{4}{8}$.

![Image of a circle divided into eight equal parts, with three parts shaded, representing $\frac{1}{2}$ and $\frac{4}{8}$]

The fraction $\frac{1}{2}$ is in **simplest form** because the numerator and denominator do not have any common factors except 1. To simplify a fraction, divide the numerator and denominator by the **greatest common factor (GCF)**.

**Example 2**
What is $\frac{6}{8}$ in simplest form?

**Strategy** Write an equivalent fraction in simplest form.

**Step 1** Identify the common factors of 6 and 8.
- The factors of 6 are 1, 2, 3, and 6.
- The factors of 8 are 1, 2, 4, and 8.
- 2 is the GCF.

**Step 2** Divide the numerator and denominator by 2.

$$\frac{6}{8} = \frac{6 \div 2}{8 \div 2} = \frac{3}{4}$$

**Solution** $\frac{6}{8}$ written in simplest form is $\frac{3}{4}$.

You can use number lines to find equivalent fractions. On the number lines below, the fractions $\frac{3}{4}$ and $\frac{6}{8}$ are equivalent fractions, since both $\frac{3}{4}$ and $\frac{6}{8}$ are the same distance from 0.
You can also find equivalent fractions by multiplying the numerator and denominator by the same number. Multiplying the numerator and denominator by the same number is the same as multiplying by 1, so the value of the fraction is unchanged.

**Example 3**

Write two equivalent fractions for $\frac{2}{3}$.

**Strategy** Multiply the numerator and denominator by the same number.

**Step 1**

Multiply the numerator and denominator by 2.

\[
\frac{2}{3} = \frac{2 \times 2}{3 \times 2} = \frac{4}{6}
\]

**Step 2**

Multiply the numerator and denominator by 3.

\[
\frac{2}{3} = \frac{2 \times 3}{3 \times 3} = \frac{6}{9}
\]

**Solution** Two equivalent fractions for $\frac{2}{3}$ are $\frac{4}{6}$ and $\frac{6}{9}$.

An important use for equivalent fractions is to create pairs of fractions with the same denominator. One way to find fractions with the same denominator is to multiply the denominators of the fractions.

**Example 4**

Write equivalent fractions for $\frac{5}{6}$ and $\frac{1}{4}$ that have the same denominator.

**Strategy** Multiply the two denominators to write equivalent fractions.

**Step 1**

Multiply the denominators.

\[
6 \times 4 = 24
\]

**Step 2**

Write an equivalent fraction for $\frac{5}{6}$ that has a denominator of 24.

\[
6 \times 4 = 24, \text{ so multiply the numerator and denominator by 4.} \\
\frac{5}{6} = \frac{5 \times 4}{6 \times 4} = \frac{20}{24}
\]

**Step 3**

Write an equivalent fraction for $\frac{1}{4}$ that has a denominator of 24.

\[
4 \times 6 = 24, \text{ so multiply the numerator and denominator by 6.} \\
\frac{1}{4} = \frac{1 \times 6}{4 \times 6} = \frac{6}{24}
\]

**Solution** $\frac{5}{6} = \frac{20}{24}$ and $\frac{1}{4} = \frac{6}{24}$
One way to determine if two fractions are equivalent is to rename one or both fractions using a common denominator. It is not necessary to use the least common multiple to find the least common denominator. You can multiply the denominators to find a common denominator.

**Example 5**

Determine if \( \frac{2}{6} \) and \( \frac{3}{9} \) are equivalent.

**Strategy**  
Rename each fraction using a common denominator.

**Step 1**  
Multiply the denominators.  
\[ 6 \times 9 = 54 \]

Rename the fractions using 54 as the denominator.

**Step 2**  
Write equivalent fractions.

\[ \frac{2}{6} \times \frac{9}{9} = \frac{18}{54} \]
\[ \frac{3}{9} \times \frac{6}{6} = \frac{18}{54} \]

**Solution**  
The fractions \( \frac{2}{6} \) and \( \frac{3}{9} \) are equivalent.

Another method to determine if two fractions are equivalent is to write each in simplest form.

**Example 6**

Determine if \( \frac{8}{10} \) and \( \frac{12}{15} \) are equivalent.

**Strategy**  
Write each fraction in simplest form.

**Step 1**  
Write \( \frac{8}{10} \) in simplest form.

\[ \frac{8}{10} \div \frac{2}{2} = \frac{4}{5} \]

**Step 2**  
Write \( \frac{12}{15} \) in simplest form.

\[ \frac{12}{15} \div \frac{3}{3} = \frac{4}{5} \]

**Solution**  
The fractions \( \frac{8}{10} \) and \( \frac{12}{15} \) are equivalent.
Write $\frac{8}{10}$ in simplest form. Then write another equivalent fraction for $\frac{8}{10}$.

First, write the fraction in simplest form.

The factors of 8 are __________, __________, __________, __________.

The factors of 10 are __________, __________, __________, __________.

The greatest common factor of 8 and 10 is __________.

Divide the numerator and denominator by __________.

\[
\frac{8}{10} = \frac{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}{\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_}\]

Next, find another equivalent fraction.

Multiply the numerator and denominator of $\frac{8}{10}$ by __________.

Show your work.

In simplest form, $\frac{8}{10}$ is __________.

Another fraction equivalent to $\frac{8}{10}$ is __________.
A **mixed number** can be written as an **improper fraction**. In an improper fraction, the numerator is greater than the denominator. For example, the mixed number $1\frac{1}{4}$ can be written as an improper fraction. The model below shows that $1\frac{1}{4} = \frac{5}{4}$.
Each rectangle is divided into 4 equal parts, which is the denominator.
There are 5 parts shaded, so that is the numerator.

### Example 1

In simplest form, what mixed number is modeled below?

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**Strategy**  
Find the whole-number part. Then find the fraction part.

**Step 1**  
Count the number of completely shaded figures.
There are 2 rectangles completely shaded.
The whole-number part is 2.

**Step 2**  
Find the fraction of the figure that is partially shaded.
$\frac{2}{4}$ of the figure is shaded.

**Step 3**  
Write the fraction part in simplest form.
The greatest common factor of 2 and 4 is 2.

$\frac{2}{4} = \frac{2 \div 2}{4 \div 2} = \frac{1}{2}$

**Step 4**  
Add the whole-number part and the fraction part.

$2 + \frac{1}{2} = 2\frac{1}{2}$

**Solution**  
The mixed number shown by the model is $2\frac{1}{2}$. 
Example 2
What improper fraction does the model represent?

Strategy
Identify the numerator and the denominator.

Step 1
To find the numerator, count the number of shaded parts.
There are 13 shaded parts.

Step 2
To find the denominator, count the number of equal parts in each rectangle.
Each rectangle is divided into 6 equal parts.

Solution
The model represents the improper fraction \(\frac{13}{6}\).

Example 3
What mixed number and improper fraction does this model represent?

Strategy
Identify the whole-number part and the fraction part.

Step 1
Identify the whole-number part of the mixed number.
One circle is completely shaded. The whole-number part is 1.

Step 2
Identify the fraction part of the mixed number.
In the second circle, 3 parts are shaded out of 8 equal parts.
The fraction part is \(\frac{3}{8}\). The mixed number is \(1\frac{3}{8}\).

Step 3
Identify the numerator of the improper fraction.
There are 11 shaded parts. The numerator is 11.

Step 4
Identify the denominator of the improper fraction.
Each circle is divided into 8 equal parts. The denominator is 8.
The improper fraction is \(\frac{11}{8}\).

Solution
The model represents the mixed number \(1\frac{3}{8}\) and the improper fraction \(\frac{11}{8}\).
Lesson 18: Improper Fractions and Mixed Numbers

You can convert a mixed number to an improper fraction. A mixed number represents the sum of a whole number and a fraction less than 1. Writing the whole-number part as an equivalent fraction can help you to write a mixed number as an improper fraction.

For example, convert \(2 \frac{1}{6}\) to an improper fraction.

Write the whole number as a fraction with a denominator of 1.

\[
2 \frac{1}{6} = \frac{2}{1} + \frac{1}{6}
\]

Write an equivalent fraction for it with the same denominator as the fraction part of the mixed number.

\[
\frac{2}{1} + \frac{1}{6} = \frac{12}{6} + \frac{1}{6}
\]

Add the two fractions to represent the mixed number as an improper fraction.

\[
\frac{12}{6} + \frac{1}{6} = \frac{13}{6}
\]

So, \(2 \frac{1}{6}\) written as an improper fraction is \(\frac{13}{6}\).

Example 4
Convert \(3 \frac{1}{2}\) to an improper fraction.

Strategy  Write the whole-number part as an equivalent fraction.

Step 1  Write the whole number as a fraction with a denominator of 1.

\[
3 = \frac{3}{1}
\]

Step 2  Write \(\frac{3}{1}\) as an equivalent fraction with a denominator of 2.

Multiply the numerator and the denominator by 2.

\[
\frac{3}{1} = \frac{3 \times 2}{1 \times 2} = \frac{6}{2}
\]

Step 3  Add the equivalent fraction and the fraction part of the mixed number.

\[
\frac{6}{2} + \frac{1}{2} = \frac{7}{2}
\]

Solution  \(3 \frac{1}{2} = \frac{7}{2}\)
You could also follow these steps to convert a mixed number to an improper fraction.

1. Multiply the whole number by the denominator of the fraction part.
2. Add the numerator of the fraction part to the product. This sum is the numerator of the improper fraction.
3. The denominator of the improper fraction is the same as the denominator of the fraction part in the mixed number.

Example 5
Write $4 \frac{7}{10}$ as an improper fraction.

Strategy  
Multiply the whole-number part by the denominator. Then add the numerator to the product.

Step 1  
Multiply the whole number by the denominator.  
$4 \times 10 = 40$

Step 2  
Add the numerator of the fraction part to the product.  
$40 + 7 = 47$

Step 3  
Write the improper fraction.  
The numerator is 47.  
The denominator remains 10.

Solution  
$4 \frac{7}{10} = \frac{47}{10}$

To convert an improper fraction to a mixed number, divide the numerator by the denominator. The quotient (without the remainder) is the whole-number part of the mixed number. The remainder is the numerator of the fraction part. The denominator remains the same if the mixed number is in simplest form.

Example 6
Write $\frac{16}{6}$ as a mixed number in simplest form.

Strategy  
Divide the numerator by the denominator.

Step 1  
Divide the numerator by the denominator.  
$\frac{16}{6} = 16 \div 6 = 2 R4$
Step 2  Write the mixed number.
   The quotient, 2, is the whole-number part.
   The remainder, 4, is the numerator of the fraction part.
   The denominator, 6, stays the same.
   The mixed number is $2 \frac{4}{6}$.

Step 3  Write the fraction in simplest form.
   The greatest common factor of 4 and 6 is 2.
   $\frac{4}{6} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3}$

Solution  $\frac{16}{6}$ written as a mixed number in simplest form is $2 \frac{2}{3}$.

Note: You can also use the fact that $\frac{6}{6} = 1$ to convert the improper fraction in Example 6 to a mixed number. $\frac{16}{6} = \frac{6}{6} + \frac{6}{6} + \frac{4}{6} = 1 + 1 + \frac{4}{6} = 2 \frac{2}{3}$

Coached Example

Ms. Rossi’s class had a pizza party. The shaded parts of the diagram show the amount of pizza that the class ate.

What mixed number in simplest form represents the amount of pizza the class ate?

How many pizzas are completely shaded? ______

Each pizza is divided into ________ equal parts.

How many parts are shaded in the partially shaded circle? ______

What fraction of the last circle is shaded? ______

Write the fraction in simplest form. ______

Add the whole-number part and the fraction part. ______ + ______ = ______

A total of _________ pizzas were eaten.
Add Fractions

Getting the Idea

To find the sum of fractions that have like denominators, add the numerators. The denominator remains the same. Write the sum in simplest form.

Example 1

Add.

\[
\frac{5}{12} + \frac{9}{12} = \square
\]

Strategy Use fraction strips to find the sum.

Step 1 Shade fraction strips to show \( \frac{5}{12} \) and \( \frac{9}{12} \).

Step 2 Count the total number of shaded parts.

Write 14 as the numerator. The denominator stays the same.

\[
\frac{5}{12} + \frac{9}{12} = \frac{5 + 9}{12} = \frac{14}{12}
\]

Step 3 Convert the improper fraction to a mixed number.

\[
\frac{14}{12} = \frac{12}{12} + \frac{2}{12} = 1\frac{2}{12}
\]

Step 4 Write the mixed number in simplest form.

\[
\frac{2}{12} = \frac{2}{12} \div 2 = \frac{1}{6}
\]

So \( 1\frac{2}{12} = 1\frac{1}{6} \).

Solution

\[
\frac{5}{12} + \frac{9}{12} = 1\frac{1}{6}
\]

To add fractions with unlike denominators, you will need to find equivalent fractions for one or both fractions, so that they have a common denominator. One way to find a common denominator is to multiply the denominators of the fractions.
Example 2
In a science experiment, a plant grew $\frac{3}{4}$ inch one week and another $\frac{2}{3}$ inch the following week. How many inches did it grow during the two weeks?

**Strategy** Write equivalent fractions with a common denominator. Then add.

**Step 1** Find a common denominator.

Multiply the two denominators.

$4 \times 3 = 12$

**Step 2** Write equivalent fractions with 12 as the denominator.

$\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12}$

$\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$

**Step 3** Add.

$\frac{9}{12} + \frac{8}{12} = \frac{17}{12}$

**Step 4** Convert the improper fraction to a mixed number in simplest form.

$\frac{17}{12} = 1 \frac{5}{12}$

**Solution** The plant grew $1 \frac{5}{12}$ inches.

Add mixed numbers in the same way that fractions are added. First add the fraction parts and then add the whole-number parts.

When the denominator of one fraction is a factor of the other fraction, use the greater number as the common denominator.

Example 3
Robert hiked $3 \frac{1}{5}$ miles Saturday and $4 \frac{3}{10}$ miles Sunday. How many miles did he hike in all?

**Strategy** Add the whole numbers and then add the fractions.

**Step 1** Find a common denominator.

Since 10 is a multiple of 5, a common denominator is 10.
Step 2  Find the fraction equivalent to $\frac{1}{5}$ with a denominator of 10.

\[
\frac{1 \times 2}{5 \times 2} = \frac{2}{10}
\]

\[
3 \frac{1}{5} = 3 \frac{2}{10}
\]

Step 3  Add the fractions in simplest form.

\[
\frac{2}{10} + \frac{3}{10} = \frac{5}{10}
\]

\[
\frac{5}{10} \div \frac{5}{5} = \frac{1}{2}
\]

Step 4  Add the whole numbers.

\[
3 + 4 = 7
\]

Step 5  Add the sums.

\[
\frac{1}{2} + 7 = 7 \frac{1}{2}
\]

Solution  Robert hiked $7 \frac{1}{2}$ miles in all.

You can use benchmarks to make an estimate. A benchmark is a common number that can be compared to another number.

Use the benchmarks 0, $\frac{1}{2}$, and 1 to make an estimate.

- If the fraction is less than $\frac{1}{4}$, round the fraction to 0.
- If the fraction is greater than or equal to $\frac{1}{4}$ and less than $\frac{3}{4}$, round to $\frac{1}{2}$.
- If the fraction is greater than or equal to $\frac{3}{4}$, round up to 1.
Example 4
Estimate the sum of $\frac{2}{3} + \frac{2}{5}$ to the nearest $\frac{1}{2}$.

**Strategy**  Round each addend to the nearest $\frac{1}{2}$.

**Step 1**  Round $\frac{2}{3}$ to the nearest $\frac{1}{2}$.
Since $\frac{1}{2} < \frac{2}{3} < \frac{3}{4}$, round $\frac{2}{3}$ to $\frac{1}{2}$.

**Step 2**  Round $\frac{2}{5}$ to the nearest $\frac{1}{2}$.
Since $\frac{1}{4} < \frac{2}{5} < \frac{1}{2}$, round $\frac{2}{5}$ to $\frac{1}{2}$.

**Step 3**  Add the rounded numbers.
$\frac{1}{2} + \frac{1}{2} = 1$

**Solution**  The sum of $\frac{2}{3} + \frac{2}{5}$ is about 1.

Another way to add fractions with unlike denominators is to write equivalent fractions with the **least common denominator (LCD)**. You can find the LCD by listing the multiples of the denominators and finding the least number that is a common multiple.

Example 5
Keira walked $\frac{5}{8}$ mile on Oak Avenue and then $\frac{7}{10}$ mile on Third Street to visit Cora. How far did Keira walk to visit Cora?

**Strategy**  Write equivalent fractions using the LCD. Then add.

**Step 1**  Find the LCD of $\frac{5}{8}$ and $\frac{7}{10}$.
List the multiples of 8: 8, 16, 24, 32, 40
List the multiples of 10: 10, 20, 30, 40
The LCD is 40.

**Step 2**  Write equivalent fractions with 40 as the denominator.
$\frac{5}{8} \times \frac{5}{5} = \frac{25}{40}$
$\frac{7}{10} \times \frac{4}{4} = \frac{28}{40}$
Step 3 Add the equivalent fractions.
\[
\frac{25}{40} + \frac{28}{40} = \frac{53}{40}
\]

Step 4 Write the sum as a mixed number.
\[
\frac{53}{40} = 1\frac{13}{40}
\]

Solution Keira walked \(1\frac{13}{40}\) miles to visit Cora.

You can add fractions with unlike denominators by multiplying the denominators to write a common denominator. For the numerators, you can multiply each numerator by the denominator of the other fraction and add the products.

Example 6
Add: \(\frac{3}{4} + \frac{5}{6}\)

Strategy Use multiplication to add fractions.

Step 1 Multiply the denominators to find a common denominator.
\[
4 \times 6 = 24
\]
Use 24 for the denominator.

Step 2 Multiply the numerator of one fraction by the denominator of the other fraction.
\[
\frac{3}{4} + \frac{5}{6} = \frac{3 \times 6}{24} + \frac{5 \times 4}{24}
\]
\[
\frac{3 \times 6}{24} + \frac{5 \times 4}{24} = \frac{18}{24} + \frac{20}{24}
\]

Step 3 Add.
\[
\frac{18}{24} + \frac{20}{24} = \frac{38}{24}
\]

Step 4 Write the sum as a mixed number in simplest form.
\[
\frac{38}{24} = \frac{38 \div 2}{24 \div 2} = \frac{19}{12}
\]
\[
\frac{19}{12} = 1\frac{7}{12}
\]

Solution \(\frac{3}{4} + \frac{5}{6} = 1\frac{7}{12}\)
Lesson 19: Add Fractions

Coached Example

Suki rides her bicycle 5/6 mile before seeing a sign that reads “Tybee Island: 3/4 mile.” If Suki rides to Tybee Island, how many miles will she travel in all?

Add to find the total miles.

The denominators of the fractions are _______ and _______.

Multiples of 6: _________________________________________

Multiples of 4: _________________________________________

The least number that is a common multiple of 6 and 4 is _______.

Find equivalent fractions with _______ as the common denominator.

\[
\frac{5}{6} = \frac{5 \times 2}{6 \times 2} = \frac{\Box}{\Box}
\]

\[
\frac{3}{4} = \frac{3 \times \Box}{4 \times \Box} = \frac{\Box}{\Box}
\]

Add.

\[
\frac{\Box}{12} + \frac{\Box}{12} = \frac{\Box}{\Box}
\]

Write your answer in simplest form: _________

Suki will ride _______ miles in all to reach Tybee Island.
To subtract fractions that have like denominators, subtract the numerators. The denominator remains the same. Write the difference in simplest form.

Example 1

Subtract.
\[ \frac{7}{8} - \frac{5}{8} = \]

Strategy  Use fraction strips to find the difference.

Step 1  Shade fraction strips to show \( \frac{7}{8} \).

\[ \begin{array}{cccccccc}
\frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\
\end{array} \]

Step 2  Cross out \( \frac{5}{8} \) of the shaded parts.

\[ \begin{array}{cccccccc}
\frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} & \frac{1}{8} \\
\end{array} \]

Step 3  Count the remaining shaded parts.

There are 2 shaded parts.

Write 2 as the numerator. The denominator stays the same.

\[ \frac{7}{8} - \frac{5}{8} = \frac{2}{8} \]

Step 4  Write the fraction in simplest form.

\[ \frac{2}{8} = \frac{2}{8} \div 2 = \frac{1}{4} \]

Solution  \[ \frac{7}{8} - \frac{5}{8} = \frac{1}{4} \]

To subtract fractions with unlike denominators, rename one or both fractions so that they have like denominators.
Example 2
Subtract: \( \frac{5}{9} - \frac{1}{6} \)

Strategy Write equivalent fractions using a common denominator. Then subtract.

Step 1 Find a common denominator of \( \frac{5}{9} \) and \( \frac{1}{6} \).
\[ 9 \times 6 = 54 \]

Step 2 Write equivalent fractions with 54 as the denominator.
\[
\begin{align*}
\frac{5}{9} &= \frac{5 \times 6}{9 \times 6} = \frac{30}{54} \\
\frac{1}{6} &= \frac{1 \times 9}{6 \times 9} = \frac{9}{54}
\end{align*}
\]

Step 3 Subtract.
\[
\frac{30}{54} - \frac{9}{54} = \frac{21}{54}
\]

Step 4 Write the difference in simplest form.
\[
\frac{21}{54} = \frac{21 \div 3}{54 \div 3} = \frac{7}{18}
\]

Solution \( \frac{5}{9} - \frac{1}{6} = \frac{7}{18} \)

You can use the least common denominator (LCD) to rename fractions. When the LCD is used, the difference will be in simplest form.

Example 3
Subtract: \( \frac{5}{6} - \frac{3}{8} \)

Strategy Use the LCD to write equivalent fractions. Then subtract.

Step 1 Find the LCD of \( \frac{5}{6} \) and \( \frac{3}{8} \).
Multiples of 6: 6, 12, 18, 24
Multiples of 8: 8, 16, 24
The LCD is 24.

Step 2 Write equivalent fractions with 24 as the denominator.
\[
\begin{align*}
\frac{5}{6} &= \frac{5 \times 4}{6 \times 4} = \frac{20}{24} \\
\frac{3}{8} &= \frac{3 \times 3}{8 \times 3} = \frac{9}{24}
\end{align*}
\]
Step 3
Subtract.
\[
\frac{20}{24} - \frac{9}{24} = \frac{11}{24}
\]

Solution
\[
\frac{5}{6} - \frac{3}{8} = \frac{11}{24}
\]

To subtract mixed numbers, you can subtract the fraction parts and then the whole-number parts.

Example 4
Leo walked a total of 3 \(\frac{1}{4}\) miles before and after school yesterday. He walked 1 \(\frac{7}{8}\) miles before school. How many miles did he walk after school?

Strategy
Use the LCD to write equivalent mixed numbers. Then subtract.

Step 1
Find the LCD of \(\frac{1}{4}\) and \(\frac{7}{8}\).
Since 8 is a multiple of 4, the LCD is 8.

Step 2
Rename \(\frac{1}{4}\), so that is has 8 as the denominator.
\[
\frac{1}{4} = \frac{1}{4} \times \frac{2}{2} = \frac{2}{8}
\]

Step 3
Subtract \(3\frac{2}{8} - 1\frac{7}{8}\).
There are not enough eighths to subtract.
Regroup 1 whole as \(\frac{8}{8}\).
\[
3\frac{2}{8} = 2\frac{10}{8}
\]

Step 4
Subtract the fractions and then the whole numbers.
\[
\frac{10}{8} - \frac{7}{8} = \frac{3}{8}
\]
\[
2 - 1 = 1
\]

Step 5
Add the differences.
\[
\frac{3}{8} + 1 = 1\frac{3}{8}
\]

Solution
Leo walked 1 \(\frac{3}{8}\) miles after school.

You can subtract mixed numbers by renaming them as improper fractions.
Example 5
Mr. Ramos bought two packages of chicken. One package weighed $2\frac{5}{8}$ pounds and the other weighed $4\frac{1}{2}$ pounds. How much more does the heavier package weigh?

Strategy Rename the mixed numbers as improper fractions. Then subtract.

Step 1 Find a common denominator for $\frac{5}{8}$ and $\frac{1}{2}$.
Since 2 is a multiple of 8, the LCD is 8.

Step 2 Rename $4\frac{1}{2}$ so that it has 8 as a denominator.

Step 3 Rename both mixed numbers as improper fractions.

Step 4 Subtract. Write the difference as a mixed number.

Solution The heavier package weighs $1\frac{7}{8}$ pounds more.

You can subtract fractions with unlike denominators by multiplying the denominators to write a common denominator. For the numerators, you can multiply each numerator by the denominator of the other fraction and subtract the products.

Example 6
Subtract: $\frac{7}{9} - \frac{3}{5}$

Strategy Use multiplication to subtract fractions.

Step 1 Multiply the denominators to find a common denominator.

Step 2 Multiply the numerator of one fraction by the denominator of the other fraction. These provide the numerators.
Step 3  Subtract.

\[
\frac{35}{45} - \frac{27}{45} = \frac{8}{45}
\]

Solution  \[
\frac{7}{9} - \frac{3}{5} = \frac{8}{45}
\]

Coached Example

Jillian poured milk into a glass \(\frac{9}{10}\) full. When she finished drinking, the glass was \(\frac{1}{4}\) full. How much of the milk in the glass did she drink?

Find the difference.

Find the least common denominator (LCD).

Multiples of 10: ___________________________

Multiples of 4: ___________________________

The least number that is a common multiple of 10 and 4 is ________.

Find equivalent fractions with ________ as the denominator.

\[
\frac{9}{10} = \frac{9 \times 2}{10 \times 2} = \boxed{} \\
\frac{1}{4} = \frac{1 \times \boxed{}}{4 \times \boxed{}} = \boxed{}
\]

Subtract.

\[
\boxed{} - \boxed{} = \boxed{}
\]

Jillian drank ________ of the milk in the glass.
To multiply a fraction by a fraction, multiply the numerators and then multiply the denominators. Write the product in simplest form. For example:

\[
\frac{3}{4} \times \frac{2}{5} = \frac{3 \times 2}{4 \times 5} = \frac{6}{20} = \frac{6 \div 2}{20 \div 2} = \frac{3}{10}
\]

When both factors in a multiplication sentence are fractions less than 1, the product is less than either of its factors. Because a fraction is a part of a whole and the product is a part of a factor, the product must be less than either factor.

**Example 1**

The product of \(\frac{3}{4} \times \frac{1}{2}\) is less than \(\frac{3}{4}\). Without multiplying, is the product greater than or less than \(\frac{1}{2}\)?

**Strategy**  Compare the factors to 1.

**Step 1**  Compare the factors \(\frac{3}{4}\) and \(\frac{1}{2}\) to 1.

\[
\frac{3}{4} < 1 \quad \text{and} \quad \frac{1}{2} < 1
\]

**Step 2**  Compare the factors to the product.

Both factors are fractions less than 1.

So, the product must be less than either factor.

The product is less than \(\frac{3}{4}\).

The product must also be less than \(\frac{1}{2}\).

**Solution**  The product is less than \(\frac{1}{2}\).

When a whole number is multiplied by a fraction greater than 1, the product is greater than the whole number. When a whole number is multiplied by a fraction less than 1, the product is less than the whole number.
Example 2
The length of Jeff’s new garden is $2\frac{1}{2}$ times its old length. The old garden was 8 feet long. What is the length of the new garden?

Strategy  Multiply a whole number by a mixed number.

**Step 1**  Compare the factors 8 and $2\frac{1}{2}$ to 1.

- $8 > 1$ and $2\frac{1}{2} > 1$
- So, the product will be greater than the whole number factor, 8.
- The new garden is longer than the old garden.

**Step 2**  Multiply the fractions.

$$8 \times 2\frac{1}{2} = \frac{8}{1} \times \frac{5}{2}$$

Write each factor as an improper fraction.

$$= \frac{8 \times 5}{1 \times 2}$$

Multiply the numerators and the denominators.

$$= \frac{40}{2}$$

Simplify.

$$= 20$$

Write the product in simplest form.

**Solution**  The new garden is 20 feet long.

When a mixed number is multiplied by a fraction less than 1, the product is less than the mixed number. When a mixed number is multiplied by a fraction greater than 1, the product is greater than the mixed number.

Example 3
A recipe calls for $6\frac{1}{2}$ cups of vegetable stock to make soup. A chef needs to make $\frac{3}{4}$ of the recipe. How much vegetable stock does she need?

Strategy  Multiply a mixed number by a fraction.

**Step 1**  Compare the factors $6\frac{1}{2}$ and $\frac{3}{4}$ to 1.

- $6\frac{1}{2} > 1$ and $\frac{3}{4} < 1$
- The product will be less than the mixed number factor, $6\frac{1}{2}$.
- The chef needs less than $6\frac{1}{2}$ cups of stock.
Lesson 21: Understand Multiplication of Fractions

Step 2  Multiply the fractions.

\[ 6 \frac{1}{2} \times \frac{3}{4} = \frac{13}{2} \times \frac{3}{4} \]
Write the mixed number as an improper fraction.

\[ = \frac{13 \times 3}{2 \times 4} \]
Multiply the numerators and the denominators.

\[ = \frac{39}{8} \]
Simplify.

\[ = 4 \frac{7}{8} \]
Write the product in simplest form.

Solution  She needs \( \frac{47}{8} \) cups of stock.

Coached Example

A spool has \( 8 \frac{3}{4} \) feet of ribbon. Amber needs \( 1 \frac{1}{3} \) times the ribbon that is on the spool. How much ribbon does she need?

To decide if the product is greater or less than \( 8 \frac{3}{4} \) feet, compare the factors ______ and ______ to 1.

Compare: \( 8 \frac{3}{4} \) _____ 1 and \( 1 \frac{1}{3} \) _____ 1.

The product of \( 8 \frac{3}{4} \times 1 \frac{1}{3} \) is __________________ than \( 8 \frac{3}{4} \).

Convert \( 8 \frac{3}{4} \) to an improper fraction. ____________

Convert \( 1 \frac{1}{3} \) to an improper fraction. ____________

Multiply \( 8 \frac{3}{4} \times 1 \frac{1}{3} \).

In simplest form, \( 8 \frac{3}{4} \times 1 \frac{1}{3} = \) ____________.

Amber needs ____________ feet of ribbon.
When you multiply a fraction by a whole number, first rename the whole number as an improper fraction with a denominator of 1.

Example 1
Isabella drank $\frac{2}{3}$ quart of iced tea each day for 5 days. How many quarts of iced tea did she drink in all?

**Strategy** Convert the whole number to an improper fraction.

**Step 1** Write an equation for the problem.
Let $n$ represent the number of quarts she drank in all.
$$5 \times \frac{2}{3} = n$$

**Step 2** Rename the whole number as an improper fraction.
$$5 = \frac{5}{1}$$

**Step 3** Multiply the numerators and the denominators.
$$\frac{5}{1} \times \frac{2}{3} = \frac{5 \times 2}{1 \times 3} = \frac{10}{3}$$

**Step 4** Use models to check.

```
\[
\begin{array}{cccc}
\Box & \Box & \Box & \Box \\
\Box & \Box & \Box & \Box \\
\Box & \Box & \Box & \Box \\
\Box & \Box & \Box & \Box \\
\end{array}
\]
```

**Step 5** Convert the improper fraction to a mixed number in simplest form.
$$\frac{10}{3} = 3 R1 \text{ or } 3\frac{1}{3}$$

**Solution** Isabella drank $3\frac{1}{3}$ quarts of iced tea in 5 days

Remember that when multiplying fractions that are between 0 and 1, the product will be less than either of the fractions.
Example 2
In a flower garden, $\frac{3}{5}$ of the flowers are tulips. Of the tulips, $\frac{1}{2}$ are yellow tulips.
What fraction of the flowers are yellow tulips?

**Strategy**  Use a model.

**Step 1**  Make a model to show $\frac{3}{5}$.

**Step 2**  Divide and shade the rectangle to show $\frac{1}{2}$.

**Step 3**  Write a fraction to show the parts that were shaded twice.

3 parts were shaded twice out of 10 equal parts.
So, $\frac{3}{10}$ of the parts are shaded twice.

**Solution**  $\frac{3}{10}$ of the flowers are yellow tulips.

Example 3
A scientist mixed $\frac{3}{8}$ liter of salt water. He used $\frac{1}{3}$ of the salt water for an experiment. How much salt water did he use?

**Strategy**  Multiply the fractions.

**Step 1**  Write an equation for the problem.
Let $w$ represent the amount of salt water the scientist used.

$$\frac{3}{8} \times \frac{1}{3} = w$$

**Step 2**  Multiply the numerators and the denominators.

$$\frac{3}{8} \times \frac{1}{3} = \frac{3 \times 1}{8 \times 3} = \frac{3}{24}$$

**Step 3**  Write the product in simplest form.

$$\frac{3}{24} = \frac{3 \div 3}{24 \div 3} = \frac{1}{8}$$

**Solution**  The scientist used $\frac{1}{8}$ liter of salt water.
To find the **area** of a rectangle, multiply the length times the width and express the area in **square units**.

![Rectangle formula](image)

**Example 4**

A rectangle has a length of \( \frac{3}{4} \) foot and a width of \( \frac{2}{3} \) foot. What is the area of the rectangle?

**Strategy** Use a model.

**Step 1**

Draw a model.

![Rectangle model](image)

**Step 2**

Multiply the numerators and denominators.

Substitute the values for \( l \) and \( w \) into the formula \( A = l \times w \).

\[
A = \frac{3}{4} \text{ ft} \times \frac{2}{3} \text{ ft} = \frac{3 \times 2}{4 \times 3} = \frac{6}{12} \text{ square foot}
\]

**Step 3**

Write the product in simplest form.

\[
\frac{6}{12} = \frac{6 \div 6}{12 \div 6} = \frac{1}{2}
\]

**Solution** The area of the rectangle is \( \frac{1}{2} \) square foot.

To multiply mixed numbers, convert the mixed numbers to improper fractions. Multiply the numerators, multiply the denominators, then simplify the product.

**Example 5**

Asia needs \( 2 \frac{1}{4} \) cups of flour for each batch of cookies she makes. How many cups of flour will she need for \( 3 \frac{1}{2} \) batches of cookies?

**Strategy** Convert the mixed numbers to improper fractions and multiply.
Lesson 22: Multiply Fractions

Step 1  Write an equation for the problem.

Let \( f \) represent the total number of cups of flour.

\[
2\frac{1}{4} \times 3\frac{1}{2} = f
\]

Step 2  Convert the mixed numbers to improper fractions.

\[
2\frac{1}{4} = \frac{4}{4} + \frac{1}{4} = \frac{9}{4}
\]

\[
3\frac{1}{2} = \frac{2}{2} + \frac{1}{2} = \frac{7}{2}
\]

Step 3  Multiply the fractions.

\[
\frac{9}{4} \times \frac{7}{2} = \frac{63}{8}
\]

Step 4  Simplify the product.

\[
\frac{63}{8} = 7\frac{7}{8}
\]

Solution  Asia needs 7\(\frac{7}{8}\) cups of flour to make 3\(\frac{1}{2}\) batches of cookies.

You can use the properties of operations to make computation easier.

<table>
<thead>
<tr>
<th>Multiplicative identity property of 1</th>
<th>(a \times 1 = 1 \times a = a)</th>
<th>(\frac{5}{6} \times 1 = 1 \times \frac{5}{6} = \frac{5}{6})</th>
</tr>
</thead>
<tbody>
<tr>
<td>The product of any number and 1 is that number.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Commutative property of multiplication</th>
<th>(a \times b = b \times a)</th>
<th>(\frac{2}{3} \times \frac{1}{5} = \frac{1}{5} \times \frac{2}{3})</th>
</tr>
</thead>
<tbody>
<tr>
<td>The order of factors can be changed. The product does not change.</td>
<td></td>
<td>(\frac{2}{15} = \frac{2}{15})</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Associative property of multiplication</th>
<th>((a \times b) \times c = a \times (b \times c))</th>
<th>(\frac{3}{4} \times \left(\frac{1}{3} \times \frac{5}{6}\right) = \left(\frac{3}{4} \times \frac{1}{3}\right) \times \frac{5}{6})</th>
</tr>
</thead>
<tbody>
<tr>
<td>Factors can be grouped in different ways. The product does not change.</td>
<td></td>
<td>(\frac{3}{4} \times \left(\frac{1}{3} \times \frac{5}{6}\right) = \frac{15}{72})</td>
</tr>
</tbody>
</table>

\[
\frac{15}{72} \div \frac{3}{3} = \frac{5}{24}
\]
Example 6
Multiply: \(\frac{3}{4} \times \frac{5}{8} \times \frac{4}{5}\)

Strategy
Use the associative property of multiplication.

Step 1
Use the associative property of multiplication.
\[
\frac{3}{4} \times \frac{5}{8} \times \frac{4}{5} = \frac{3}{4} \times \left(\frac{5}{8} \times \frac{4}{5}\right)
\]

Step 2
Multiply inside the parentheses. Write the product in simplest form.
\[
\frac{5}{8} \times \frac{4}{5} = \frac{20}{40}
\]
\[
\frac{20}{40} = \frac{20}{20} \div \frac{20}{20} = \frac{1}{2}
\]

Step 3
Multiply the product times the other factor.
\[
\frac{3}{4} \times \frac{1}{2} = \frac{3}{8}
\]

Solution
\[
\frac{3}{4} \times \frac{5}{8} \times \frac{4}{5} = \frac{3}{8}
\]

Coached Example
A rectangle has a length of \(\frac{5}{6}\) foot and a width of \(\frac{1}{4}\) foot.

What is the area of the rectangle?

The formula for the area of a rectangle is \(A = \) ____________.

\[
A = \boxed{X} \times \boxed{Y}
\]

Multiply the numerators. \(\boxed{X} \times \boxed{Y} = \boxed{Z}\)

Multiply the denominators. \(\boxed{X} \times \boxed{Y} = \boxed{Z}\)

The product is \(\boxed{Z}\).

The area of the rectangle is \(\boxed{Z}\) square foot.
A fraction is related to division. The fraction \( \frac{2}{3} \) is the same as 2 divided by 3.

Remember that division and multiplication are inverse operations. Since \( 2 \div 3 = \frac{2}{3} \), then \( \frac{2}{3} \times 3 = 2 \).

**Example 1**
Show \( \frac{3}{4} \) as division, and show that \( \frac{3}{4} \times 4 = 3 \).

**Strategy** Use fraction strips.

**Step 1** Show the fraction as division.
\[
\frac{3}{4} = 3 \div 4
\]

**Step 2** Use fraction strips to model 3 divided by 4.

**Step 3** Show that \( \frac{3}{4} \times 4 = 3 \).

The model above shows 3 \( \div \) 4. It also shows that \( \frac{3}{4} \times 4 = 3 \), as you can see below.

**Solution** \( 3 \div 4 = \frac{3}{4} \) and \( \frac{3}{4} \times 4 = 3 \)
Example 2
Dan had 2 feet of ribbon. He cut the ribbon into 5 equal pieces. How long is each piece of ribbon?

Strategy  Use a model.

Step 1  Draw 2 wholes to represent 2 feet of ribbon. Each whole represents 1 foot.

Step 2  Divide 2 wholes into 5 equal pieces.

\[ \frac{2}{5} \]

Step 3  What fraction of the whole is each part?

Each part is \( \frac{2}{5} \).

Solution  Each piece of ribbon is \( \frac{2}{5} \) foot.

In Example 2, use multiplication to check that \( 2 \div 5 = \frac{2}{5} \).

\[ \frac{2}{5} \times 5 = \frac{2}{5} \times \frac{5}{1} = \frac{2 \times 5}{5 \times 1} = \frac{10}{5} = 2 \]

Example 3
Mohini has 60 ounces of milk. She wants to give 8 friends the same amount of milk. How many ounces of milk should each friend get? Between what two whole numbers does the answer lie?

Strategy  Write a division sentence, then divide.

Step 1  Write an equation for the problem.

Let \( o \) represent the number of ounces of milk each friend will get.

\[ 60 \div 8 = o \]
Step 2    Divide.

\[ 60 \div 8 = 7 \text{ R}4 \]

Step 3    Write the quotient as a mixed number. Then simplify.

\[ 7 \text{ R}4 \rightarrow 7 \frac{4}{8} \quad 7 \frac{4}{8} = 7 \frac{1}{2} \]

Step 4    What two whole numbers does the answer lie between?

\[ 7 \frac{1}{2} \text{ is between 7 and 8.} \]

Solution    Each friend should get \( 7 \frac{1}{2} \) ounces of milk. The answer is between the whole numbers 7 and 8.

---

**Coached Example**

If 4 friends want to share 45 ounces of jelly beans equally by weight, how many ounces should each friend get? Between what two whole numbers does your answer lie?

Write a division sentence for the problem.

Let \( o \) represent the number of ounces of jelly beans each friend will get.

\[ \underline{\quad} \div \underline{\quad} = \underline{\quad} \]

Divide. Show your work.

\[ 4)45 \]

Write the quotient as a mixed number. \( \underline{\quad} \)

Is the mixed number in simplest form? \( \underline{\quad} \)

Each friend should get \( \underline{\quad} \) ounces of jelly beans. The answer lies between the whole numbers \( \underline{\quad} \) and \( \underline{\quad} \).
Divide Fractions

Getting the Idea
Fraction strips can help you divide a whole number by a fraction.

Example 1
Samara has 3 yards of ribbon. She wants to cut the ribbon into pieces that are each $\frac{1}{4}$ yard long. How many pieces of ribbon will Samara have?

Strategy Use fraction strips.

Step 1 Write an equation for the problem.
Let $p$ represent the number of $\frac{1}{4}$ yard long pieces.

$$3 \div \frac{1}{4} = p$$

Step 2 Use three whole fraction strips to model 3 yards.

Step 3 Use $\frac{1}{4}$ fraction strips below the wholes to model $\frac{1}{4}$ yard.

Step 4 Count the number of $\frac{1}{4}$ fraction strips.
There are 12.

$$3 \div \frac{1}{4} = 12$$

Solution Samara will have 12 pieces of $\frac{1}{4}$ yard long ribbon.

Dividing a whole number by a fraction is the same as multiplying by the reciprocal of the divisor. To find the reciprocal, switch the numerator and the denominator of the divisor. In Example 1, the reciprocal of $\frac{1}{4}$ is $\frac{4}{1}$, or 4, so $3 \div \frac{1}{4} = 3 \times 4 = 12$. 

Domain 3 • Lesson 24
Getting the Idea • Divide Fractions

Domain 3 • Lesson 24
Getting the Idea • Divide Fractions

Domain 3 • Lesson 24
Getting the Idea • Divide Fractions
In a unit fraction, which is a fraction with a numerator of 1, the reciprocal will always be a whole number. When you divide a whole number by a unit fraction, the quotient will always be a greater whole number than the dividend.

Example 2
Caleb bought 2 pounds of trail mix in a large bag. He wants to put $\frac{1}{8}$ pound of trail mix into small bags. How many small bags of trail mix will Caleb have?

**Strategy**  Write the reciprocal of the divisor and multiply.

**Step 1**  Write an equation for the problem.
Let $b$ represent the number of small bags of trail mix.

\[2 \div \frac{1}{8} = b\]

**Step 2**  Write the reciprocal of $\frac{1}{8}$.
Switch the numerator and denominator of $\frac{1}{8}$.

\[\frac{8}{1} \text{ or } 8\]

**Step 3**  Multiply 2 by the reciprocal.

\[2 \times 8 = 16\]
\[2 \div \frac{1}{8} = 16\]

**Solution**  Caleb will have 16 small bags of trail mix.

You can use multiplication to check division. In Example 2, multiply the quotient by the divisor.

\[16 \times \frac{1}{8} = \frac{16}{1} \times \frac{1}{8} = \frac{16}{8} = 2\]

Since $16 \times \frac{1}{8} = 2$, the quotient is correct.
When you divide a unit fraction by a whole number, the quotient will always be a unit fraction less than the dividend. For example, \( \frac{1}{2} \div 5 = \frac{1}{10} \) because when \( \frac{1}{2} \) is divided into 5 equal parts, the size of each part is \( \frac{1}{10} \).

**Example 3**

What is \( \frac{1}{6} \div 2 \)?

**Strategy**  
Write the reciprocal of the divisor and multiply.

**Step 1**  
Write the reciprocal of 2.  
The reciprocal of 2 is \( \frac{1}{2} \).

**Step 2**  
Multiply \( \frac{1}{6} \) by the reciprocal.  
\[
\frac{1}{6} \times \frac{1}{2} = \frac{1}{12}
\]

**Solution**  
\( \frac{1}{6} \div 2 = \frac{1}{12} \)

**Example 4**

What is the missing number?

\[
\square \div \frac{1}{3} = 15
\]

**Strategy**  
Use the inverse operation of division.

**Step 1**  
Multiplication and division are inverse operations.  
Since \( \square \div \frac{1}{3} = 15 \), \( 15 \times \frac{1}{3} = \square \).

**Step 2**  
Multiply.  
\[
15 \times \frac{1}{3} = \frac{15}{1} \times \frac{1}{3} = \frac{15}{3} = 5
\]
So \( \square = 5 \).

**Step 3**  
Check your answer.  
\[
\square \div \frac{1}{3} = 15
\]
\[
5 \div \frac{1}{3} = 5 \times \frac{3}{1} = \frac{5}{1} \times \frac{3}{1} = \frac{15}{1} = 15
\]

**Solution**  
The missing number is 5.
Lesson 24: Divide Fractions

Lucy baked a loaf of banana bread. She took half of the loaf to school. She wants to share the banana bread equally among 6 friends. What fraction of the loaf of banana bread will each friend get?

Write an equation for the problem. ___________________________

When dividing a unit fraction by a whole number, the quotient is always a unit fraction _________________ than the dividend.

Divide $\frac{1}{2}$ by 6 to find the size of each slice of banana bread.

To divide fractions, multiply the dividend by the _________________ of the divisor.

The reciprocal of 6 is _________.

Multiply.

$$\frac{1}{2} \times \text{______} = \text{______}$$

Each friend will get _________ of the banana bread.
When you want to know how long or tall something is, you measure its **length**. Units of length in the customary system include **inches**, **feet**, **yards**, and **miles**.

When measuring the length of an object, more units are needed when smaller units are used. For example, a piece of paper that is 12 inches long also has a length of 1 foot. More inches than feet are used to measure the length of the paper. This is because a foot is a longer unit than an inch.

You can convert units if you know their equivalent measures. For example, since there are 24 hours in a day, 48 hours is equivalent to 2 days. The table shows the conversions for length in the customary system.

<table>
<thead>
<tr>
<th>Customary Units of Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 foot (ft) = 12 inches (in.)</td>
</tr>
<tr>
<td>1 yard (yd) = 3 feet</td>
</tr>
<tr>
<td>1 mile (mi) = 1,760 yards</td>
</tr>
</tbody>
</table>

To convert a smaller unit to a larger unit, divide.

**Example 1**

Ms. Richards’s car is 198 inches long. How many feet is that?

**Strategy**  Divide to convert a smaller unit to a larger unit.

**Step 1**  Write the relationship between feet and inches.

\[
1 \text{ foot} = 12 \text{ inches}
\]

**Step 2**  Divide the number of inches by 12 to find the number of feet.

\[
198 \div 12 = 16 \text{ R}6
\]

**Step 3**  Interpret the remainder.

The remainder means there are 6 inches left over.

**Solution**  Ms. Richards’s car is 16 feet 6 inches long.

If the quotient of 198 ÷ 12 is written with the remainder as a fraction, the quotient is 16\(\frac{1}{2}\). So the length of the car can also be written as 16\(\frac{1}{2}\) feet, because 6 inches is \(\frac{1}{2}\) foot.
When you want to know how heavy something is, you measure its **weight**. Units of weight in the customary system include **ounces**, **pounds**, and **tons**.

The table shows the conversions for weight in the customary system.

<table>
<thead>
<tr>
<th>Customary Units of Weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 pound (lb) = 16 ounces (oz)</td>
</tr>
<tr>
<td>1 ton (T) = 2,000 pounds</td>
</tr>
</tbody>
</table>

To convert a larger unit to a smaller unit, multiply.

**Example 2**

Henry weighed 7 pounds 9 ounces when he was born. How many ounces is that?

**Strategy**  Multiply to convert a larger unit to a smaller unit. Then add.

**Step 1**  Write the relationship between ounces and pounds.

\[ 1 \text{ pound} = 16 \text{ ounces} \]

**Step 2**  Multiply the number of pounds by 16.

\[ 7 \times 16 = 112 \text{ ounces} \]

**Step 3**  Add the extra ounces to the product

\[ 112 + 9 = 121 \text{ ounces} \]

**Solution**  Henry weighed 121 ounces.

**Capacity**  measures the amount of dry or liquid volume a container can hold. Units of capacity in the customary system include **fluid ounces**, **cups**, **pints**, **quarts**, and **gallons**.

The table shows the conversions for capacity in the customary system. Fluid ounces are not the same as ounces, although they are often called ounces.

<table>
<thead>
<tr>
<th>Customary Units of Capacity</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 cup (c) = 8 fluid ounces (fl oz)</td>
</tr>
<tr>
<td>1 pint (pt) = 2 cups</td>
</tr>
<tr>
<td>1 quart (qt) = 2 pints</td>
</tr>
<tr>
<td>1 gallon (gal) = 4 quarts</td>
</tr>
</tbody>
</table>
Example 3
Regina made 10 quarts of fruit punch. How many gallons of fruit punch did she make?

Strategy   Divide to convert a smaller unit to a larger unit.

Step 1   Write the relationship between gallons and quarts.
          \[1 \text{ gallon} = 4 \text{ quarts}\]

Step 2   Divide the number of quarts by 4.
          \[10 \div 4 = 2 \text{ R}2\]
          The remainder represents \(\frac{2}{4}\) of a gallon.

Step 3   Write the remainder in simplest form.
          \[\frac{2}{4} = \frac{1}{2}\]

Solution   Regina made \(2 \frac{1}{2}\) gallons of fruit punch.

Example 4
At lunch, a group of students drank 5 quarts of milk in all. Each student in the group drank 1 cup of milk. How many students were in the group?

Strategy   Multiply to convert a larger unit to a smaller unit.

Step 1   Write the relationship between quarts and pints.
          \[1 \text{ quart} = 2 \text{ pints}\]
          Multiply the number of quarts by 2.
          \[5 \times 2 = 10 \text{ pints}\]
          5 quarts = 10 pints

Step 2   Write the relationship between pints and cups.
          \[1 \text{ pint} = 2 \text{ cups}\]
          Multiply the number of pints by 2.
          \[10 \times 2 = 20 \text{ cups}\]
          10 pints = 20 cups

Solution   There were 20 students in the group.
Lesson 25: Convert Customary Units

Coached Example

Luanne needs to fill a pot with 1 gallon of water. She only has a 1-pint measuring cup. How many times must Luanne fill the 1-pint measuring cup to have 1 gallon of water?

Use the relationships between the different units to find how many times Luanne must fill the 1-pint measuring cup.

How many pints are in 1 quart? ____________

How many quarts are in 1 gallon? ____________

Multiply to find how many pints are equal to 1 gallon.

___________ \times ___________ = ___________

Luanne must fill the 1-pint measuring cup ___________ times to have 1 gallon of water.
Units of length in the metric system include **millimeters**, **centimeters**, **meters**, and **kilometers**.

When measuring the length of an object, more units are needed when smaller units are used. For example, a desk that has a length of 1 meter also has a length of 100 centimeters. It takes more centimeters than meters to measure the length of the desk because a centimeter is a shorter unit than a meter.

You can convert units if you know their equivalent measures. The table shows the conversions for length in the metric system.

### Metric Units of Length

<table>
<thead>
<tr>
<th>Conversion</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 centimeter (cm) = 10 millimeters (mm)</td>
</tr>
<tr>
<td>1 meter (m) = 100 centimeters</td>
</tr>
<tr>
<td>1 kilometer (km) = 1,000 meters</td>
</tr>
</tbody>
</table>

**Example 1**
Sanjay lives 3 kilometers from school. How many meters does he live from school?

**Strategy**  
Multiply to convert a larger unit to a smaller unit.

**Step 1**  
Write the relationship between kilometers and meters.  
1 kilometer = 1,000 meters

**Step 2**  
Multiply the number of kilometers by 1,000.  
3 × 1,000 = 3,000 meters

**Solution**  
Sanjay lives 3,000 meters away from school.
Example 2
Benny cut a piece of string that is 2 meters long. Rina cut a piece of string that is 80 centimeters long. How many centimeters longer is Benny’s piece of string than Rina’s?

**Strategy**  Convert the units to centimeters, and then subtract.

**Step 1**  Convert 2 meters to centimeters.

\[100 \text{ centimeters} = 1 \text{ meter}\]

There are 200 centimeters in 2 meters since \(100 \times 2 = 200\).

**Step 2**  Subtract 80 centimeters from 200 centimeters.

\[200 - 80 = 120\]

**Solution**  Benny’s string is 120 centimeters longer than Rina’s.

Mass is the measure of how much matter an object has. Unlike weight, which can change according to gravity, mass never changes. Mass can be measured in milligrams, grams, kilograms, and metric tons in the metric system. As with weight, you can use a balance or a scale to measure mass. The table shows the conversions for mass in the metric system.

<table>
<thead>
<tr>
<th>Metric Units of Mass</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 gram (g) = 1,000 milligrams (mg)</td>
</tr>
<tr>
<td>1 kilogram (kg) = 1,000 grams</td>
</tr>
<tr>
<td>1 metric ton (t) = 1,000 kilograms</td>
</tr>
</tbody>
</table>

Example 3
A book has a mass of 690 grams. What is the mass of the book in kilograms?

**Strategy**  Divide to convert a smaller unit to a larger unit.

**Step 1**  Write the relationship between kilograms and grams.

\[1 \text{ kilogram} = 1,000 \text{ grams}\]

**Step 2**  Divide the number of grams by 1,000.

Dividing by 1,000 is the same as moving the decimal point 3 places to the left.

\[690 \div 1000 = 0.690 = 0.69 \text{ kilogram}\]

**Solution**  The book has a mass of 0.69 kilogram.
Example 4

Ling bought a 450-gram box of strawberries, a 2.2-kilogram watermelon, and 0.75 kilogram of apples. What is the total mass, in grams, of the fruit that Ling bought?

**Strategy** Convert the units to grams. Then add.

**Step 1** Find the mass of the strawberries in grams.

The strawberries have a mass of 450 grams.

**Step 2** Find the mass of the watermelon in grams.

1 kilogram = 1,000 grams

Multiply the number of kilograms of the watermelon by 1,000.

\[2.2 \times 1,000 = 2,200\] grams

The watermelon has a mass of 2,200 grams.

**Step 3** Find the mass of the apples in grams.

Multiply the number of kilograms of apples by 1,000.

\[0.75 \times 1,000 = 750\] grams

The apples have a mass of 750 grams.

**Step 4** Find the total mass of the fruit.

Add the masses.

\[450 + 2,200 + 750 = 3,400\]

**Solution** The total mass of the fruit is 3,400 grams.

Metric units of capacity include **milliliters** and **liters**. The table shows the conversions for capacity in the metric system.

**Metric Units of Capacity**

\[
\begin{align*}
1 \text{ liter (L)} &= 1,000 \text{ milliliters (mL)}
\end{align*}
\]
Example 5
Quinn’s punch bowl has a capacity of 575 milliliters. How many liters is that?

Strategy  Multiply to convert a larger unit to a smaller unit.

Step 1  Write the relationship between liters and milliliters.
        1 liter = 1,000 milliliters

Step 2  Divide the number of milliliters by 1,000.
        575 ÷ 1,000 = 0.575

Solution  Quinn’s punch bowl has a capacity of 0.575 liter.

Coached Example

Alex wants to drink 2 liters of water today. So far, he has drunk five 250-milliliter glasses of water. How many more milliliters of water does Alex need to drink today to reach his goal?

Use the relationship between liters and milliliters.

1 liter = ___________ milliliters, so 2 liters = ___________ milliliters

Alex wants to drink ___________ milliliters of water today.

The amount of water Alex drank so far can be found by multiplying ____ × ____.

How many milliliters of water did Alex drink so far? ______________

Subtract: _________ mL − _________ mL = _________ mL

Alex needs to drink ___________ milliliters more of water today to reach his goal.
Recall that capacity is a measure of how much a container can hold. Capacity is measured in units such as cups and milliliters. The **volume** of a three-dimensional figure is the number of **cubic units** that fit inside it. A cubic unit is a **cube** with each **edge** measuring 1 unit. For the cube below, let $u$ represent 1 unit. The volume of this cube can then be expressed as $u^3$. The notation $u^3$ means $u \times u \times u$.

Common units of volume are the cubic inch (in.$^3$) and the cubic centimeter (cm$^3$). When measuring the volume of an object, more units are needed when smaller cubic units are used.

To find the volume of a rectangular prism or a cube, you can count the number of cubic units that would fit inside the figure.
Example 1
What is the volume of the rectangular prism in cubic units?

![Rectangular prism]

**Strategy** Find the area of each layer. Then multiply by the number of layers.

**Step 1** Find the number of cubes in the bottom layer.
There are 2 rows and 6 columns of cubes.

**Step 2** Find the area of the bottom layer.
\[2 \times 6 = 12\]
The area of each layer is 12 square units.

**Step 3** Multiply the area of one layer by the number of layers.
There are 3 layers.
\[3 \times 12 = 36\]
Since each cube represents 1 cubic unit, the volume is 36 cubic units.

**Solution** The volume of the rectangular prism is 36 cubic units.

You can also find the volume of a rectangular prism by multiplying the number of cubic units needed to cover the base by the number of layers of cubes needed to fill the height of the prism.

The number of cubes needed to cover the base tells you the area of the base. This is the same as multiplying the edge lengths to find the area of the base. Then multiply the area of the base by the height of the rectangular prism to find the volume of the prism. Remember, area is the number of square units needed to cover a two-dimensional figure.
Example 2
The rectangular prism below has a height of 10 centimeters.

What is the volume of the rectangular prism in cubic centimeters?

**Strategy** Use cubes to find the area of the base. Then multiply the area of the base by the height of the prism.

**Step 1** Find the area of the bottom layer.

There are 8 rows and 15 columns in the bottom layer.
Multiply: $8 \times 15 = 120$

The area of the base of the prism is 120 square centimeters.
Step 2  Multiply the area of the base by the height of the prism.

The area of the base is 120 square centimeters.
The height of the prism is 10 centimeters.

\[ 120 \times 10 = 1,200 \]

The volume is 1,200 cubic centimeters.

Solution  The volume of the rectangular prism is 1,200 cubic centimeters.

Coached Example

The first layer of a box is filled with cubes. The height of the prism is 6 centimeters.

What is the volume of the box?

Find the number of 1-centimeter cubes in the bottom layer.
There are _________ rows and _________ columns of cubes in the bottom layer.
Multiply to find the total number of cubes in the bottom layer.

\[ \underline{\text{________}_1} \times \underline{\text{________}_2} = \underline{\text{________}_3} \]

The area of the base of the prism is _________ square centimeters.

Multiply the area of the base by the height of the prism.
The height of the prism is _________ centimeters.

\[ \underline{\text{________}_4} \times 6 = \underline{\text{________}_5} \]

The volume of the cube is _________ cubic centimeters.
To find the volume of a rectangular prism you can use the formula \( V = lwh \), where \( l \) is the length, \( w \) is the width, and \( h \) is the height.

**Example 1**

What is the volume of the fish tank?

![Fish tank diagram](image)

**Strategy** Use the formula \( V = lwh \) for the volume of a rectangular prism.

**Step 1** Substitute the values into the formula \( V = lwh \).

\[
V = 4 \times 2 \times 3
\]

**Step 2** Multiply using the associative property.

\[
V = 4 \text{ ft} \times 2 \text{ ft} \times 3 \text{ ft}
\]

\[
= 8 \text{ ft}^2 \times 3 \text{ ft}
\]

\[
= 24 \text{ ft}^3
\]

**Solution** The volume of the fish tank is 24 cubic feet.

Another formula that you can use to find the volume of a rectangular prism is \( V = Bh \), where \( B \) is the area of the base and \( h \) is the height of the prism. No matter which of the two formulas you use, the result will be the same.
Example 2
A storage unit in the shape of a rectangular prism has a length of 12 feet, a width of 8 feet, and a height of 8 feet. What is the volume of the storage unit?

Strategy  Use the formula $V = Bh$ for the volume of a rectangular prism.

Step 1  Find the area of the base.

$B = 12 \times 8$

$= 96$ square feet

Step 2  Multiply the area of the base times the height.

$V = 96 \text{ ft}^2 \times 8 \text{ ft}$

$= 768 \text{ ft}^3$

Solution  The volume of the storage unit is 768 cubic feet.

A cube is a rectangular prism with square faces. To find the volume of a cube, you can use the formula $V = e^3$, where $e$ is the length of each edge of the cube.

Example 3
What is the volume of this cube?

Strategy  Use the formula for the volume of a cube.

Step 1  Substitute the value into the formula

$V = e^3$

$V = 5 \text{ cm} \times 5 \text{ cm} \times 5 \text{ cm}$

Step 2  Multiply using the associative property.

$V = 25 \text{ cm}^2 \times 5 \text{ cm}$

$V = 125 \text{ cm}^3$

Solution  The volume of the cube is 125 cubic centimeters.
Example 4
What is the volume of this solid figure?

Strategy  Separate the figure into rectangular prisms and find the volume of each part.

Step 1  Separate the figure into two rectangular prisms.

Step 2  Find the volume of the larger prism.
\[ V = 15 \text{ m} \times 6 \text{ m} \times 9 \text{ m} \]
\[ = 90 \text{ m}^2 \times 9 \text{ m} \]
\[ = 810 \text{ m}^3 \]

Step 3  Find the volume of the smaller prism.
\[ V = 6 \text{ m} \times 6 \text{ m} \times 6 \text{ m} \]
\[ = 36 \text{ m}^2 \times 6 \text{ m} \]
\[ = 216 \text{ m}^3 \]

Step 4  Add the volumes.
\[ 810 \text{ m}^3 + 216 \text{ m}^3 = 1,026 \text{ m}^3 \]

Solution  The volume of the figure is 1,026 cubic meters.
Lesson 28: Volumes of Rectangular Prisms

Coached Example

This figure is made up of 1-foot cubes.

What is the volume of the figure?

Separate the figure into 2 rectangular prisms, one on the left and one on the right.

Use the formula for finding the volume of a rectangular prism.

\[ V = \text{length} \times \text{width} \times \text{height} \]

Start with the prism on the left.

The length is ________ feet. The width is ________ feet. The height is ________ feet.

Substitute the values into the formula.

\[ V = \text{length} \times \text{width} \times \text{height} = \text{cubic feet} \]

The volume of the prism on the left is ________________________.

Next find the volume of the prism on the right.

The length is ________ feet. The width is ________ feet. The height is ________ feet.

Substitute the values into the formula.

\[ V = \text{length} \times \text{width} \times \text{height} = \text{cubic feet} \]

The volume of the prism on the right is _______________________ cubic feet.

Add to find the total volume.

\[ \text{cubic feet} + \text{cubic feet} = \text{cubic feet} \]

The volume of the figure is ____________________.
A line plot uses a number line and Xs or dots to organize data. The number of Xs above each number indicates how many times that value occurs in a data set.

Example 1
The line plot shows beakers of water in a lab.

Water in Beakers (in pints)

0 1 2 3 4 5 6 7 8

How many beakers contain less than $\frac{1}{2}$ pint of water?

Strategy  Find the total number of beakers that contain less than $\frac{1}{2}$ pint of water.

Step 1  Look at the line plot to find all the fractions that are less than $\frac{1}{2}$.

$\frac{1}{2} = \frac{4}{8}$

So 0, $\frac{1}{8}$, $\frac{2}{8}$, and $\frac{3}{8}$ are less than $\frac{4}{8}$.

Step 2  Identify the number of Xs above $\frac{1}{8}$, $\frac{2}{8}$, and $\frac{3}{8}$ pints.

There are no Xs above 0.

There are 3 Xs above $\frac{1}{8}$.

There are 4 Xs above $\frac{2}{8}$.

There is 1 X above $\frac{3}{8}$.

Step 3  Add to find the total number of beakers.

$0 + 3 + 4 + 1 = 8$

Solution  8 beakers contain less than $\frac{1}{2}$ pint of water.
Data can be collected using a line plot.

**Example 2**
The weights, in pounds, of different amounts of bagged trail mix that were sold at a health food store is shown below.

\[ \frac{1}{4}, \frac{1}{2}, \frac{1}{4}, \frac{1}{2}, \frac{1}{4}, \frac{1}{2}, \frac{1}{2}, \frac{3}{4}, 1, \frac{3}{4}, \frac{1}{2}, \frac{1}{2}, \frac{3}{4}, \frac{1}{2} \]

Make a line plot to represent the data.

**Strategy**  
Make an X for each value.

**Step 1**  
Draw a number line from 0 to 2 by fourths.

![Number line from 0 to 2 with marks at 1/4, 1/2, 3/4, 1, 1 1/4, 1 1/2, 1 3/4, 2]

**Step 2**  
Give the line plot a title.

**Step 3**  
Write an X for each value.

![Line plot with Xs at the appropriate weights]

**Solution**  
The line plot is shown in Step 3.
Example 3

Using the line plot in Example 2, find the total weight of the bags of trail mix that were sold.

![Trail Mix in Bags (in pounds)](image)

**Strategy**  Add to find the total number of pounds.

**Step 1**  Write an equation to represent the total number of pounds, $p$.  
\[
\frac{1}{4} + \left(2 \times \frac{1}{2}\right) + \frac{3}{4} + 1 + \left(2 \times \frac{11}{4}\right) + \left(4 \times \frac{1}{2}\right) + 1\frac{3}{4} = p
\]

**Step 2**  Rename the mixed numbers as improper fractions.  
\[
\frac{1}{4} + \left(2 \times \frac{1}{2}\right) + \frac{3}{4} + 1 + \left(2 \times \frac{5}{4}\right) + \left(4 \times \frac{3}{2}\right) + \frac{7}{4} = p
\]

**Step 3**  Work inside the parentheses.  
\[
\frac{1}{4} + \frac{2}{2} + \frac{3}{4} + 1 + \frac{10}{4} + \frac{12}{2} + \frac{7}{4} = p
\]

**Step 4**  Rename the addends using the common denominator, 4.  
\[
\frac{1}{4} + \frac{4}{4} + \frac{3}{4} + \frac{4}{4} + \frac{10}{4} + \frac{24}{4} + \frac{7}{4} = p
\]

**Step 5**  Add the numerators.  
\[
\frac{1}{4} + \frac{4}{4} + \frac{3}{4} + \frac{4}{4} + \frac{10}{4} + \frac{24}{4} + \frac{7}{4} = \frac{53}{4}
\]

**Step 6**  Write the sum as a mixed number.  
\[
\frac{53}{4} = 13\frac{1}{4}
\]

**Solution**  The total weight of the bags of trail mix was $13\frac{1}{4}$ pounds.
Example 4

The line plot shows beakers of liquid.

<table>
<thead>
<tr>
<th>Liquid in Beakers (in liters)</th>
</tr>
</thead>
<tbody>
<tr>
<td>X</td>
</tr>
<tr>
<td>X</td>
</tr>
<tr>
<td>X</td>
</tr>
<tr>
<td>X</td>
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<tr>
<td>X</td>
</tr>
<tr>
<td>X</td>
</tr>
<tr>
<td>X</td>
</tr>
</tbody>
</table>

How much liquid would be in each beaker if the total amount in all the beakers was redistributed equally?

**Strategy** Find the total amount of liquid. Then divide by the number of beakers.

**Step 1** Find the total number of liters.

\[
\begin{align*}
\frac{1}{10} + \left(2 \times \frac{2}{10}\right) + \frac{4}{10} + \left(2 \times \frac{5}{10}\right) + \frac{6}{10} + \left(2 \times \frac{8}{10}\right) + \frac{9}{10} \\
\frac{1}{10} + \frac{4}{10} + \frac{4}{10} + \frac{10}{10} + \frac{6}{10} + \frac{16}{10} + \frac{9}{10} = \frac{50}{10}
\end{align*}
\]

There are \(\frac{50}{10}\) liters of liquid in all the beakers.

**Step 2** Divide to redistribute the liquid equally.

There are 10 beakers, so divide by 10.

\[
\frac{50}{10} \div 10 = \frac{50}{10} \times \frac{1}{10} = \frac{50}{100}
\]

**Step 3** Simplify.

\[
\frac{50}{100} = \frac{50 \div 50}{100 \div 50} = \frac{1}{2}
\]

**Solution** There would be \(\frac{1}{2}\) liter in each beaker if the liquid was redistributed equally.
The line plot shows the weight of each piece of fruit that Logan bought today.

What is the total weight of the fruit that Logan bought?

Find the total weight of the pieces of fruit that weigh \( \frac{1}{4} \) pound.

\[ \text{_____} \times \frac{1}{4} = \text{_____} \]

Find the total weight of the pieces of fruit that weigh \( \frac{3}{8} \) pound.

\[ \text{_____} \times \frac{3}{8} = \text{_____} \]

Find the total weight of the pieces of fruit that weigh \( \frac{1}{2} \) pound.

\[ \text{_____} \times \frac{1}{2} = \text{_____} \]

Find the total weight of the pieces of fruit that weigh \( \frac{5}{8} \) pound.

\[ \text{_____} \times \frac{5}{8} = \text{_____} \]

Rename the products, so they all have a denominator of _____.

Add.

\[ \text{_____} + \text{_____} + \text{_____} + \text{_____} = \text{_____} \]

Write the sum as a mixed number in simplest form.

\[ \text{_____} = \text{_____} \]

The total weight of the fruit that Logan bought is _____ pounds.
An ordered pair is a pair of numbers used to locate a point on a coordinate plane. The left-right or horizontal number line on the coordinate plane is the $x$-axis. The up-down or vertical number line is the $y$-axis. The $x$-axis and $y$-axis are perpendicular to each other. The point where the axes meet is called the origin and is named by the ordered pair $(0, 0)$.

The first number in an ordered pair is the $x$-coordinate. The $x$-coordinate tells the distance from the origin along the $x$-axis.

The second number in an ordered pair is the $y$-coordinate. The $y$-coordinate tells the distance from the origin along the $y$-axis.

For example, the ordered pair $(3, 5)$ is shown on the coordinate plane above. The $x$-coordinate is 3 and the $y$-coordinate is 5.
Example 1
What is the location of point $P$?

Strategy  
Find the location of each coordinate.

**Step 1**  
Find the number directly below point $P$.  
5 is directly below point $P$.  
The $x$-coordinate is 5.

**Step 2**  
Find the number directly to the left of point $P$.  
2 is directly to the left of point $P$.  
The $y$-coordinate is 2.

**Step 3**  
Write the ordered pair.  
$(5, 2)$

Solution  
Point $P$ is located at $(5, 2)$.  

\[ P \]
Example 2

Three points are graphed on the coordinate plane below.

Which letter is not represented by an ordered pair that has the same $x$- and $y$-coordinates?

**Strategy**  Write the ordered pair for each point.

**Step 1** Write the ordered pair for point $A$.
- Point $A$ is 4 units to the right of 0.
- Point $A$ is 4 units above 0.
- Point $A$ is located at $(4, 4)$.

**Step 2** Write the ordered pair for point $B$.
- Point $B$ is at the origin, which is $(0, 0)$.

**Step 3** Write the ordered pair for point $C$.
- Point $C$ is 8 units to the right of 0.
- Point $C$ is 6 units above 0.
- Point $C$ is located at $(8, 6)$.

**Solution** Point $C$ does not have the same $x$- and $y$-coordinates.
Coached Example

Name the location of point $A$.

Place your finger on point $A$.

The number directly below point $A$ is ______.
This is the number of units to the ___________ of the origin.

The number directly to the left of point $A$ is ____.
This is the number of units ___________ the origin.

The ordered pair _____ gives the location of point $A$. 
The first number in an ordered pair is the $x$-coordinate. The second number is the $y$-coordinate. The point where the two axes meet is the origin (0, 0).

To locate a point on the part of the coordinate plane that is shown in Example 1, you follow these steps:

- Start at the origin.
- For the $x$-coordinate, move to the right.
- For the $y$-coordinate, move up.

You can plot a point on a coordinate plane using an ordered pair.

**Example 1**

Plot a point at (5, 8) on the coordinate plane.

**Strategy**  Use each number in the ordered pair to find the exact location on the coordinate plane.

**Step 1**  Start at the origin.

**Step 2**  Look at the $x$-coordinate and the $y$-coordinate.

The $x$-coordinate is 5, so move 5 units to the right of the origin.

The $y$-coordinate is 8, so move 8 units up.

**Step 3**  Label the point (5, 8).

**Solution**  The coordinate plane with a point plotted at (5, 8) is shown in Step 3.
To find the distance between two points, count the number of units to the left or right (west and east) and the number of units up or down (north or south).

**Example 2**
Emile made the map below.

![Map of city blocks with points labeled Post Office, Library, School, and Supermarket. The x-axis and y-axis are labeled with numbers from 0 to 10.](image)

What is the distance from the post office to the library?
You can only go right and left and up and down.

**Strategy**  Name the ordered pairs. Then use subtraction.

**Step 1** Find the location of the post office and the library.
- The post office is located at (4, 8).
- The library is located at (4, 1).

**Step 2** Find the distance between the post office and the library.
- Both the post office and the library have the same x-coordinate.
- They have different y-coordinates.

**Step 3** Subtract the y-coordinates of each ordered pair.
- \(8 - 1 = 7\)
- The post office is 7 city blocks from the library.

**Solution**  The distance from the post office to the library is 7 city blocks.
In Example 2, you could also have solved the problem by counting the units between the post office and the library. There are 7 units between the post office and the library.

You can use the lengths of line segments to find the perimeter and area.

**Example 3**

What is the area of rectangle $ABCD$?

**Strategy** Find the length of each side.

- **Step 1** Find the length of line segment $AB$.
  - Points $A$ and $B$ have the same $y$-coordinates.
  - Subtract the $x$-coordinates: $7 - 2 = 5$
  - The length of line segment $AB$ is 5 units.

- **Step 2** Find the length of line segment $BC$.
  - Points $B$ and $C$ have the same $x$-coordinates.
  - Subtract the $y$-coordinates: $8 - 2 = 6$
  - The length of line segment $BC$ is 6 units.

- **Step 3** Use the formula for the area of a rectangle.
  - $A = lw$
  - $A = 5 \times 6$
  - $A = 30$ square units

**Solution** The area of rectangle $ABCD$ is 30 square units.
Rectangle $JKLM$ is shown on the coordinate plane below.

What is the perimeter of rectangle $JKLM$?

Look at line segment $JK$.
Points $J$ and $K$ have the same _____-coordinates.
To find the length of line segment $JK$, subtract the _____-coordinates.

$$______ - ______ = ______$$

Look at line segment $KL$.
Points $K$ and $L$ have the same _____-coordinates.
To find the length of line segment $KL$, subtract the _____-coordinates.

$$______ - ______ = ______$$

Use the formula for the perimeter of a rectangle: $P = 2l + 2w$

$$P = 2 \times ______ + 2 \times ______$$

$$P = _____ + _____$$

$$P = _____ \text{ units}$$

The perimeter of rectangle $JKLM$ is ________ units.
A **two-dimensional figure** is a **plane figure**. A **polygon** is a closed plane figure with straight sides. A side is a **line segment**. A polygon is classified by its number of **sides**, **angles**, or **vertices**.

A **regular polygon** has all equal sides and all equal angles. Some regular polygons are shown below.

- **Triangle**: 3 sides, 3 angles
- **Square**: 4 sides, 4 angles
- **Pentagon**: 5 sides, 5 angles
- **Hexagon**: 6 sides, 6 angles
- **Heptagon**: 7 sides, 7 angles
- **Octagon**: 8 sides, 8 angles
- **Nonagon**: 9 sides, 9 angles
- **Decagon**: 10 sides, 10 angles

A **circle** is a plane figure with all points an equal distance from a point called the **center**. A circle is not a polygon because it does not have straight sides.
Example 1
How can you classify this polygon? Is the figure a regular polygon?

Strategy Count the number of sides.

Step 1 A side is a line segment.
There are 6 line segments.

Step 2 A regular polygon has all equal sides and equal angles.
The sides are different lengths and the angles have different measures.

Solution The figure is a hexagon, but it is not a regular hexagon.

An irregular polygon is a polygon that does not have all equal sides and all equal angles.

Example 2
Jerome saw this traffic sign while walking to school.

Is the sign a regular or irregular polygon?

Strategy Identify the two-dimensional figure.

Step 1 Count the number of sides.
A side is a line segment.
There are 5 line segments.
The sign is a pentagon.

Step 2 A regular polygon has all equal sides and equal angles.
The sides are not all the same length and the angles do not all have the same measure.
The sign is an irregular polygon.

Solution The traffic sign is an irregular pentagon.
Example 3
Sort the figures below.

Strategy  Determine how the figures are alike and different.

Step 1  Determine how the figures are alike.
- All of the figures are polygons.
- Some figures have 3 sides.
- Some figures have 5 sides.
- Some figures are regular polygons.

Step 2  Determine how the figures are different.
- Some figures have unequal side lengths.
- They are irregular polygons.

Solution  The figures are shown sorted in the steps above.
Lesson 32: Plane Figures

Look at the figures below.

A                      B                      C                      D

Which are irregular polygons?

Sort the figures.

Figure A is a ________________________.
   Do all of its sides appear equal? ______________
   Do all of its angles appear equal? ______________
   Figure A is a(n) ________________________ polygon.

Figure B is a ________________________.
   Do all of its sides appear equal? ______________
   Do all of its angles appear equal? ______________
   Figure B is a(n) ________________________ polygon.

Figure C is a ________________________.
   Do all of its sides appear equal? ______________
   Do all of its angles appear equal? ______________
   Figure C is a(n) ________________________ polygon.

Figure D is a ________________________.
   Do all of its sides appear equal? ______________
   Do all of its angles appear equal? ______________
   Figure D is a(n) ________________________ polygon.

Figure _________ and Figure _________ are irregular polygons.
You can classify and sort triangles into different groups.

You can classify a triangle by the number of equal sides. The length of the longest side of a triangle is less than the sum of the lengths of the two shorter sides.

- **scalene triangle**
  No sides are equal.

- **isosceles triangle**
  At least 2 sides are equal.

- **equilateral triangle**
  All sides are equal.

The sum of the angle measures for any triangle is 180 degrees (°). You can classify a triangle by the measure of its greatest angle.

- **acute triangle**
  3 acute angles

- **right triangle**
  One angle is a right angle.

- **obtuse triangle**
  One angle is an obtuse angle.

**Example 1**

Classify this triangle by the number of equal sides.

**Strategy**

Identify the lengths of the sides.

There are two equal sides.

An isosceles triangle has at least two equal sides.

**Solution**

The triangle is an isosceles triangle.
Example 2
Classify this triangle by the measures of its angles.

\[
\begin{align*}
45^\circ & \quad 110^\circ & \quad 25^\circ
\end{align*}
\]

**Strategy** Identify the greatest angle measure.

**Step 1** List the measures of the angles.

\[25^\circ, 45^\circ, 110^\circ\]

**Step 2** Classify the measure of the greatest angle.

The greatest angle measure is 110°.
The greatest angle is an obtuse angle.

**Solution** The triangle is an obtuse triangle.

Example 3
Ellie designed a triangular flower garden. A diagram of her garden is shown below.

Classify the triangle Ellie used to design her flower garden by the number of equal sides and by the measure of its angles.

**Strategy** Compare the angles to a right angle, then compare the side lengths.

**Step 1** Decide if any of the angles are right angles.

None of the angles are right angles.
Step 2  Compare each angle measure to a right, or 90°, angle.
   Each of the angles measures less than 90°.

Step 3  Classify each of the angles.
   Each angle is an acute angle.

Step 4  Compare the side lengths.
   Each side is a different length.
   The triangle is scalene.

Solution  Ellie used an acute, scalene triangle to design her flower garden.

Coached Example

Classify the triangle by its number of equal sides and by the measures of its angles.

Classify the triangle by its sides.
Measure the lengths of the sides to the nearest centimeter.
The lengths of the sides are _________ cm, _________ cm, and _________ cm.
A(n) ________________________ triangle has _______ equal sides.

Classify the triangle by the measure of its greatest angle.
The triangle has a(n) ________________________ angle, so the triangle is a ________________________ triangle.
The triangle is a(n) ________________________, ________________________ triangle.
A quadrilateral is a plane figure with 4 sides and 4 angles. There are many different kinds of quadrilaterals, some of which are shown in the chart below. You can classify and sort quadrilaterals into different groups.

<table>
<thead>
<tr>
<th>Quadrilateral</th>
<th>Figure</th>
<th>Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>parallelogram</td>
<td><img src="image" alt="Parallelogram" /></td>
<td>A parallelogram is a quadrilateral in which both pairs of opposite sides are parallel. Opposite sides of a parallelogram have the same length, and opposite angles have the same measure.</td>
</tr>
<tr>
<td>rhombus</td>
<td><img src="image" alt="Rhombus" /></td>
<td>A rhombus is a parallelogram with four sides that have the same length.</td>
</tr>
<tr>
<td>rectangle</td>
<td><img src="image" alt="Rectangle" /></td>
<td>A rectangle is a parallelogram with four right angles.</td>
</tr>
<tr>
<td>square</td>
<td><img src="image" alt="Square" /></td>
<td>A square is a rectangle with four sides that have the same length.</td>
</tr>
<tr>
<td>trapezoid</td>
<td><img src="image" alt="Trapezoid" /></td>
<td>A trapezoid is a quadrilateral with exactly one pair of parallel sides.</td>
</tr>
<tr>
<td>kite</td>
<td><img src="image" alt="Kite" /></td>
<td>A kite is a quadrilateral with two different pairs of connected sides that have the same length.</td>
</tr>
</tbody>
</table>
Some quadrilaterals can be classified in different ways.

![Quadrilateral classification diagram]

**Example 1**
Identify all the ways to classify this quadrilateral.

**Strategy**  Use the properties to classify the quadrilateral.

- **Step 1** Identify any pairs of opposite sides that are parallel.
  
  Both pairs of opposite sides are parallel, so it is a parallelogram.

- **Step 2** Identify any sides that have the same length.
  
  All of the sides have the same length, so it is a rhombus or a square.

- **Step 3** Identify any right angles.
  
  All of the angles are right angles, so it is a rectangle or a square.

**Solution**  The quadrilateral can be classified as a parallelogram, a rhombus, a rectangle, and a square.
Example 2
Renee designed a quadrilateral deck to be built behind her house. She drew a sketch of the deck, which is shown below.

What is the best way to classify the shape of the deck?

Strategy Identify the properties of the quadrilateral.

Step 1 Identify any pairs of opposite sides that are parallel.
Exactly one pair of opposite sides is parallel.

Step 2 Identify any sides that have the same length.
None of the sides have the same length.

Step 3 Identify any right angles.
There are two right angles.

Step 4 Analyze the properties.
There are two right angles, but four right angles are needed for a quadrilateral to be a square or rectangle. Since there is exactly one pair of opposite sides that are parallel, the quadrilateral is a trapezoid.

Solution The best way to classify the shape of the deck is as a trapezoid.
What are two ways you can classify quadrilateral $JKLM$?

Determine if quadrilateral $JKLM$ is a trapezoid or a parallelogram.

A trapezoid has exactly one pair of ________________________ sides.

A parallelogram has both pairs of opposite sides ________________________.

$JK$ is parallel to ________.

$JM$ is parallel to ________. 

Is quadrilateral $JKLM$ a trapezoid or a parallelogram? ______________________

Determine if quadrilateral $JKLM$ is a rectangle, rhombus, and/or square.

Which quadrilaterals have 4 right angles? ________________________________

Does quadrilateral $JKLM$ have 4 right angles? _____________

Which quadrilaterals have 4 equal sides? ________________________________

Does quadrilateral $JKLM$ have 4 equal sides? _____________

The quadrilateral that has 4 right angles, but does not have 4 equal sides, is a _________________.

**Quadrilateral $JKLM$ can be classified as a ______________________ and as a ______________________.**
Lesson 1
Coached Example
Write a numerical expression for “divide 30 by 5.”
30 ÷ 5
Write a numerical expression for “then add 12.”
+ 12
Combine the parts.
(30 ÷ 5) + 12
The expression is (30 ÷ 5) + 12.

Lesson 2
Coached Example
100 − 60 ÷ 5 × 8 + 17
100 − 12 × 8 + 17
100 − 96 + 17
4 + 17
21
100 − 60 ÷ 5 × 8 + 17 = 21

Lesson 3
Coached Example
[7 × 2 − 8] ÷ 3
[14 − 8] ÷ 3
6 ÷ 3
Divide.
6 ÷ 3
2
[(4 + 3) × 2 − 8] ÷ 3 = 2

Lesson 4
Coached Example
0, 4, 8, 12, 16
The first 5 terms of Pattern A are 0, 4, 8, 12, 16.
0, 12, 24, 36, 48
The first 5 terms of Pattern B are 0, 12, 24, 36, 48.

<table>
<thead>
<tr>
<th>Pattern A</th>
<th>0</th>
<th>4</th>
<th>8</th>
<th>12</th>
<th>16</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pattern B</td>
<td>0</td>
<td>12</td>
<td>24</td>
<td>36</td>
<td>48</td>
</tr>
</tbody>
</table>

12 ÷ 4 = 3
24 ÷ 8 = 3
36 ÷ 12 = 3
48 ÷ 16 = 3

The relationship is that each term in Pattern B is 3 times the value as each corresponding term in Pattern A.

Lesson 5
Coached Example
0, 1, 2, 3, 4
0, 4, 8, 12, 16
(0, 0)
(1, 4)
(2, 8)
(3, 12)
(4, 16)

The value of the y-coordinate is 4 times the value of the x-coordinate.

Lesson 6
Coached Example
329 × $26 = m

1
15
329
× 26
1974
+ 6580
8,554

What is the partial product? 1974
Use a 0 as a placeholder in the ones place of the second partial product.
What is the partial product? 6580
Add the partial products to find the product.
What is the product? 8,554
The theater took in $8,554 for this performance.

Lesson 7
Coached Example

\[
\begin{array}{c}
47 \text{ R}4 \\
12\underline{568} \\
-48 \\
\underline{88} \\
-84 \\
\underline{4} \\
\end{array}
\]

The quotient is 47.
The remainder is 4.
The quotient means that 47 bags can be filled with 12 oranges.
The remainder means that there will be 4 oranges left over.
The question asks how many bags Katie needs for all the oranges, so round the quotient up to the nearest whole number.
You can check your answer by multiplying 12 times 47 and adding 4.
Katie needs 48 bags for all the oranges.

Lesson 8
Coached Example

\[
\begin{array}{c}
28 \\
34\underline{963} \\
-68 \\
\underline{283} \\
-272 \\
\underline{11} \\
\end{array}
\]

What is the dividend? 963
What is the quotient? 28
What is the divisor? 34
What is the remainder? 11
963 = 28 \times 34 + 11
28 + \frac{11}{34} = 28\frac{11}{34}
The equation for 963 \div 34 is 963 = 28 \times 34 + 11 and the mixed number is 28\frac{11}{34}.

Lesson 9
Coached Example
To write the number name, first write the decimal in a place-value chart.

<table>
<thead>
<tr>
<th>Ones</th>
<th>.</th>
<th>Tenths</th>
<th>Hundredths</th>
<th>Thousandths</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>.</td>
<td>8</td>
<td>3</td>
<td>7</td>
</tr>
</tbody>
</table>

Write the number name for 6. six
Write the word that separates the whole-number part from the decimal part. and
Write the decimal part as you would a whole number. eight hundred thirty-seven
What is the least place value of the decimal part? thousandths
The number name for 6.837 is six and eight hundred thirty-seven thousandths.
Find the value of each digit.
6 ones = 6 \times 1 = 6
8 tenths = 8 \times 0.1 = 0.8
3 hundredths = 3 \times 0.01 = 0.03
7 thousandths = 7 \times 0.001 = 0.007
Write the expanded form.
6.837 = 6 + 0.8 + 0.03 + 0.007
Write the expanded form with multiplication.
6.837 = 6 \times 1 + 8 \times 0.1 + 3 \times 0.01 + 7 \times 0.001

Lesson 10
Coached Example
Compare the whole-number parts first.
The whole-number parts are the same.
Next compare the digits in the tenths place.
Use >, <, or = to compare.
2 < 3, so 4.295 < 4.3
The lesser distance is 4.295 kilometers.

Lesson 11
Coached Example
Round 18.27 to the nearest whole second.
Look at the digit to the right of the place you are rounding to.
The digit in that place is 2, which means you round down.
Rounded to the nearest whole second, 18.27 is 18.
Lesson 12
Coached Example
Will you multiply or divide 0.9 by a power of 10? divide
When you divide by a power of 10, do you move the decimal point to the right or to the left? left
The exponent tells how many places to move the decimal point.
What is the exponent, or the power of 10? 3
Move the decimal point in 0.9 three places to the left to find the quotient.
Fill the empty places with zeros.

\[
0.9 \div 10^3 = 0.0009
\]

\[
0.9 \div 10^3 = 0.0009
\]

Lesson 13
Coached Example
5.64 rounds up to 6.
9.5 rounds up to 10.
2.07 rounds down to 2.
Add the rounded numbers: \(6 + 10 + 2 = 18\)
The estimated mass of the three samples is 18 grams.
Do all the digits have the same number of places to the right of the decimal point? no
To write the problem, you need to insert a 0 to the right of 5.6 and 15.5.
Next, subtract the sum of those two days from the number of miles that Kobe ran in all.

\[
\begin{array}{c}
0.15410 \\
\times 27 \\
\hline
9765 \\
27900 \\
\hline
376.65
\end{array}
\]

What is the result? 6.15
Kobe ran 6.15 miles on Sunday.

Lesson 14
Coached Example
Do all the digits have the same number of places to the right of the decimal point? no
To write the problem, you need to insert a 0 to the right of 5.6 and 15.5.
First, add to find the total number of miles Kobe ran on Friday and Saturday.

\[
\begin{array}{c}
1 \\
3.75 \\
+ 5.60 \\
\hline
9.35
\end{array}
\]

Kobe ran 9.35 miles on Friday and Saturday.

Lesson 15
Coached Example
Multiply by the ones:

\[
9765
\]
Multiply by the tens:

\[
27900
\]
Add the partial products: \(9765 + 27900 = 37665\)
There are 2 decimal places in the factors, so the product will have 2 decimal places.
Write the decimal point and the $ in the product:

\[
$376.65
\]
The total cost of the tickets is $376.65.

Lesson 16
Coached Example
Write an equation for the problem.

\[
28.12 \div 9.5 = g
\]
What is the dividend? 28.12
What is the divisor? 9.5
How many decimal places are after the decimal point in the divisor? 1
By what number should you multiply both the divisor and dividend? 10
10 \times 28.12 = 281.2
10 \times 9.5 = 95
    \[
    2.96
    \]
    \[
    \underline{912}
    \]
    \[
    -855
    \]
    \[
    570
    \]
    \[
    -570
    \]
    \[
    0
    \]
Madison paid $2.96 for each gallon of gas.

Lesson 17
Coached Example
The factors of 8 are 1, 2, 4, 8.
The factors of 10 are 1, 2, 5, 10.
The greatest common factor of 8 and 10 is 2.
Divide the numerator and denominator by 2.
\[
\frac{8}{10} = \frac{8 \div 2}{10 \div 2} = \frac{4}{5}
\]
Multiply the numerator and denominator of \(\frac{8}{10}\) by Possible answer: 2.
Possible work:
\[
\frac{8}{10} = \frac{8 \times 2}{10 \times 2} = \frac{16}{20}
\]
In simplest form, \(\frac{8}{10}\) is \(\frac{4}{5}\).
Another fraction equivalent to \(\frac{8}{10}\) is Possible answer: \(\frac{16}{20}\).

Lesson 18
Coached Example
How many pizzas are completely shaded? 3
Each pizza is divided into 8 equal parts.
How many parts are shaded in the partially shaded circle? 6
What fraction of the last circle is shaded? \(\frac{6}{8}\)
Write the fraction in simplest form. \(\frac{3}{4}\)
Add the whole-number part and the fraction part.
\(3 + \frac{3}{4} = 3\frac{3}{4}\)
A total of 3\(\frac{3}{4}\) pizzas were eaten.

Lesson 19
Coached Example
The denominators of the fractions are 6 and 4.
Multiples of 6: 6, 12, 18, 24, 30, 36
Multiples of 4: 4, 8, 12, 16, 20, 24
The least number that is a common multiple of 6 and 4 is 12.
Find equivalent fractions with 12 as the common denominator.
\[
\frac{5}{6} \times \frac{2}{2} = \frac{10}{12}
\]
\[
\frac{3}{4} \times \frac{3}{3} = \frac{9}{12}
\]
\[
\frac{10}{12} + \frac{9}{12} = \frac{19}{12}
\]
Write your answer in simplest form: \(1\frac{7}{12}\)
Suki will ride \(1\frac{7}{12}\) miles in all to reach Tybee Island.

Lesson 20
Coached Example
Multiples of 10: 10, 20, 30, 40, 50, 60
Multiples of 4: 4, 8, 12, 16, 20, 24
The least number that is a common multiple of 10 and 4 is 20.
Find equivalent fractions with 20 as the denominator.
\[
\frac{9}{10} \times \frac{2}{2} = \frac{18}{20}
\]
\[
\frac{1}{4} \times \frac{5}{5} = \frac{5}{20}
\]
\[
\frac{18}{20} - \frac{5}{20} = \frac{13}{20}
\]
Jillian drank \(\frac{13}{20}\) of the milk in the glass.

Lesson 21
Coached Example
To decide if the product is greater or less than \(8\frac{3}{4}\) feet, compare the factors \(8\frac{3}{4}\) and \(1\frac{1}{3}\) to 1.
Compare: \(8\frac{3}{4} > 1\) and \(1\frac{1}{3} > 1\).
The product of \(8\frac{3}{4} \times 1\frac{1}{3}\) is greater than \(8\frac{3}{4}\).
Convert \(8\frac{3}{4}\) to an improper fraction. \(\frac{35}{4}\)
Convert \(1\frac{1}{3}\) to an improper fraction. \(\frac{4}{3}\)
Multiply \(8\frac{3}{4} \times 1\frac{1}{3}\).
In simplest form, \( \frac{35}{4} \times \frac{3}{3} = \frac{105}{12} = \frac{35}{4} = \frac{11}{3} \).

Amber needs \( 11 \frac{2}{3} \) feet of ribbon.

**Lesson 22**

*Coached Example*

Substitute \( \frac{5}{6} \) for the length and \( \frac{1}{4} \) for the width.

\[
A = \frac{5}{6} \times \frac{1}{4}
\]

Multiply the numerators. \( 5 \times 1 = 5 \)

Multiply the denominators. \( 6 \times 4 = 24 \)

The product is \( \frac{5}{24} \).

The area of the rectangle is \( \frac{5}{24} \) square foot.

**Lesson 23**

*Coached Example*

Write a division sentence for the problem.

Let \( o \) represent the number of ounces of jellybeans each friend will get.

\[
45 \div 4 = o
\]

\[
\underline{11} R1
\]

\[
\underline{05}
\]

\[
\underline{\text{1}}
\]

Write the quotient as a mixed number. \( 11 \frac{1}{4} \)

Is the mixed number in simplest form? yes

Each friend should get \( 11 \frac{1}{4} \) ounces of jelly beans. The answer lies between the whole numbers \( 11 \) and \( 12 \).

**Lesson 24**

*Coached Example*

Write an equation for the problem.

\[
\frac{1}{2} \div 6 = s
\]

When dividing a unit fraction by a whole number, the quotient is always a unit fraction less than the dividend.

To divide fractions, multiply the dividend by the reciprocal of the divisor.

The reciprocal of 6 is \( \frac{1}{6} \).

\[
\frac{1}{2} \times \frac{1}{6} = \frac{1}{12}
\]

Each friend will get \( \frac{1}{12} \) of the banana bread.

**Lesson 25**

*Coached Example*

How many pints are in 1 quart? 2

How many quarts are in 1 gallon? 4

\( 2 \times 4 = 8 \)

Luanne must fill the 1-pint measuring cup 8 times to have 1 gallon of water.

**Lesson 26**

*Coached Example*

1 liter = 1,000 milliliters,

so 2 liters = 2,000 milliliters

Alex wants to drink 2,000 milliliters of water today.

The amount of water Alex drank so far can be found by multiplying \( 5 \times 250 \).

How many milliliters of water did Alex drink so far? 1,250

Subtract: \( 2,000 \text{ mL} - 1,250 \text{ mL} = 750 \text{ mL} \)

Alex needs to drink 750 milliliters more of water today to reach his goal.

**Lesson 27**

*Coached Example*

There are 9 rows and 7 columns of cubes in the bottom layer.

\[
9 \times 7 = 63
\]

The area of the base of the prism is 63 square centimeters.

The height of the prism is 6 centimeters.

\[
63 \times 6 = 378
\]

The volume of the cube is 378 cubic centimeters.

**Lesson 28**

*Coached Example*

The length is 2 feet.

The width is 2 feet.

The height is 2 feet.

\[
V = l \times w \times h
\]

\[
V = 2 \times 2 \times 2 = 8 \text{ cubic feet}
\]
The volume of the prism on the left is **8** cubic feet.
The length is **2** feet.
The width is **4** feet.
The height is **2** feet.
Substitute the values into the formula.
\[ V = 2 \times 4 \times 2 = 16 \] cubic feet
The volume of the prism on the right is **16** cubic feet.
8 cubic feet + 16 cubic feet = **24** cubic feet
The volume of the figure is **24 cubic feet**.

**Lesson 29**
Coached Example

\[ 3 \times \frac{1}{4} = \frac{3}{4} \]
\[ 2 \times \frac{1}{2} = \frac{6}{8} \]
\[ 5 \times \frac{1}{4} = \frac{5}{8} \]
\[ 1 \times \frac{1}{8} = \frac{5}{8} \]
Rename the products, so they all have a denominator of **8**.
\[ \frac{6}{8} + \frac{6}{8} + \frac{20}{8} + \frac{5}{8} = \frac{37}{8} \]
\[ \frac{37}{8} = 4\frac{5}{8} \]
The total weight of the fruit that Logan bought is **4\frac{5}{8}** pounds.

**Lesson 30**
Coached Example
The number directly below point **A** is **4**.
This is the number of units to the right of the origin.
The number directly to the left of point **A** is **6**.
This is the number of units above the origin.
The ordered pair (4, 6) gives the location of point **A**.

**Lesson 31**
Coached Example
Points **J** and **K** have the same **y**-coordinate.
To find the length of line segment **JK**, subtract the **x**-coordinates.
\[ 8 - 2 = 6 \]
Points **K** and **L** have the same **x**-coordinate.
To find the length of line segment **KL**, subtract the **y**-coordinates.
\[ 8 - 4 = 4 \]
P = \( 2 \times 6 + 2 \times 4 \)
P = \( 12 + 8 \)
P = **20** units
The perimeter of rectangle **JKLM** is **20 units**.

**Lesson 32**
Coached Example
Figure **A** is a **square**.
Do all of its sides appear equal? **yes**
Do all of its angles appear equal? **yes**
Figure **A** is a(n) **regular** polygon.
Figure **B** is a **triangle**.
Do all of its sides appear equal? **no**
Do all of its angles appear equal? **no**
Figure **B** is a(n) **irregular** polygon.
Figure **C** is a **pentagon**.
Do all of its sides appear equal? **yes**
Do all of its angles appear equal? **yes**
Figure **C** is a(n) **regular** polygon.
Figure **D** is a **hexagon**.
Do all of its sides appear equal? **no**
Do all of its angles appear equal? **no**
Figure **D** is a(n) **irregular** polygon.
Figure **B** and Figure **D** are irregular polygons.

**Lesson 33**
Coached Example
The lengths of the sides are **5** cm, **5** cm, and **7** cm.
A(n) **isosceles** triangle has **2** equal sides.
The triangle has a(n) **right** angle, so the triangle is a **right** triangle.
The triangle is a(n) **isosceles, right** triangle.
Lesson 34
Coached Example
A trapezoid has exactly one pair of parallel sides. A parallelogram has both pairs of opposite sides parallel.
$\overline{JK}$ is parallel to $\overline{ML}$.
$\overline{JM}$ is parallel to $\overline{KL}$.
Is quadrilateral $JKLM$ a trapezoid or a parallelogram? parallelogram
Which quadrilaterals have 4 right angles? rectangle and square
Does quadrilateral $JKLM$ have 4 right angles? yes
Which quadrilaterals have 4 equal sides? rhombus and square
Does quadrilateral $JKLM$ have 4 equal sides? no
The quadrilateral that has 4 right angles, but does not have 4 equal sides, is a rectangle.
Quadrilateral $JKLM$ can be classified as a parallelogram and as a rectangle.