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Factors and Multiples

Getting the Idea

A **multiple** of a number is the product of that number and any of the counting numbers (1, 2, 3, 4, 5, …). For example, some multiples of 7 are shown below.

7, 14, 21, 28, 35, …

The **least common multiple (LCM)** of two numbers is the least number that is a multiple of both numbers.

Example 1

What is the LCM of 8 and 12?

**Strategy**  List the multiples of each number. Find the least number that is in both lists.

**Step 1**  List the first six multiples of 8.

8, 16, 24, 32, 40, 48

**Step 2**  List the first six multiples of 12.

12, 24, 36, 48, 60, 72

**Step 3**  Determine the LCM.

24 is the least number in both lists.

**Solution**  The LCM of 8 and 12 is 24.

You can use what you know about the LCM to solve a real-world problem.

Example 2

The students in the drama club can be divided into 3 equal groups or 5 equal groups, with no students left over. What is the least number of students that could be in the drama club?

**Strategy**  Use what you know about the LCM.
Step 1  How can the LCM help you solve this problem?
Since the drama club can be divided into equal groups of 3 or 5, the number of students in the drama club must be a multiple of both 3 and 5. To find the least possible number of students in the drama club, find the LCM of 3 and 5.

Step 2  List the first six multiples of 3 and 5.
- Multiples of 3: 3, 6, 9, 12, 15, 18
- Multiples of 5: 5, 10, 15, 20, 25, 30

Step 3  Find the least number that is common to both lists.
The LCM is 15.

Solution  The least possible number of students in the drama club is 15.

The factors of a number are the whole numbers that can be multiplied to get that number. The greatest common factor (GCF) of two numbers is the greatest number that is a factor of both numbers.

Example 3
What is the GCF of 20 and 50?

Strategy  List the factors of each number. Find the greatest number that is in both lists.

Step 1  List the factors of 20.
- 1, 2, 4, 5, 10, 20

Step 2  List the factors of 50.
- 1, 2, 5, 10, 25, 50

Step 3  Determine the GCF.
- 1, 2, 5, and 10 appear in both lists.
The GCF is 10.

Solution  The GCF of 20 and 50 is 10.
Numbers can be expressed in different ways. For example, the number 10 can be written as the sum 4 + 6. You can use the **distributive property** to write this sum in another way.

<table>
<thead>
<tr>
<th><strong>Distributive Property</strong></th>
</tr>
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<tr>
<td>When you multiply the sum of two numbers by another number, you can multiply each addend by the number, and then add the products. The property also applies to subtraction.</td>
</tr>
<tr>
<td>[ a(b + c) = ab + ac ]</td>
</tr>
<tr>
<td>[ a(b - c) = ab - ac ]</td>
</tr>
</tbody>
</table>

Using the distributive property:  
\[ 2(2 + 3) = (2 \times 2) + (2 \times 3) \]
\[ = 4 + 6 \]
\[ = 10 \]

The number 10 can be written as 4 + 6 and as 2(2 + 3). The numbers in the sum (2 + 3) have no common factors greater than 1.

**Example 4**
The number 99 can be expressed as the sum 45 + 54. Use the distributive property to rewrite that sum as a multiple of a sum whose addends have no common factors greater than 1.

**Strategy**  
Find the GCF of 45 and 54. Use the distributive property to rewrite the sum.

**Step 1**  
Find the GCF of 45 and 54.  
Factors of 45: 1, 3, 5, 9, 15, and 45  
Factors of 54: 1, 2, 3, 6, 9, 18, 27, and 54  
The common factors are 1, 3, and 9. So, the GCF is 9.

**Step 2**  
Use the GCF and the distributive property to rewrite the sum.  
45 = 5 \times 9  
54 = 6 \times 9  
45 + 54 = 9(5 + 6)

**Step 3**  
Check that the addends in the sum have no common factors, other than 1.  
Factors of 5: 1, 5  
Factors of 6: 1, 2, 3, 6  
The numbers 5 and 6 have no common factors, other than 1.

**Solution**  
The sum 45 + 54 can be expressed as 9(5 + 6).
Coached Example

Two P.E. classes are participating in Field Day. One class has 24 students. The other class has 27 students. The P.E. teachers want to divide the students in each class into the largest possible equal groups, with no students left over. If all the groups have the same number of students, how many students are in each group?

If the classes are divided into equal groups, the number of students in each group must be a ______________ of 24 and 27.

List the factors of 24: 1, _____, _____, _____, _____, _____, 24
List the factors of 27: 1, _____, _____, 27
What are the common factors of 24 and 27? __________
What is the greatest common factor of 24 and 27? _____
If all the groups have the same number of students, there will be _____ students in each group.
Divide Whole Numbers

Getting the Idea

In division, the number that is divided is the **dividend**. The number that divides the dividend is the **divisor**. The result of dividing the dividend by the divisor is the **quotient**. Some division problems will have a remainder. A **remainder** is a number that is left over when two counting numbers are divided. A remainder is always less than the divisor.

To divide by a 2-digit number, you may estimate first to help you find the quotient.

Example 1

What is $64,015 \div 74$?

**Strategy**  
**Estimate the first digit in the quotient and work from there.**

**Step 1**  
Decide where to place the first digit in the quotient.

\[
74 \overline{)64,015} 
\]

The first digit of the quotient will be in the hundreds place.

**Step 2**  
Divide 640 by 74.

\[
8 
74 \overline{)64,015} 
\]

\[
\begin{array}{c}
-592 \\
48 \text{ Multiply: } 8 \times 74 = 592 \\
\end{array}
\]

**Step 3**  
Bring down the 1 and divide.

\[
86 
74 \overline{)64,015} 
\]

\[
\begin{array}{c}
-592 \\
481 \text{ Multiply: } 6 \times 74 = 444 \\
-444 \text{ Subtract: } 481 - 444 = 37 \\
\end{array}
\]
Step 4 Bring down the 5 and divide.

\[
\begin{array}{c}
865 \div 74 = 64,015 \\
74 \overline{)64,015} \\
-59 \quad 2 \\
4 \quad 81 \\
-4 \quad 44 \\
375 \\
-370 \\
-370 \quad \text{Multiply: } 5 \times 74 = 370 \\
5 \quad \text{Subtract: } 375 - 370 = 5 \\
\text{The remainder is 5.}
\end{array}
\]

Solution \[ 64,015 \div 74 = 865 \text{ R5} \]

Example 2
There are 1,288 seats in an auditorium. Each of the 23 rows in the auditorium has the same number of seats. How many seats are in each row?

Strategy Divide to find the solution.

Step 1 Decide where to place the first digit in the quotient.

\[
\begin{array}{c}
23 \overline{)1,288} \\
\text{The first digit of the quotient will be in the tens place.}
\end{array}
\]

Step 2 Divide 128 by 23.

\[
\begin{array}{c}
5 \\
23 \overline{)1,288} \\
-1 \quad 15 \\
-1 \quad 38 \\
13 \quad \text{Multiply: } 5 \times 23 = 115 \\
\text{Subtract: } 128 - 115 = 13
\end{array}
\]

Step 3 Bring down the 8 and divide.

\[
\begin{array}{c}
56 \\
23 \overline{)1,288} \\
-1 \quad 15 \\
-1 \quad 38 \\
138 \quad \text{Multiply: } 6 \times 23 = 138 \\
0 \quad \text{Subtract: } 138 - 138 = 0
\end{array}
\]

Solution There are 56 seats in each row of the auditorium.
Example 3

Mindy’s annual salary as a physical therapist is $59,796. How much does Mindy earn per month?

Strategy  
Divide each place, going from left to right.

Step 1  
There are 12 months in a year, so the divisor is 12.

\[
\begin{array}{c}
12\overline{)59,796} \\
-48 \\
117 \\
-108 \\
99 \\
-96 \\
3
\end{array}
\]

Multiply: \(4 \times 12 = 48\)  
Subtract: \(59 - 48 = 11\)

Step 2  
Divide 59 by 12.

\[
\begin{array}{c}
4 \\
12\overline{)59,796} \\
-48 \\
117 \\
-108 \\
99 \\
-96 \\
3
\end{array}
\]

Multiply: \(9 \times 12 = 108\)  
Subtract: \(117 - 108 = 9\)

Step 3  
Bring down the 7. Divide.

\[
\begin{array}{c}
4 \ 9 \\
12\overline{)59,796} \\
-48 \\
117 \\
-108 \\
99 \\
-96 \\
3
\end{array}
\]

Step 4  
Bring down the 9. Divide.

\[
\begin{array}{c}
4 \ 98 \\
12\overline{)59,796} \\
-48 \\
117 \\
-108 \\
99 \\
-96 \\
3
\end{array}
\]

Multiply: \(8 \times 12 = 96\)  
Subtract: \(99 - 96 = 3\)
**Lesson 2: Divide Whole Numbers**

**Step 5**
Bring down the 6. Divide.

\[
\begin{array}{c}
4983 \\
\hline
12 | 59796 \\
\hline
-48 \\
\hline
117 \\
-108 \\
\hline
99 \\
-96 \\
\hline
36 \\
\hline
-36 \\
0
\end{array}
\]

Multiply: \(3 \times 12 = 36\)

Subtract: \(36 - 36 = 0\)

**Solution**
Mindy earns $4,983 per month.

---

**Coached Example**

Divide: \(32 \div 89824\)

\[
\begin{array}{c}
2 \underline{0} \\
\hline
32 | 89824 \\
\hline
-64 \\
\hline
\underline{8} \\
\hline
22 \\
\hline
\underline{0} \\
\hline
4 \\
\hline
\underline{0}
\end{array}
\]

Multiply: \(2 \times 32 = \)___

Multiply: \(\underline{8} \times 32 = \)___

Multiply: \(\underline{2} \times 32 = \)___

Multiply: \(\underline{4} \times 32 = \)___

Check your answer. Multiply the quotient and the divisor.

\[
\begin{array}{c}
\underline{89824} \times 32 = \underline{2874384}
\end{array}
\]

\[
89824 \div 32 = \underline{2874384}
\]
Integers include the set of whole numbers and their opposites. The number line below shows the integers from −5 to 5. Negative integers have values less than zero, so they are to the left of zero on the number line. Positive integers have values greater than zero, so they are to the right of zero on the number line. Zero is neither negative nor positive.

You can use integers to describe opposite situations. Here are some uses for integers:

Positive integers
- A bank deposit (adding money to an account)
- An elevation above sea level
- A rise in temperature

Negative integers
- A bank withdrawal (taking money out of an account)
- An elevation below sea level
- A drop in temperature

**Example 1**
A bird is flying 25 feet above sea level and a fish is swimming 10 feet below sea level. Use integers to represent the elevation of the fish and the bird.

**Strategy**  Use an integer to describe each situation.

**Step 1**  What elevation would the number 0 represent?
Zero represents sea level, or the surface of the water.

**Step 2**  Find a signed number for the elevation of the bird.
The bird is above sea level, so use +25 or 25.

**Step 3**  Find a signed number for the elevation of the fish.
The fish is below sea level, so use −10.

**Solution**  The elevation of the bird is 25 feet. The elevation of the fish is −10 feet.
You can show negative integers by extending to the left a number line that shows the numbers (0, 1, 2, 3, . . .). Number lines showing positive and negative integers can be either horizontal or vertical, such as a thermometer.
Example 2
What temperatures are indicated on the Fahrenheit thermometers below?

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>100°F</td>
<td>100°F</td>
</tr>
<tr>
<td>90°</td>
<td>90°</td>
<td>90°</td>
</tr>
<tr>
<td>80°</td>
<td>80°</td>
<td>80°</td>
</tr>
<tr>
<td>70°</td>
<td>70°</td>
<td>70°</td>
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<tr>
<td>60°</td>
<td>60°</td>
<td>60°</td>
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<td>10°</td>
<td>10°</td>
<td>10°</td>
</tr>
<tr>
<td>0°</td>
<td>0°</td>
<td>0°</td>
</tr>
<tr>
<td>−10°</td>
<td>−10°</td>
<td>−10°</td>
</tr>
<tr>
<td>−20°</td>
<td>−20°</td>
<td>−20°</td>
</tr>
</tbody>
</table>

Thermometer A   Thermometer B

Strategy  Use integers to describe the temperature.

Step 1  Find the temperature on thermometer A.
        The temperature is above 0, so use a positive number.
        Thermometer A shows a temperature of 40°F.

Step 2  Find the temperature on thermometer B.
        The temperature is below 0, so use a negative number.
        Thermometer B shows a temperature of −10°F.

Solution  The temperature on thermometer A is 40°F. The temperature on thermometer B is −10°F.

Integers that are the same distance from 0 on a number line are called **opposites**. For example, 5 and −5 are opposites of each other.

```
-8  -7  -6  -5  -4  -3  -2  -1  0  1  2  3  4  5  6  7  8
```

```
opposites
```

So, the opposite of 5 is −5, and the opposite of −5, written as −(−5), is 5. The opposite of 0 is 0.
Example 3
Find the opposites of 6 and of −2.

Strategy  Use a number line.

Step 1  Plot a point for 6 on a number line.

Step 2  Find the integer that is the same distance from 0 in the opposite direction.

   The integer 6 is 6 units to the right of 0.

   Count 6 units to the left of 0. Plot a point.

   The opposite of 6 is −6.

Step 3  Plot a point for −2 on a number line.

Step 4  Find the integer that is the same distance from 0 in the opposite direction.

   The integer −2 is 2 units to the left of 0.

   Count 2 units to the right of 0. Plot a point.

   The opposite of −2 is 2.

Solution  The opposite of 6 is −6. The opposite of −2 is 2.
Coached Example

On its first play of the game, a football team gained 6 yards. On its next two plays, the team lost 2 yards and then gained 7 yards. Use integers to describe these three plays.

What integer represents a play in which the team neither gains yards nor loses yards? ____
A play that gains yards would be represented by a _________________ integer.
A play that loses yards would be represented by a _________________ integer.
On the first play, the team gained _____ yards.
A gain of 6 yards is represented by the integer ______.
On the second play, the team lost _____ yards.
A loss of 2 yards is represented by the integer ______.
On the third play, the team gained _____ yards.
A gain of 7 yards is represented by the integer ______.

The three plays can be described by the integers _____, _____, and _____.
The absolute value of a number is its distance from 0 on a number line. Since a distance must be either a positive number or zero, the absolute value of a number is always a positive number or zero. The absolute value of a number $x$ is written as $|x|$.

The integers $-4$ and $4$ are opposites. You can use the number line below to see that each number is the same distance from 0. So, $|-4| = 4$ and $|4| = 4$.

Example 1
Find the value of $|-7|$.

Strategy Use a number line.

Step 1 Plot a point for $-7$ on a number line.

Step 2 Count the number of units from $-7$ to 0.

The distance is 7 units.

Solution $|-7| = 7$
You can use absolute values to represent and help you understand real-world situations.

For example, if a diver is 20 meters below the ocean’s surface, that depth, in meters, can be shown as $-20$ meters. But the distance the diver would have to swim to get to the surface of the water cannot be represented by a negative number. You can use absolute value instead. The diver must swim $|{-20}|$ meters, or 20 meters, to reach the surface.

**Example 2**

Hannah wrote a check for more money than she has in her bank account. The balance in her account is now $-60$. How much does Hannah owe the bank, in dollars?

**Strategy** Use an absolute value to represent the situation.

**Step 1** Is the amount she owes a positive or negative number?

The balance in Hannah’s account is $-60$, but she cannot owe the bank a negative amount of money.

The amount Hannah owes must be shown as a positive number.

**Step 2** Use an absolute value.

The amount she owes, in dollars, is $|{-60}|$, or 60.

The number line below shows that Hannah owes the bank $60$.

![Number line showing $60$ units from $-80$ to $80$]

**Solution** Hannah owes the bank $60$. 
Absolute values can also help you understand situations in which an exact number is not known.

**Example 3**
A team of mountaineers has climbed to the summit of Mount Everest. The temperature at the summit is less than $-15^\circ F$. Describe how many degrees Fahrenheit below $0^\circ F$ the temperature is.

**Strategy**
Use an absolute value to represent and understand the situation.

**Step 1**
Is the number of degrees below $0^\circ F$ a positive or negative number?
- An actual temperature may be negative, but the number of degrees Fahrenheit below $0^\circ F$ must be a positive number.

**Step 2**
Use a number line to represent the situation.
- The temperature is less than $-15^\circ F$.
- On a number line, a number less than $-15$ is to the left of $-15$.
- The arrow below shows all the numbers less than $-15$.

**Step 3**
Use absolute value to describe the number of degrees Fahrenheit below $0^\circ F$.
- $|-15| = 15$
- All the numbers less than $-15$ are more than 15 units from $0$.

**Solution**
The temperature at the summit is more than $15^\circ F$ below $0^\circ F$. 
Yesterday, Marcus bought two different stocks, A and B, each at the same price. From yesterday to today, the change in the price of Stock A was $-12, and the change in the price of Stock B was $9. From yesterday to today, which stock's price changed by the greater amount?

The price change with the greater __________ _________ is the greater change.

On the number line below, plot points for $-12$ and 9.

Count the units from each integer to 0 to determine its absolute value.

$|{-12}| = _____$

$|9| = _____$

Which number has the greater absolute value, $-12$ or 9? _____

The stock with the price change of _______ dollars changed by the greater amount.

That stock was Stock ______.
A rational number is a number that can be expressed as the ratio of two integers in the form $\frac{a}{b}$, where $b$ is not equal to 0. The set of rational numbers includes integers, fractions, mixed numbers, percents, terminating decimals, and repeating decimals. Some examples of rational numbers are shown below.

$$8\% \quad \frac{4}{5} \quad 0.35 \quad 1\frac{3}{8} \quad -7 \quad 1.\overline{6}$$

Fractions and decimals have opposites, just as integers do. For example, $\frac{5}{8}$ and $-\frac{5}{8}$ are opposites, and so are $-3.25$ and $3.25$.

**Example 1**

Explain why $4$, $\frac{2}{3}$, and $0.9$ are rational numbers.

**Strategy**  
Express the numbers in the form $\frac{a}{b}$.

**Step 1**  
Show that $4$ is a rational number.  
$4 = \frac{4}{1}$, which is in the form $\frac{a}{b}$.

**Step 2**  
Show that $\frac{2}{3}$ is a rational number.  
$\frac{2}{3}$ is in the form $\frac{a}{b}$.

**Step 3**  
Show that $0.9$ is a rational number.  
$0.9 = \frac{9}{10}$, which is in the form $\frac{a}{b}$.

**Solution**  
The numbers $4$, $\frac{2}{3}$, and $0.9$ are rational numbers because each can be written in the form $\frac{a}{b}$. 
Example 2
Plot the rational numbers $\frac{3}{5}$ and $-\frac{2}{5}$ on the number line shown below.

Strategy
Identify what fractional units the number line is divided into. Then use absolute value to plot each point.

Step 1
Determine what each mark on the number line stands for.
There are 5 spaces between 0 and 1.
The number line is divided into fifths. Each mark stands for $\frac{1}{5}$.

Step 2
Plot $\frac{3}{5}$ on the number line.
$\frac{3}{5}$ is three units away from 0 on this number line.
Since $\frac{3}{5}$ is positive, count 3 units to the right of 0. Plot the point.

Step 3
Plot $-\frac{2}{5}$ on the number line.
$-\frac{2}{5}$ is two units away from 0 on this number line.
Since $-\frac{2}{5}$ is negative, count 2 units to the left of 0. Plot the point.

Solution
The rational numbers $\frac{3}{5}$ and $-\frac{2}{5}$ are shown on the number line in Step 3 above.
Example 3
The floor of the valley in which Griffin lives is $7\frac{1}{2}$ feet below sea level. Write that elevation as a rational number. Then plot a point for it on the number line below.

Strategy
Write a rational number representing $7\frac{1}{2}$ feet below sea level. Then identify the value of each mark on the number line.

Step 1
Determine whether the elevation is a positive or negative number.
In this case, the number 0 represents sea level.
The valley floor is below sea level, so the number will be less than 0.
The rational number $-7\frac{1}{2}$ represents the elevation.

Step 2
Determine what each mark on the number line stands for.
There are 20 spaces between 0 and 10 and 20 spaces between 0 and $-10$.
Each mark between integers stands for $\frac{1}{2}$.

Step 3
Plot $-7\frac{1}{2}$ on the number line.

$| -7\frac{1}{2} | = 7\frac{1}{2} \quad | -7 | = 7$

$7\frac{1}{2} > 7$, so $-7\frac{1}{2}$ will be farther from 0 than $-7$.
Plot a point for $-7\frac{1}{2}$ at the mark to the left of $-7$.

Solution
The rational number $-7\frac{1}{2}$ represents the elevation.
The location of $-7\frac{1}{2}$ on a number line is shown in Step 3 above.
Example 4
What decimals do points $P$ and $Q$ represent on the number line shown?

![Number line with points P and Q]

**Strategy**  Identify what units the number line is divided into.

**Step 1**  Determine what each mark on the number line stands for.
- There are 10 spaces between 0 and 1.
- The number line is divided into tenths. Each mark stands for 0.1.

**Step 2**  Find the number of tenths that points $P$ and $Q$ represent.
- Count from 0. It may help to label the marks as shown below.

![Number line with labeled marks]

**Solution**  Point $P$ represents $-0.8$ and point $Q$ represents $0.5$.

**Coached Example**

The number line below shows points for several rational numbers. Which points represent $-\frac{1}{5}$, $\frac{4}{5}$, $\frac{2}{5}$, and $-\frac{3}{5}$?

![Number line with points A, B, C, and D]

There are ____ spaces between 0 and 1.
- The number line is divided into ______. Each mark stands for ____.
- $-\frac{1}{5}$ is between _____ and _____ . It is located at point ____.
- $\frac{4}{5}$ is between _____ and _____ . It is located at point ____.
- $\frac{2}{5}$ is between _____ and _____ . It is located at point ____.
- $-\frac{3}{5}$ is between _____ and _____ . It is located at point ____.

Point ___ represents $-\frac{1}{5}$, point ___ represents $\frac{4}{5}$, point ___ represents $\frac{2}{5}$, and point ___ represents $-\frac{3}{5}$.
Getting the Idea

To compare numbers, you can use the symbols $>$ (is greater than), $<$ (is less than), or $=$ (is equal to). The inequality $p > q$ means that $p$ is located to the right of $q$ on a number line. The inequality $p < q$ means that $p$ is located to the left of $q$ on a number line.

Example 1

Walter and four friends decided to compare the balances in their bank accounts. The table below shows each person’s balance.

<table>
<thead>
<tr>
<th>Bank Balances</th>
</tr>
</thead>
<tbody>
<tr>
<td>Person</td>
</tr>
<tr>
<td>Walter</td>
</tr>
<tr>
<td>Ellen</td>
</tr>
<tr>
<td>Christine</td>
</tr>
<tr>
<td>Randy</td>
</tr>
<tr>
<td>Peter</td>
</tr>
</tbody>
</table>

Order the account balances from greatest to least.

Strategy

Use a number line.

**Step 1** Write each account balance as an integer.

$35, -10, -5, 40, -20$

**Step 2** Plot each integer on a number line.

The negative integers will be to the left of 0. The positive integers will be to the right of 0.
Step 3  Order the integers from greatest to least.

List the integers as they appear on the number line, going from right to left.

40, 35, −5, −10, and −20

Solution  From greatest to least, the account balances are $40, $35, −$5, −$10, and −$20.

In Example 1, Peter’s account balance is less than Christine’s balance, but his debt is greater than Christine’s debt.

To compare and order fractions, you will need fractions with common denominators. One way to find a common denominator is to multiply the denominators of the fractions.

To compare mixed numbers, first look at the whole-number parts. If the whole-number parts are equal, then compare the fraction parts.

Example 2

Which symbol makes this sentence true? Use >, <, or =.

\[2 \frac{3}{4} \bigcirc 2 \frac{2}{3}\]

Strategy  Compare the whole-number parts. If necessary, use a common denominator to compare the fraction parts.

Step 1  Compare the whole-number parts.

\[2 = 2\]

Step 2  Find a common denominator for the fraction parts.

Multiply the denominators to find a common denominator.

\[4 \times 3 = 12\]

Step 3  Write the fraction parts as equivalent fractions with a common denominator.

\[\frac{3}{4} = \frac{3 \times 3}{4 \times 3} = \frac{9}{12}\]

\[\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}\]

Step 4  Compare the fractions.

\[\frac{9}{12} > \frac{8}{12}\]

Solution  \[2 \frac{3}{4} \bigcirc 2 \frac{2}{3}\]
When comparing decimals, align the digits on the decimal point, then compare from left to right. The number of decimal places does not affect whether a decimal is greater than or less than another decimal.

**Example 3**
Kelly owns two dogs. Bonus weighs 73.59 pounds and Riley weighs 73.6 pounds. Write a number sentence to compare the dogs’ weights. Which dog weighs more?

**Strategy**  
Write a number sentence.

**Step 1**  
Align the numbers on the decimal point. Compare from left to right.

\[
\begin{align*}
73.59 \\
73.6
\end{align*}
\]

**Step 2**  
Find the first place that has different digits.

The tens are both 7.
The ones are both 3.

Compare the next greatest place: tenths.

**Step 3**  
Compare the tenths.

\[
\begin{align*}
5 &< 6 \\
73.59 &< 73.6
\end{align*}
\]

Bonus weighs less than Riley.

**Solution**  
Riley weighs more than Bonus.

**Example 4**
Last winter, Cedric recorded the low temperature, in °F, at his farm over 5 days. His data is shown below.

\[
0.5, 1.3, -2, 1, -1.5
\]

Order the temperatures from least to greatest.

**Strategy**  
Use a number line.

**Step 1**  
Plot the numbers on a number line divided into tenths.
Step 2

List the numbers as they appear from left to right on the number line.

\(-2, -1.5, 0.5, 1, 1.3\)

This can be written as: \(-2 < -1.5 < 0.5 < 1 < 1.3\)

Solution

From least to greatest, the temperatures, in °F, are \(-2, -1.5, 0.5, 1, 1.3\).

Coached Example

Order the following numbers from greatest to least:

\(-3, 2.6, 3, 2\frac{3}{10}, -3\frac{1}{4}\)

Separate the positive numbers from the negative numbers.

The positive numbers are \(\), \(\), and \(\).

Rename 2.6 as a mixed number with a denominator of 10.

\(2.6 = \) \(\)

The greatest positive number is \(\).

Compare the remaining two positive numbers. \(\) > \(\)

From greatest to least, the positive numbers are \(\), \(\), and \(\).

The negative numbers are \(\) and \(\).

Which negative number is greater? \(\)

From greatest to least, the numbers are \(\), \(\), \(\), \(\), \(\), and \(\).
Adding and subtracting decimals is like adding and subtracting integers. Write the problem vertically, lining up the decimal points. Add or subtract from right to left. Regroup if necessary.

Example 1
Sheila and Leslie hiked 8.76 kilometers in the morning and 4.29 kilometers in the afternoon. How many kilometers did they hike in all?

Strategy  
Align the decimal points. Add from right to left. Regroup if necessary.

Step 1  
Align the decimal points.  
Place the decimal point in the sum.  
\[ \begin{align*} 
8.76 \\
+ 4.29 
\end{align*} \]

Step 2  
Add the hundredths.  
\[ 6 + 9 = 15 \]  
Write the 5.  
Regroup 10 hundredths as 1 tenth.  
\[ \begin{align*} 
1 \\
8.76 \\
+ 4.29 
\end{align*} \]

Step 3  
Add the tenths.  
\[ 1 + 7 + 2 = 10 \]  
Write the 0.  
Regroup 10 tenths as 1 one.  
\[ \begin{align*} 
1 \\
8.76 \\
+ 4.29 
\end{align*} \]

Solution  
Sheila and Leslie hiked a total of 13.05 kilometers.
Writing one or more zeros to the right of the last digit in a decimal does not change the value of the decimal. For example:

\[ 1.2 = 1.20 = 1.200 \]

You can use this to help you add and subtract decimals with different numbers of places.

**Example 2**

A plumber has two metal pipes. The first pipe is 2.35 meters long. The second pipe is 1.725 meters long. How much longer is the first pipe than the second pipe?

**Strategy** Subtract from right to left. Regroup if necessary.

**Step 1**
Align the decimal points. Write a 0 in the thousandths place for 2.35. Place the decimal point in the difference.

\[
\begin{align*}
2.350 \\
- 1.725 \\
\hline
\end{align*}
\]

**Step 2**
Regroup 1 hundredth as 10 thousandths. Subtract the thousandths.

\[
\begin{align*}
4 & \quad 10 \\
2.3 & \quad \underline{\phantom{0}} \\
- 1.7 & \quad 2.5 \\
\hline
\quad 5 \\
\end{align*}
\]

**Step 3**
Subtract the hundredths.

\[
\begin{align*}
4 & \quad 10 \\
2.3 & \quad \underline{\phantom{0}} \\
- 1.7 & \quad 2.5 \\
\hline
\quad 2.5 \\
\end{align*}
\]

**Step 4**
Regroup 1 one as 10 tenths. Subtract the tenths.

\[
\begin{align*}
1 & \quad 13 \quad 4 \quad 10 \\
2.3 & \quad \underline{\phantom{0}} \quad \underline{\phantom{0}} \\
- 1.7 & \quad 2.5 \\
\hline
\quad .6 \quad 2.5 \\
\end{align*}
\]

**Step 5**
Subtract the ones.

\[
\begin{align*}
1 & \quad 13 \quad 4 \quad 10 \\
2.3 & \quad \underline{\phantom{0}} \quad \underline{\phantom{0}} \\
- 1.7 & \quad 2.5 \\
\hline
0 & \quad .6 \quad 2.5 \\
\end{align*}
\]

**Solution** The first pipe is 0.625 meter longer than the second pipe.
Example 3
Liana and Terrence each bought orange juice for a school brunch. Liana bought a container with 1.89 liters of orange juice. Terrence bought 3 small containers, each of which held 0.473 liter of orange juice. Who bought more orange juice? How much more orange juice, in liters, did that student buy?

Strategy  Add. Compare. Then subtract.

Step 1  Add to find the total number of liters Terrence bought.

\[
\begin{array}{c}
1.2 \\
0.473 \\
0.473 \\
+ 0.473 \\
\hline
1.419 \\
\end{array}
\]

Step 2  Compare Liana’s total to Terrence’s total.

Liana bought 1.89 liters. Terrence bought 1.419 liters.

\[1.89 > 1.419,\text{ so Liana bought more orange juice than Terrence.}\]

Step 3  Subtract Terrence’s total from Liana’s total.

First write a 0 in the thousandths place for 1.89.

\[
\begin{array}{c}
810 \\
1.890 \\
- 1.419 \\
\hline
0.471 \\
\end{array}
\]

Solution  Liana bought 0.471 liter of orange juice more than Terrence did.
Melanie had a budget of $225 for costumes for the school play. She spent $86.56 on jackets and $45.38 on hats. How much money does Melanie have left in the budget?

First add to find how much money Melanie has spent so far.

The jackets cost $______.
The hats cost $______.

Add. Show your work in the space below.

Then subtract the total amount spent from $225.
Remember that 225 = 225.00

Subtract. Show your work in the space below.

Melanie has $______ left in the budget.
Multiplying decimals is similar to multiplying whole numbers. Multiply the factors as if there were no decimal points. Then, add the number of decimal places in the factors. That sum is the number of decimal places in the product.

Example 1
What is the product of $0.4 \times 0.6$?

**Strategy** Multiply the factors. Then place the decimal point in the product.

**Step 1** Multiply as you would with whole numbers.

\[
\begin{array}{c}
2 \\
0.4 \\
\times 0.6 \\
\hline
24
\end{array}
\]

You do not need to multiply by 0 since the product would be 0.

**Step 2** Add the number of decimal places in the factors. Insert the decimal point in the product.

\[
\begin{array}{c}
2 \\
0.4 \quad \text{← 1 decimal place} \\
\times 0.6 \quad \text{← 1 decimal place} \\
\hline
0.24 \quad \text{← 1 + 1, or 2, decimal places}
\end{array}
\]

**Solution** $0.4 \times 0.6 = 0.24$
Example 2
Karin buys fancy yarn at a cost of $0.36 per yard. She uses 0.5 yard of that yarn to make a necklace. How much did the yarn used to make the necklace cost?

**Strategy** Multiply the factors. Then place the decimal point in the product.

\[
\begin{array}{c}
3 \\
0.36 \quad \Rightarrow \text{2 decimal places} \\
\times 0.5 \quad \Rightarrow \text{1 decimal place} \\
\hline
0.180 \quad \Rightarrow \text{3 decimal places}
\end{array}
\]

Drop the zero after the 8 since this is an amount of money:

\[0.180 = 0.18\]

**Solution** The cost of the yarn used to make the necklace is $0.18.

Example 3
Multiply.

\[
\begin{array}{c}
1.748 \\
\times 4.6
\end{array}
\]

**Strategy** Multiply the factors. Then place the decimal point in the product.

**Step 1** Multiply by the 6 tenths.

\[
\begin{array}{c}
4 \quad 24 \\
1.748 \\
\times 4.6 \\
\hline
10488
\end{array}
\]

**Step 2** Multiply by the 4 ones.

Write a 0 in the partial product so that you begin writing the partial product in the correct place. Then multiply.

\[
\begin{array}{c}
2 \quad 13 \\
4 \quad 24 \\
1.748 \\
\times \quad 4.6 \\
\hline
10488 \\
69920
\end{array}
\]
Lesson 8: Multiply and Divide Decimals

Step 3  Add the partial products and insert the decimal point.

\[
\begin{align*}
213 \\
424 \\
1.748 & \quad \text{3 decimal places} \\
\times 4.6 & \quad \text{1 decimal place} \\
10488 \\
+ 69920 & \\
80408 & \quad \text{4 decimal places}
\end{align*}
\]

Solution  \(1.748 \times 4.6 = 8.0408\)

Dividing decimals is similar to dividing whole numbers.

When dividing a decimal by a whole number, place the decimal point in the quotient above the decimal point in the dividend. Then divide as you would with whole numbers. If necessary, insert zeros after the last nonzero digit in the dividend until you find a decimal quotient that terminates or repeats.

Example 4
A scientist has 18.6 milliliters of a chemical solution. She divides the solution evenly among 8 different cylinders for an experiment. How many milliliters of solution will be in each cylinder?

Strategy Divide from left to right. If necessary, insert zeros in the dividend.

Step 1  Place the decimal point in the quotient and divide as you would divide whole numbers.

\[
\begin{align*}
2.3 \\
8)18.6 \\
-16 & \quad \text{Multiply: } 2 \times 8 = 16 \\
26 & \quad \text{Subtract: } 18 - 16 = 2. \text{ Bring down the 6.} \\
-24 & \quad \text{Multiply: } 3 \times 8 = 24 \\
2 & \quad \text{Subtract: } 26 - 24 = 2
\end{align*}
\]
Step 2  There is a remainder of 2. Insert a 0 in the dividend and continue dividing.

\[
\begin{array}{c}
2.32 \\
8)18.60 \\
\hline
-16 \\
\hline
26 \\
-24 \\
\hline
20 \\
\hline
16 \\
\hline
4 \\
\end{array}
\]

Bring down the 0.

Multiply: \(2 \times 8 = 16\)

Subtract: \(20 - 16 = 4\)

Step 3  There is a remainder of 4. Insert another 0 in the dividend and continue dividing.

\[
\begin{array}{c}
2.325 \\
8)18.600 \\
\hline
-16 \\
\hline
26 \\
-24 \\
\hline
20 \\
\hline
16 \\
\hline
40 \\
\hline
40 \\
\hline
0 \\
\end{array}
\]

Bring down the 0.

Multiply: \(5 \times 8 = 40\)

Subtract: \(40 - 40 = 0\)

There is no remainder. The quotient is a terminating decimal.

Solution  Each cylinder will have 2.325 milliliters of the chemical solution.

To divide with decimal divisors, multiply the dividend and the divisor by the same power of 10 to ensure a whole-number divisor. A power of 10 is the number 10 written as a base with an exponent. Multiplying by a power of 10 is like moving the decimal point to the right.
Example 5
Divide: 94.575 ÷ 3.9

**Strategy**  Multiply by a power of 10 to ensure a whole-number divisor, then divide.

**Step 1**  Multiply the dividend and divisor by a power of 10.
Since the divisor goes to tenths, multiply by 10.

\[
3.9)\ 94.575 \\
3.9 \times 10 = 39 \\
94.575 \times 10 = 945.75
\]

**Step 2**  Rewrite the problem with the new dividend and divisor.
Place the decimal point in the quotient.

\[
\begin{array}{c|c}
39 & 945.75 \\
\end{array}
\]

**Step 3**  Divide each place from left to right.

\[
\begin{array}{c|c}
24.25 & 945.75 \\
39 & 945.75 \\
-78 & \underline{165} \\
-156 & \underline{195} \\
-97 & 0
\end{array}
\]

**Solution**  \( 94.575 \div 3.9 = 24.25 \)
Coached Example

Mr. Giamatti bought 7.5 pounds of tea leaves. He spent a total of $142.50. How much did each pound of tea leaves cost?

To rename the divisor as a whole number, ______ the dividend and divisor by ______.

\[ 7.5 \times \_\_\_\_ = \_\_\_\_ \]
\[ $142.50 \times \_\_\_\_ = \_\_\_\_ \]

Divide: \[ \_\_\_\_ \div \_\_\_\_ \]

Write the problem vertically with the new divisor and dividend. Then divide. Show your work.

The quotient is \_\_\_\_\_.

Each pound of tea leaves cost $\_\_\_\_.


Divide Fractions and Mixed Numbers

Getting the Idea

To divide a number by a fraction, multiply the number by the reciprocal of the fraction. Two numbers are reciprocals if their product is 1.

To find the reciprocal of a fraction, switch its numerator and denominator. For example, the reciprocal of \( \frac{3}{5} \) is \( \frac{5}{3} \) since \( \frac{3}{5} \times \frac{5}{3} = 1 \).

You can use models to help you divide fractions.

Example 1

Jeffrey has a wooden board that is \( \frac{2}{3} \) yard long. He wants to cut the board into pieces that are \( \frac{1}{9} \) yard long. How many pieces will Jeffrey cut?

Strategy Write a division sentence. Use models to find the quotient.

Step 1 Write a division sentence to represent the situation.

Let \( p \) represent the number of \( \frac{1}{9} \)-yard pieces Jeffrey will cut.

\[
\frac{2}{3} \div \frac{1}{9} = p
\]

Step 2 Draw a rectangle. Divide it into 3 equal parts. Shade the model to represent \( \frac{2}{3} \).

Step 3 Find the number of \( \frac{1}{9} \)s in \( \frac{2}{3} \).

Divide the rectangle into 9 equal parts.
Step 4  Count the number of shaded sections.

There are 6 shaded sections.

Solution  Jeffrey will cut the board into 6 pieces.

Multiplication and division are inverse operations. For example, the division sentence \(\frac{3}{4} \div \frac{1}{12} = 9\) is related to the multiplication sentence \(\frac{1}{12} \times 9 = \frac{3}{4}\).

You can use related multiplication sentences to check your work in a division sentence.

You can cancel like terms by dividing a numerator and a denominator by the GCF or a common factor before performing the multiplication. If you use the GCF, the fraction part of the quotient will be in simplest form.

Example 2

\[ \frac{3}{10} \div \frac{5}{6} = \boxed{} \]

Strategy  Find the reciprocal of the divisor and multiply.

Step 1  Write the reciprocal of the divisor, \(\frac{5}{6}\).

\[
\frac{5}{6} \times \frac{6}{5} = 1, \text{ so the reciprocal of } \frac{5}{6} \text{ is } \frac{6}{5}.
\]

Step 2  Rewrite the division as multiplication.

\[
\frac{3}{10} \div \frac{5}{6} = \frac{3}{10} \times \frac{6}{5}
\]

Step 3  Simplify and multiply.

Simplify: divide 10 and 6 by the GCF, 2. Multiply.

\[
\frac{3}{10} \times \frac{6}{5} = \frac{3}{10} \times \frac{6}{5} = \frac{9}{25}
\]

Solution  \(\frac{3}{10} \div \frac{5}{6} = \frac{9}{25}\)

Example 3

Write a story problem using \(\frac{3}{4} \div \frac{1}{8}\) involving a street and stop signs on that street. Then find the quotient of the problem.

Strategy  Write a story problem by breaking the division problem into parts.

Step 1  Look at the dividend \(\frac{3}{4}\).

This is the number that the divisor will be divided by.

A street is \(\frac{3}{4}\) mile long.
Lesson 9: Divide Fractions and Mixed Numbers

Step 2
Look at the divisor $\frac{1}{8}$.

This is the what $\frac{3}{4}$ will be divided by.

There are sets of stop signs every $\frac{1}{8}$ mile.

Step 3
Determine your question.

How many sets of stop signs are on the street?

Step 4
Answer your question by dividing $\frac{3}{4} \div \frac{1}{8}$.

Rewrite the division as multiplication.

Step 5
Simplify and multiply.

Divide 4 and 8 by the GCF, 4.

Solution

A story problem could be as follows: A street is $\frac{3}{4}$ mile long. There are sets of stop signs every $\frac{1}{8}$ mile. How many sets of stop signs are there on the street? The answer is 6 sets of stop signs.

Another way to divide fractions is to rewrite the division problem as one fraction and then simplify:

\[
\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}
\]

Example 4
Divide $\frac{3}{7} \div \frac{5}{8}$

Strategy
Rewrite the problem as one fraction.

Solution

$\frac{3}{7} \div \frac{5}{8} = \frac{3 \times 8}{7 \times 5} = \frac{24}{35}$
To divide mixed numbers, rename the mixed numbers as **improper fractions**, and then multiply the dividend by the reciprocal of the divisor.

**Example 5**

Jessica worked a total of $8\frac{1}{2}$ hours at the local soup kitchen as a volunteer over the holidays. She worked $2\frac{3}{4}$ hours on her first day. What fraction of the total time did Jessica work that first day?

**Strategy**  Write a division sentence. Rename mixed numbers as improper fractions. Then solve.

**Step 1**  Write a division sentence to represent the situation.

Let $f$ represent the fraction of the total time she worked on the first day.

$$\frac{2\frac{3}{4}}{8\frac{1}{2}} = f$$

**Step 2**  Rename the mixed numbers as improper fractions.

$$2\frac{3}{4} = \frac{11}{4} \quad \text{and} \quad 8\frac{1}{2} = \frac{17}{2}$$

**Step 3**  Rewrite the division problem as a multiplication problem, using the reciprocal of the divisor.

$$\frac{11}{4} \div \frac{17}{2} = \frac{11}{4} \times \frac{2}{17}$$

**Step 4**  Simplify and multiply.

Divide 4 and 2 by the GCF, 2.

$$\frac{11}{4} \times \frac{2}{17} = \frac{11}{4} \times \frac{1}{17} = \frac{11}{68}$$

**Solution**  Jessica worked $\frac{11}{34}$ of the total number of hours she worked at the soup kitchen on her first day.
Mrs. Castillo’s son and his friends were helping her clean up the backyard after a storm. She made \( \frac{7}{8} \) gallon of lemonade for her son and his friends. If each serving of lemonade is 1 cup, how many servings can Mrs. Castillo offer?
(Note: 1 cup = \( \frac{1}{16} \) gallon)

Write a division sentence to represent the situation.
Let \( s \) represent the number of servings.
\[
\frac{7}{8} \div \frac{3}{16} = s
\]

Write the reciprocal of the divisor.
\[
\frac{1}{16} \times \frac{1}{3} = 1, \text{ so the reciprocal of } \frac{1}{16} \text{ is } \frac{1}{3}.
\]

Rewrite the division as multiplication.
\[
\frac{7}{8} \times \frac{16}{3} = \frac{7}{1} \times \frac{16}{3} = \frac{7 \times 16}{3} = \frac{112}{3}
\]

Mrs. Castillo can offer \( \frac{112}{3} \) servings of lemonade.
You can use a coordinate plane to locate points. A coordinate plane is formed by a horizontal number line, called the **x-axis**, and a vertical number line, called the **y-axis**. Each axis includes both positive and negative numbers. The coordinate plane is divided into four sections called **quadrants**. They are numbered with Roman numerals in a counterclockwise direction, as shown below.

By looking at whether the **x**- and **y**-coordinates are positive or negative, you can tell which quadrant contains a given point without seeing it graphed on a coordinate plane. Use the table below to help you.

<table>
<thead>
<tr>
<th>Quadrant</th>
<th>x-coordinate</th>
<th>y-coordinate</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>+</td>
<td>+</td>
</tr>
<tr>
<td>II</td>
<td>-</td>
<td>+</td>
</tr>
<tr>
<td>III</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>IV</td>
<td>+</td>
<td>-</td>
</tr>
</tbody>
</table>

Points on the **x**-axis or the **y**-axis are not in any quadrant.

An **ordered pair** names a point on a coordinate plane. The first number represents the **x-coordinate**, which tells the number of units to the left or right of the **origin**, (0, 0). The second number represents the **y-coordinate**, which tells the number of units above or below the origin.
Example 1
What is the location of point A?

Strategy
Find the value of the coordinates.

Step 1
Find the value of the x-coordinate.
Point A is 4 units to the left of the origin, which is \(-4\).

Step 2
Find the value of the y-coordinate.
Point A is 6 units above the origin, which is \(6\).

Step 3
Write the ordered pair.
Point A is located at \((-4, 6)\).

Solution
The location of point A is \((-4, 6)\).
Example 2

Plot (5.5, −3.5) on a coordinate plane. Label the point $M$.

**Strategy**  Use ordered pairs to plot a point.

**Step 1**  Start at the origin. Find the $x$-coordinate for point $M$.
- The $x$-coordinate is 5.5.
- The point will be halfway between 5 and 6 on the $x$-axis.

**Step 2**  From 5.5 on the $x$-axis, find the $y$-coordinate for point $M$.
- The $y$-coordinate is −3.5.
- The point will be halfway between −3 and −4 on the $y$-axis.

**Solution**  Point $M$ is shown on the graph below.

![Coordinate Plane with Point M](image-url)
Coached Example

The coordinate plane below represents the streets in Brad and Cara’s town.

Brad’s house is at \((-6, \frac{6}{2})\) and Cara’s house is at \((4\frac{1}{2}, -5)\). Plot and label the points for both houses.

Start with Brad’s house.
The ___-coordinate is negative and the ___-coordinate is positive.
Start at the origin, which is the point (____, ____).
Move 6 units to the __________ of the origin.
From ____ on the x-axis, move __________ \(\frac{6}{2}\) units.
Plot the point and label it “B” for Brad.

Now locate Cara’s house.
The ___-coordinate is positive and the ___-coordinate is negative.
Start at the origin and move \(4\frac{1}{2}\) units to the __________.
From ____ on the x-axis, move __________ 5 units.
Plot the point and label it “C” for Cara.
Getting the Idea

A point can be flipped over a line. The new point is a reflection of the original point. When a point is reflected across the $x$-axis, the $y$-axis, or both axes on a coordinate plane, the signs of one or both coordinates will change. The new point will be in a different quadrant.

Example 1

Plot point $A$ at $(-2, 4)$ on the coordinate plane. Then plot point $B$ at $(-2, -4)$. Compare the coordinates and locations of points $B$ and $A$.

Strategy

Plot points $A$ and $B$ on a coordinate plane. Compare their coordinates. Describe how one point is a reflection of the other.

Step 1

Plot point $A$ at $(-2, 4)$ and point $B$ at $(-2, -4)$.

Step 2

Compare their coordinates.

The $x$-coordinates are the same and the $y$-coordinates are opposites.

Step 3

Describe the locations of the points.

Both points lie on the same vertical line, $x = -2$.

Point $A$ is in Quadrant II and is 4 units above the $x$-axis.

Point $B$ is in Quadrant III and is 4 units below the $x$-axis.

So, point $B$ is a reflection of point $A$ over the $x$-axis.

Solution

The coordinate plane above shows points $A$ and $B$. The points are reflections of each other across the $x$-axis, and their $y$-coordinates are opposites.
The statements below show the relationship between reflected points and the signs of their coordinates.

- If a point \((x, y)\) is reflected across the \(x\)-axis, the sign of its \(y\)-coordinate changes.
  \((x, y) \rightarrow (x, -y)\)

- If a point \((x, y)\) is reflected across the \(y\)-axis, the sign of its \(x\)-coordinate changes.
  \((x, y) \rightarrow (-x, y)\)

- If a point \((x, y)\) is reflected across both axes, the signs of both of its coordinates change.
  \((x, y) \rightarrow (-x, -y)\)

You can use absolute value to find the distance between two points on a coordinate plane. Each axis on a coordinate plane is a number line. Since the absolute value of a number is its distance from zero on a number line, you can use absolute values to find horizontal and vertical distances on the coordinate plane.

**Example 2**

A map of some places in Carr County is shown at the right. What is the distance, in kilometers, between City Hall and the library?

**Strategy** Use absolute values to find the distance between points. Then use the scale to find the distance in kilometers.

**Step 1** Identify the ordered pairs.
- City Hall is at \((-6, 3)\).
- The library is at \((4, 3)\).

**Step 2** Use absolute value to find the distance from City Hall to the \(y\)-axis.
- The \(x\)-coordinate for City Hall is \(-6\).
- \(|-6| = 6\)
- The distance between City Hall and the \(y\)-axis is 6 units.
Step 3  Use absolute value to find the distance from the public library to the y-axis.

The x-coordinate for the library is 4.

\[ |4| = 4 \]

The distance between the library and the y-axis is 4 units.

Step 4  Add the absolute values to find the total distance.

The total distance between City Hall and the library on the map is:

\[ 6 + 4 = 10 \text{ units} \]

Each unit on the map represents 1 kilometer, so the actual distance is 10 kilometers.

Solution  The distance between City Hall and the library is 10 kilometers.

You can also use absolute values to find the perimeter of a figure on a coordinate plane.

Example 3
A rectangular garden is shown below. What is the perimeter of the garden?

Strategy  Use absolute values to find the length of one vertical side and one horizontal side. Then find the perimeter.
Step 1  Use absolute value to find the length of a vertical side.

The endpoints of one vertical side are \((3, 6)\) and \((3, -6)\). The \(y\)-coordinates for the endpoints are 6 and \(-6\).

\[|6| = 6 \quad |{-6}| = 6\]

The total length of a vertical side is: \(6 + 6 = 12\) units

Step 2  Use absolute value to find the length of a horizontal side.

The endpoints of one horizontal side are \((-5, 6)\) and \((3, 6)\). The \(x\)-coordinates of the endpoints are \(-5\) and 3.

\[|{-5}| = 5 \quad |3| = 3\]

The total length of the horizontal side is: \(5 + 3 = 8\) units

Step 3  Find the perimeter.

\[P = 2(12 + 8) = 40\] units

Each unit on the coordinate plane stands for 1 yard, so the actual perimeter is 40 yards.

Solution  The perimeter of the garden is 40 yards.
Coached Example

Point \(J\) below at \((4, 3)\) can be reflected across one or more axes to form a point at \((-4, -3)\). Describe the reflection(s) that are needed.

Compare the signs of the coordinates in \((4, 3)\) and \((-4, -3)\).
The \(x\)-coordinates of the ordered pairs have different signs.
The ____-coordinates of the ordered pairs have different signs too.

Would point \(J\) need to be reflected across the \(x\)-axis, the \(y\)-axis, or both axes to move to \((-4, -3)\) on the coordinate plane?

Since the signs of both coordinates are different, point \(J\) would need to be reflected across ________________.

Check your answer by reflecting \((4, 3)\) across the \(x\)-axis only.
The coordinates of that reflected point are \((4, ____).\)
Plot that point on the grid above and label it point \(K\).
Now, reflect point \(K\) across the \(y\)-axis.
The coordinates of that reflected point are (____, ____).
Plot that point above and label it point \(L\).

Point \(L\) is at (____, ____). Both of its coordinates have different signs than the coordinates of point \(J\). Point \(L\) is a reflection of point \(J\) across ________________.
A ratio is a comparison of two numbers using division. Ratios are used to show relationships between quantities and can be written in three ways.

<table>
<thead>
<tr>
<th>As a fraction</th>
<th>With a colon</th>
<th>In words</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{a}{b} )</td>
<td>( a:b )</td>
<td>( a \ to \ b )</td>
</tr>
</tbody>
</table>

You can show different kinds of comparisons with a ratio.

- Part to part: The ratio of white dots to gray dots is \( \frac{6}{4} \), 6:4, or 6 to 4.
- Part to whole: The ratio of white dots to the total number of dots is \( \frac{6}{10} \), 6:10, or 6 to 10.
- Whole to whole: The ratio of the number of triangles to the number of dots is \( \frac{0}{10} \), 0:10, or 0 to 10.

As with fractions, you can simplify ratios. For example, the ratio of white dots to gray dots in the diagram above can be simplified as \( \frac{3}{4} \), 3:2, or 3 to 2. A ratio should not be written as a mixed number, so do not write \( \frac{3}{4} \) as \( 1 \frac{1}{2} \).

**Example 1**
What is the ratio of cats to dogs in a neighborhood that has 9 dogs and 3 cats?

**Strategy**
Write a ratio to compare the number of cats to dogs.

**Step 1**
Identify the numbers of cats and dogs.
There are 3 cats and 9 dogs.

**Step 2**
Write the ratio of cats to dogs.
The ratio of cats to dogs is \( \frac{3}{9} \).

**Step 3**
Simplify.
\[
\frac{3}{9} = \frac{3 \div 3}{9 \div 3} = \frac{1}{3}
\]

**Solution**
The ratio of cats to dogs in the neighborhood is \( \frac{1}{3} \), or there is 1 cat for every 3 dogs.
Example 2
What is the ratio of girls to boys in a class of 20 students that has 12 boys? Write the ratio using a colon.

Strategy Write a ratio to compare the number of girls to boys.

Step 1 Find the numbers of boys and girls in the class.
There are 12 boys in the class.
\[
20 - 12 = 8
\]
There are 8 girls in the class.

Step 2 Write the ratio of girls to boys.
The ratio of girls to boys is 8:12.

Step 3 Simplify.
Simplify the ratio as you would if it were written in fraction form.
\[
\frac{8}{12} = \frac{8 \div 4}{12 \div 4} = \frac{2}{3}
\]
So, 8:12 = 2:3.

Solution The ratio of girls to boys in the class is 2:3. There are 2 girls for every 3 boys.

Example 3
The table shows the eye colors of the students in Mr. Matthew’s class.

<table>
<thead>
<tr>
<th>Eye Colors of Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>Color</td>
</tr>
<tr>
<td>Brown</td>
</tr>
<tr>
<td>Blue</td>
</tr>
<tr>
<td>Green</td>
</tr>
<tr>
<td>Hazel</td>
</tr>
</tbody>
</table>

For every student with green eyes, how many students have brown eyes?

Strategy Find the ratio in simplest form.

Step 1 Find the ratio of students with green eyes to students with brown eyes.
There are 3 students with green eyes and 15 students with brown eyes.
The ratio of students with green eyes to students with brown eyes is \(\frac{3}{15}\).

Step 2 Simplify.
\[
\frac{3}{15} = \frac{3 \div 3}{15 \div 3} = \frac{1}{5}
\]

Solution For every student with green eyes, there are 5 students with brown eyes.
Coached Example

Matt’s movie DVD collection contains 8 comedies, 6 dramas, 3 adventures, and 7 science fictions. What is the ratio of comedies and dramas to the total number of DVDs in Matt’s collection? Write the ratio using the word to.

The ratio compares the total number of ____________ and ____________ to the total number of ____________.

There are _____ comedies and _____ dramas.

Add: _____ + _____ = ______

There are _____ + _____ + _____ + _____ = _____ DVDs in all.

Write the ratio. ______ to ______

Simplify the ratio.
\[
\frac{14}{24} = \frac{14 \div \_}{24 \div \_}
\]

So, _____ to _____ can be simplified as _____ to _____.

The ratio of comedies and dramas to total DVDs is ____________.
Equivalent Ratios

Getting the Idea

A ratio can be written in simplest form. For example, \( \frac{8}{6} \) or 8:6 can be written in simplest form as \( \frac{4}{3} \) or 4:3. The ratios \( \frac{8}{6} \) and \( \frac{4}{3} \) are equivalent ratios.

Models can help you tell if two ratios are equivalent.

Example 1

Use models to determine if the ratios below are equivalent.

3:4 and 9:12

Strategy

Model each ratio. Then try to separate 9:12 into groups of 3:4.

Step 1

Model 3:4 using white dots and gray dots.

Step 2

Model 9:12 using white dots and gray dots.

Step 3

Try to separate the model for 9:12 into groups of 3:4.

Circle groups of 3 white dots and 4 gray dots.

Solution

The ratios 3:4 and 9:12 are equivalent.
A second way to determine if two ratios or rates form equivalent ratios is to find a common denominator. A **rate** is a ratio that compares two quantities with different units of measure. When comparing rates, it does not matter which is the numerator and which is the denominator, but the comparison must be consistent.

**Example 2**
There are two photocopy machines in an office. The first machine produces 5 copies in 8 seconds. The second machine produces 16 copies in 24 seconds. Are the two machines making copies at the same rate?

**Strategy**  
Use a common denominator.

**Step 1**  
Express each rate as a fraction.

\[
\begin{align*}
5 \text{ copies in } 8 \text{ seconds} &= \frac{5}{8} \\
16 \text{ copies in } 24 \text{ seconds} &= \frac{16}{24}
\end{align*}
\]

**Step 2**  
Find a common denominator.

Because 24 is a multiple of 8, the LCD is 24.

**Step 3**  
Rename \(\frac{5}{8}\) with 24 as the denominator.

\[
\frac{5 \times 3}{8 \times 3} = \frac{15}{24}
\]

\[
\frac{15}{24} \neq \frac{16}{24}
\]

**Solution**  
The two machines are not making copies at the same rate.

Two other ways to determine if two ratios or rates are equivalent ratios are 1) to write each rate or ratio in simplest form or 2) to **cross multiply**. To cross multiply two fractions, multiply each numerator by the denominator of the other fraction. If the **cross products** are equivalent, the ratios or rates are equivalent.

To cross multiply the problem above:

\[
\begin{align*}
\frac{5}{8} & \neq \frac{16}{24} \\
5 \times 24 & \neq 8 \times 16 \\
120 & \neq 128
\end{align*}
\]

The cross products are not equivalent, so the ratios are not equivalent.
You can cross multiply to find a missing value in a ratio or rate.

**Example 3**

Emma is driving at a constant speed. She drives 4 miles in 5 minutes. If she continues to drive at the same speed, how many miles will she drive in 15 minutes?

**Strategy**

Write two ratios to represent the situation. Cross multiply to find the missing term.

**Step 1**

Write two ratios.

The first ratio is: \( \frac{4 \text{ miles}}{5 \text{ minutes}} \) or \( \frac{4}{5} \).

You want to find the number of miles she will drive in 15 minutes.

The second ratio is: \( \frac{x \text{ miles}}{15 \text{ minutes}} \) or \( \frac{x}{15} \).

**Step 2**

Cross multiply to find the missing term.

\[
\frac{4}{5} = \frac{x}{15} \\
4 \times 15 = 5 \times x \\
60 = 5x \\
\frac{60}{5} = 5x \\
12 = x
\]

**Solution**

Emma will drive 12 miles in 15 minutes.

You could also use a double number line to solve Example 3. The double number line shows the distances and times produced by a constant speed.

The double number line above shows that 4 miles in 5 minutes and 12 miles in 15 minutes are equivalent rates of speed.
You can look for patterns in a table to find equivalent ratios.

**Example 4**
A baker made this table to show the number of cups of flour and the number of eggs he needs to make a carrot cake recipe.

<table>
<thead>
<tr>
<th>Carrot Cake Recipe</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cups of Flour (x)</td>
</tr>
<tr>
<td>Number of Eggs (y)</td>
</tr>
</tbody>
</table>

How many eggs are needed if he uses 60 cups of flour?

**Strategy** Find and apply a rule to find an equivalent ratio.

**Step 1** Determine if the ratios in the table are equivalent.

Write each ratio in simplest form. If they simplify to the same fraction, they are equivalent.

\[
\begin{align*}
\frac{2}{4} & \div \frac{2}{2} = \frac{1}{2} \\
\frac{3}{6} & \div \frac{3}{3} = \frac{1}{2} \\
\frac{6}{12} & \div \frac{6}{6} = \frac{1}{2}
\end{align*}
\]

The ratios are equivalent.

**Step 2** Write a rule.

The number of eggs (y) is equal to 2 times the number of cups of flour (x).

\[y = 2x\]

**Step 3** Use the rule for 60 cups of flour.

Substitute 60 for x.

\[y = 2 \times 60\]

\[y = 120\]

**Solution** If the baker uses 60 cups of flour, he needs 120 eggs.
You can also plot ordered pairs, \((x, y)\), on a coordinate grid to find equivalent ratios. If the ratios of \(x:y\) are equivalent, the points will lie along the same straight line.

**Example 5**  
The table shows the cost of shipping a package for different package weights.

<table>
<thead>
<tr>
<th>Weight in Pounds ((x))</th>
<th>1</th>
<th>2</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cost in Dollars ((y))</td>
<td>3</td>
<td>6</td>
<td>12</td>
</tr>
</tbody>
</table>

Plot each ordered pair on a coordinate grid. Then use that graph to find two more equivalent rates.

**Strategy**  
Plot the ordered pairs on a coordinate grid. Graph the line for the ordered pairs. Find two more ordered pairs on the line.

**Step 1**  
Plot points for each pair of values in the table.  
Plot \((1, 3)\), \((2, 6)\), and \((4, 12)\) on a coordinate grid.

**Step 2**  
Draw a line through the points. Extend the line.  
Name two other points on the line.  
The line through points \((1, 3)\), \((2, 6)\), and \((4, 12)\) shows that the ratios \(\frac{1}{3}\), \(\frac{2}{6}\), and \(\frac{4}{12}\) are equivalent.  
The points \((3, 9)\) and \((5, 15)\) are also on the line.  
This shows that \(\frac{3}{9}\) and \(\frac{5}{15}\) are equivalent to the other ratios.  
The points represent these rates:  
3 pounds for \$9\) and 5 pounds for \$15\).

**Solution**  
Two additional equivalent rates are 3 pounds for \$9\) and 5 pounds for \$15\).
Hudson bought several cans of tennis balls. Each can contained both green and yellow tennis balls. He purchased 10 green tennis balls and 5 yellow tennis balls in all. If each can has 2 green tennis balls in it, how many yellow tennis balls are in each can?

Write two ratios for this problem.

Write the first ratio.

\[
\frac{10 \text{ green tennis balls}}{\text{yellow tennis balls}} \quad \text{or} \quad \frac{10}{\text{yellow tennis balls}}
\]

You want to find the number of yellow tennis balls in a can with 2 green tennis balls.

Write the second ratio.

\[
\frac{2 \text{ green tennis balls}}{x \text{ yellow tennis balls}} \quad \text{or} \quad \frac{2}{x}
\]

Set the ratios equal to each other. Cross multiply to find the number of yellow tennis balls in each can.

\[
\frac{10}{x} = \frac{2}{x}
\]

\[10 \times x = \underline{\quad} \times 2\]

\[10x = \underline{\quad}\]

\[\frac{10x}{10} = \underline{\quad}\]

\[x = \underline{\quad}\]

Each can contains 2 green tennis balls and _____ yellow tennis ball(s).
Getting the Idea

Some examples of rates are shown below:

- Miles per gallon: 540 miles on 18 gallons of gas, \( \frac{540 \text{ miles}}{18 \text{ gallons}} \)
- Cost: $3.60 for 4 pounds, or \( \frac{$3.60}{4 \text{ pounds}} \)
- Pay rate: $285 for 30 hours, or \( \frac{$285}{30 \text{ hours}} \)

Rates are often given as a unit rate, which is a rate in which the second measure is 1 unit. Each of the rates listed above can be simplified as unit rates.

- Miles per gallon: \( \frac{540 \text{ miles}}{18 \text{ gallons}} = \frac{30 \text{ miles}}{1 \text{ gallon}} \)
- Cost: \( \frac{$3.60}{4 \text{ pounds}} = \frac{$0.90}{1 \text{ pound}} \)
- Pay rate: \( \frac{$285}{30 \text{ hours}} = \frac{$9.50}{1 \text{ hour}} \)

In general, for every ratio \( a:b \), the corresponding unit rate is \( \frac{a}{b} \), where \( b \neq 0 \).

For example, if there are 4 cups of cranberry juice to every 5 cups of orange juice in a punch recipe, the ratio of cranberry juice to orange juice is 4:5, or \( \frac{4}{5} \).

**Example 1**

A recipe for trail mix uses 5 ounces of mixed nuts, 6 ounces of dried fruit, and 4 ounces of granola. How many ounces of granola are there for every ounce of dried fruit?

**Strategy**

Write a ratio. Then find the unit rate.

**Step 1**

Write the ratio of granola to dried fruit.

For every 4 ounces of granola, there are 6 ounces of dried fruit.

The ratio of granola to dried fruit is 4:6, or \( \frac{2}{3} \).

**Step 2**

Interpret the ratio as a unit rate.

The ratio 2:3 means that there is \( \frac{2}{3} \) ounce of granola for every ounce of dried fruit.

**Solution**

There is \( \frac{2}{3} \) ounce of granola for each ounce of dried fruit.
To find a unit price, identify the quantities you want to compare and write a rate. Then simplify the rate to find the unit price.

Example 2
Mr. Wilson spent $252 to stay 3 nights at Pavia Pavilions. At that rate, how much will he spend to stay 7 nights?

**Strategy** Find the unit price. Then multiply by 7 nights.

**Step 1** Find the rate.
The rate is $252 for 3 nights, or \( \frac{252}{3} \).

**Step 2** Find the unit rate, or unit price.

\[
\begin{array}{c}
3 \) \ 252 \\
-24 \\
12 \\
-12 \\
0
\end{array}
\]

The unit price is $84 per night.

**Step 3** Multiply the unit price by 7.

\[7 \times 84 = 588\]

**Solution** Mr. Wilson will spend $588 to stay 7 nights at Pavia Pavilions.

In Example 2, you could also have set up equivalent ratios to solve the problem. Let \( x \) represent the cost of staying 7 nights.

\[
\frac{252}{3} = \frac{x}{7}
\]

\[3 \times x = 252 \times 7 \quad \text{Cross multiply.}\]

\[3x = 1,764 \quad \text{Divide both sides by 3.}\]

\[x = 588\]
A common use of rate is the speed formula \( r = \frac{d}{t} \), or rate = \( \frac{\text{distance}}{\text{time}} \).

**Example 3**
A train is traveling at a constant speed of 45 miles per hour. How far will the train travel in 2.5 hours?

**Strategy** Use the speed formula.

**Step 1** Substitute the known values in the speed formula \( r = \frac{d}{t} \).

\[
45 = \frac{d}{2.5} \quad \text{or} \quad 45 \times \frac{2.5}{1} \quad \text{or} \quad \frac{45}{1} = \frac{d}{2.5}
\]

**Step 2** Find an equivalent fraction for \( \frac{45}{1} \) with a denominator of 2.5.

\[
\frac{45}{1} = \frac{45 \times 2.5}{1 \times 2.5} = \frac{112.5}{2.5}
\]

**Solution** The train will travel 112.5 miles in 2.5 hours.

You can rewrite the speed formula \( r = \frac{d}{t} \) to solve for either distance, \( d \), or time, \( t \).

If \( r = \frac{d}{t} \), then \( d = r \times t \).

If \( r = \frac{d}{t} \), then \( t = \frac{d}{r} \).

In Example 3, you could have used the formula \( d = r \times t \) to solve the problem.

\[
d = r \times t \\
d = 45 \times 2.5 \\
d = 112.5
\]

---

**Coached Example**
Tanya walked 15 laps on an indoor track in 30 minutes. What was Tanya’s average speed in laps per minute?

The speed formula is \( r = \) _____.

The distance is _____ laps.

The time is _____ minutes.

Substitute the known values into the speed formula.

\( r = \) _____.

Simplify the fraction.

\( r = \) _____

Tanya’s average speed was _____ laps per minute.
Percents

Getting the Idea

A percent is a ratio that means “per hundred.” The symbol for percent is %. A percent can be written as a fraction with a denominator of 100 or as a decimal. To write a percent as a decimal, divide the percent by 100 and remove the percent sign. Dividing by 100 is the same as moving the decimal point two places to the left. For example, \(45\% = \frac{45}{100} = 0.45\).

Example 1

What fraction, percent, and decimal name the shaded part of this grid?

Strategy  Write the fraction of the grid that is shaded. Then convert the fraction to a percent and a decimal.

Step 1  Write the fraction.
  Count the number of shaded squares.
  There are 48 shaded squares. There are a total of 100 squares.
  \(\frac{48}{100}\) of the grid is shaded.

Step 2  Write the percent.
  Write the numerator of the fraction with a percent symbol after it.
  \(\frac{48}{100} = 48\%\)

Step 3  Write the decimal.
  Divide the percent by 100 and drop the percent sign.
  \(48\% \div 100 = 0.48\)

Solution  \(\frac{48}{100}, 48\%,\) and 0.48 name the shaded part of the grid.
You can use multiplication to solve percent problems. When you multiply a number by a percent, you are finding a part of a whole. The formula below will help you.

\[ \text{percent} \times \text{whole} = \text{part} \]

**Example 2**
What is 25% of 120?

**Strategy** Find a part of a whole using the formula \( \text{percent} \times \text{whole} = \text{part} \).

**Step 1** Identify the known and unknown values in the formula.
- The percent is 25%.
- The whole is 120.
- The part is unknown.

**Step 2** Write 25% as a fraction in simplest form.
\[
\frac{25}{100} \div \frac{25}{25} = \frac{1}{4}
\]

**Step 3** Substitute the values into the formula and solve.
\[
\frac{1}{4} \times 120 = \frac{120}{4} = 30
\]

**Solution** 25% of 120 is 30.

**Example 3**

There are 180 students going on a class field trip. Of those students, 40% are boys. How many students on the field trip are boys?

**Strategy** Write the percent as a fraction. Multiply.

**Step 1** Write the percent as a fraction in simplest form.
\[
40\% = \frac{40}{100} \div \frac{20}{20} = \frac{2}{5}
\]

**Step 2** Use the formula: \( \text{percent} \times \text{whole} = \text{part} \).
Multiply the total number of students by the fraction.
\[
\frac{2}{5} \times 180 = \frac{2}{5} \times \frac{180}{1} = 72
\]

**Solution** There are 72 boys on the field trip.
If the part and the percent are known, you can rewrite the formula to find the whole.
If percent \times \text{whole} = \text{part}, then \text{whole} = \text{part} \div \text{percent}.

**Example 4**

A group of students is trying out for the soccer team. Of those students, 22 are seventh graders. If 55% of the students trying out are seventh graders, how many students in all are trying out?

**Strategy**
Find the whole using the formula: \text{whole} = \text{part} \div \text{percent}.

**Step 1**
Rename the percent as a decimal.

\[
55\% = \frac{55}{100} = 0.55
\]

**Step 2**
Substitute known values into the formula.

\[
\text{whole} = \text{part} \div \text{percent}
\]
\[
\text{whole} = 22 \div 0.55
\]

**Step 3**
Multiply by a power of 10.

\[
22 \times 100 = 2,200
\]
\[
0.55 \times 100 = 55
\]

**Step 4**
Divide.

\[
2,200 \div 55 = 40
\]

**Solution**
There are a total of 40 students trying out for the soccer team.

---

**Coached Example**

A pet store has 52 freshwater fish. Of all the fish in the store, 80\% are freshwater fish. How many fish does the store have in all?

To find the total number of fish, use the formula for finding the whole.

\[
\text{whole} = \text{part} \div \text{percent}
\]

Rename the percent as a decimal.

\[
80\% = \frac{80}{100} = _______
\]

Substitute known values into the formula.

\[
\text{whole} = _____ \div _____ = _____
\]

The store has _____ fish in all.
The tables below show some conversions for units of length in both the customary system and the metric system.

<table>
<thead>
<tr>
<th>Customary Units of Length</th>
<th>Metric Units of Length</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 foot (ft) = 12 inches (in.)</td>
<td>1 centimeter (cm) = 10 millimeters (mm)</td>
</tr>
<tr>
<td>1 yard (yd) = 3 feet</td>
<td>1 meter (m) = 100 centimeters</td>
</tr>
<tr>
<td>1 yard = 36 inches</td>
<td>1 meter = 1,000 millimeters</td>
</tr>
<tr>
<td>1 mile (mi) = 5,280 feet</td>
<td>1 kilometer (km) = 1,000 meters</td>
</tr>
<tr>
<td>1 mile = 1,760 yards</td>
<td></td>
</tr>
</tbody>
</table>

You can convert measurements using equivalent ratios.

Example 1
Nancy ran 8 miles. How many yards did she run?

Strategy  
Set up equivalent ratios and cross multiply.

Step 1  
Write a ratio that compares yards to miles.

\[
\frac{\text{yards}}{\text{miles}} = \frac{1,760}{1}
\]

Step 2  
Write a ratio that compares the unknown length to the length you know.

Let \( y \) represent the number of yards.

\[
\frac{\text{yards}}{\text{miles}} = \frac{y}{8}
\]

Step 3  
Set up equivalent ratios using the two ratios.

\[
\frac{1,760}{1} = \frac{y}{8}
\]

Step 4  
Cross multiply.

\[
1,760 \times 8 = 1 \times y
\]

\[
14,080 = y
\]

Solution  
Nancy ran 14,080 yards.
The tables below show conversions among units of weight and mass.

**Customary Units of Weight**

<table>
<thead>
<tr>
<th>1 pound (lb)</th>
<th>= 16 ounces (oz)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 ton (T)</td>
<td>= 2,000 pounds</td>
</tr>
</tbody>
</table>

**Metric Units of Mass**

<table>
<thead>
<tr>
<th>1 gram (g)</th>
<th>= 1,000 milligrams (mg)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 kilogram (kg)</td>
<td>= 1,000 grams</td>
</tr>
<tr>
<td>1 metric ton (t)</td>
<td>= 1,000 kilograms</td>
</tr>
</tbody>
</table>

**Example 2**

How many grams are equal to 5 kilograms?

**Strategy**

Set up equivalent ratios and cross multiply.

**Step 1**

Write a ratio that compares grams to kilograms.

\[
\frac{\text{grams}}{\text{kilograms}} = \frac{1,000}{1}
\]

**Step 2**

Write a ratio that compares the unknown mass to the mass you know.

Let \( g \) represent the number of grams.

\[
\frac{\text{grams}}{\text{kilograms}} = \frac{g}{5}
\]

**Step 3**

Set up equivalent ratios using the two ratios.

\[
\frac{1,000}{1} = \frac{g}{5}
\]

**Step 4**

Cross multiply.

\[
1,000 \times 5 = 1 \times g
\]

\[
5,000 = g
\]

**Solution**

There are 5,000 grams in 5 kilograms.
Compound units can be used to express measurements. For example, you may express a weight in ounces, or you may express the same weight using pounds and ounces.

**Example 3**
A newborn baby weighed 133 ounces. What is the baby’s weight in pounds and ounces?

**Strategy**  Set up equivalent ratios and cross multiply.

**Step 1** Write a ratio that compares ounces to pounds.

\[
\frac{\text{ounces}}{\text{pounds}} = \frac{16}{1}
\]

**Step 2** Write a ratio that compares the unknown weight to the weight you know. Let \( p \) represent the number of pounds.

\[
\frac{\text{ounces}}{\text{pounds}} = \frac{133}{p}
\]

**Step 3** Set up equivalent ratios using the two ratios.

\[
\frac{16}{1} = \frac{133}{p}
\]

**Step 4** Cross multiply.

\[
16 \times p = 1 \times 133
\]

\[
16p = 133
\]

\[
16p \div 16 = 133 \div 16
\]

\[
p = 8 \text{ R}5
\]

The remainder is the additional number of ounces. There are 5 ounces.

**Solution**  The newborn baby weighed 8 pounds 5 ounces.
The tables below show conversions among units of capacity.

### Customary Units of Capacity

<table>
<thead>
<tr>
<th>Unit</th>
<th>Conversion</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 cup (c)</td>
<td>= 8 fluid ounces (fl oz)</td>
</tr>
<tr>
<td>1 pint (pt)</td>
<td>= 2 cups</td>
</tr>
<tr>
<td>1 quart (qt)</td>
<td>= 2 pints</td>
</tr>
<tr>
<td>1 gallon (gal)</td>
<td>= 4 quarts</td>
</tr>
</tbody>
</table>

### Metric Units of Capacity

<table>
<thead>
<tr>
<th>Unit</th>
<th>Conversion</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 liter (L)</td>
<td>= 1,000 milliliters (mL)</td>
</tr>
</tbody>
</table>

### Example 4

A fishbowl has a capacity of 192 fluid ounces. How many quarts is that?

**Strategy**  
Set up equivalent ratios and cross multiply.

**Step 1**  
Find the number of fluid ounces in a quart.

- 1 cup = 8 fluid ounces
- 2 cups = 16 fluid ounces = 1 pint
- 4 cups = 32 fluid ounces = 2 pints = 1 quart

There are 32 fluid ounces in a quart.

**Step 2**  
Write a ratio that compares fluid ounces to quarts.

\[
\frac{\text{fluid ounces}}{\text{quarts}} = \frac{32}{1}
\]

**Step 3**  
Write a ratio that compares the capacity you know to the unknown capacity.

Let \( q \) represent the number of quarts.

\[
\frac{\text{fluid ounces}}{\text{quarts}} = \frac{192}{q}
\]

**Step 4**  
Set up equivalent ratios using the two ratios.

\[
\frac{32}{1} = \frac{192}{q}
\]

**Step 5**  
Cross multiply.

\[
32 \times q = 1 \times 192
\]

\[
32q = 192
\]

\[
32q \div 32 = 192 \div 32
\]

\[
q = 6
\]

**Solution**  
The capacity of the fishbowl is 6 quarts.
**Coached Example**

A basketball player is 78 inches tall. What is his height in feet?

Write a ratio that compares inches to feet. Remember, 12 inches = 1 foot.

\[
\frac{\text{inches}}{\text{feet}} = \frac{\_\_\_\_}{1}
\]

Write a ratio that compares the height you know to the unknown height.

Let \( f \) represent the height in feet.

\[
\frac{\text{inches}}{\text{feet}} = \frac{\_\_\_\_}{f}
\]

Set up equivalent ratios using the two ratios.

\[
\frac{\_\_\_\_}{\_\_\_\_} = \frac{\_\_\_\_}{\_,\_\_\_\_}
\]

Solve. Show your work.

The remainder is the additional number of \_______________. The remainder is \___, so there are \____ additional inches.

1 foot = 12 inches, so 6 inches = \____ foot.

The basketball player is \____ feet tall.
An expression is a mathematical phrase with numbers, operation signs, and variables. You can write an expression to describe a real-world situation. For example, the numerical expression $6 \div 3$ could describe placing 6 students into 3 equal groups.

A variable is a letter or symbol that is used to represent an unknown number in an algebraic expression. In the expression $x + 1$, $x$ is the variable. If a variable occurs in an expression more than once, it refers to the same number each time.

A term is a number, variable, product, or quotient in an expression. A coefficient is the numerical factor in a term with a variable. There are two terms in the expression $3n + 5$. The terms are $3n$ and 5. In the term $3n$, the coefficient is 3.

Example 1
George had 5 apples. His mother gave him a few more apples to share with his friends. Write an expression to represent the total number of apples George has.

Strategy Choose a variable for the unknown number. Write a word expression.

Step 1 Choose a variable.
Let $x$ represent the number of apples George’s mother gave him.

Step 2 Write the word expression for the number of apples.
$x$ more than 5

Step 3 Write the expression.
$x + 5$

Solution George has a total of $5 + x$ apples.
Example 2
Write an expression for this statement: four times as much as a number \( k \).

**Strategy** Write the word expression.

Write the expression using the operation and values.

\[ 4 \times k, \text{ or } 4k \]

**Solution** An expression for “four times as much as a number \( k \)” is \( 4k \).

Example 3
Michelle writes 15 pages in her journal each week, plus an extra 5 pages on her birthday. Michelle’s birthday was this week. Write an expression to show how many pages Michelle has written this year.

**Strategy** Break the expression into parts.

**Step 1** Write an expression for the number of pages written each week.

Michelle writes 15 pages each week.

Let \( w \) represent the number of weeks.

\[ 15w \]

**Step 2** Write an expression for the extra pages.

The pages are extra, so add 5 pages.

\[ 15w + 5 \]

**Solution** Michelle has written \( 15w + 5 \) journal pages this year.
Example 4
Write an expression for this statement: the product of 8 and the difference of a number and 12.

Strategy Break the expression into parts.

Step 1 Write an expression for the difference of a number and 12.
12 is being subtracted from a number.
Let \( n \) represent the number.
\( n - 12 \)

Step 2 Write the expression for the entire statement.
Write an expression for the product of 8 and \( n - 12 \).
\( 8 \times (n - 12) \), or \( 8(n - 12) \)

Solution An expression for “the product of 8 and the difference of a number and 12” is \( 8(n - 12) \).

When you multiply a number by itself, the product is called a square number. The square of a number can be written in exponential form such as \( 3^2 \), where 3 is the base and 2 is the exponent. So, \( 3^2 = 3 \times 3 \). A number raised to the second power is said to be “squared.”

A cube number is the product of a number multiplied by itself two times. For example, 8 is a cube number because \( 2 \times 2 \times 2 = 8 \). This is written as \( 2^3 \). A number raised to the third power is said to be “cubed.”

Example 5
Write an expression for this statement: the quotient of 80 and 4 squared.

Strategy Break the expression into parts.

Step 1 Write an expression for “4 squared.”
“4 squared” can be written as \( 4^2 \).

Step 2 Write an expression for the entire statement.
Divide 80 by \( 4^2 \).
\( 80 \div 4^2 \)

Solution An expression for “the quotient of 80 and 4 squared” is \( 80 \div 4^2 \).
Coached Example

Write an expression that represents “twice a number $n$ increased by 11.”

Break the word expression into two parts.

An expression that means “twice a number $n$” is ______________.

“Increased by 11” means to ______________.

Write an expression combining the two parts. ______________

An expression that represents “twice a number $n$ increased by 11” is __________.
You can evaluate an expression with a variable by substituting a number for the variable. If an expression contains more than one operation, you need to know in which order to perform the operations. The order of operations is a set of rules that determines the correct sequence for evaluating expressions.

**Order of Operations**
1. Evaluate expressions in parentheses.
2. Evaluate exponents.
3. Multiply and divide in order from left to right.
4. Add and subtract in order from left to right.

**Example 1**
Evaluate the expression $3^2 + 5$.

**Strategy** Use the order of operations.

**Step 1** There are no parentheses, so evaluate exponents first.

\[
3^2 + 5 = 9 + 5
\]

**Step 2** There is no multiplication or division, so add next.

\[
9 + 5 = 14
\]

**Solution** $3^2 + 5 = 14$

**Example 2**
Evaluate $7x - 5$ for $x = 4$.

**Strategy** Substitute the given value of $x$ in the expression. Then evaluate.

**Step 1** Rewrite the expression, substituting 4 for $x$.

\[
7 \times 4 - 5
\]

**Step 2** Evaluate the expression using the order of operations.

\[
7 \times 4 - 5 \quad \text{Multiply first.}
\]

\[
28 - 5 \quad \text{Then subtract.}
\]

\[
23
\]

**Solution** When $x = 4$, $7x - 5 = 23$. 
**Example 3**
Evaluate \(a^2 + 6\) for \(a = 5\).

**Strategy**  
Substitute the given value of \(a\) in the expression. Then evaluate.

**Step 1**  
Rewrite the expression, substituting 5 for \(a\).
\[5^2 + 6\]

**Step 2**  
Evaluate the expression using the order of operations.
\[5^2 + 6 \quad \text{Evaluate exponents first.}\]
\[25 + 6 \quad \text{Then add.}\]
31

**Solution**  
When \(a = 5\), \(a^2 + 6 = 31\).

**Example 4**
Evaluate \(a^3 \div (2 \times b)\) for \(a = 4\) and \(b = 8\).

**Strategy**  
Substitute the given values of \(a\) and \(b\) in the expression. Then evaluate.

**Step 1**  
Rewrite the expression, substituting 4 for \(a\) and 8 for \(b\).
\[4^3 \div (2 \times 8)\]

**Step 2**  
Evaluate the expression.
\[4^3 \div (2 \times 8) \quad \text{Perform operations in parentheses first.}\]
\[64 \div 16 \quad \text{Then evaluate exponents.}\]
\[4 \quad \text{Then divide.}\]

**Solution**  
When \(a = 4\) and \(b = 8\), \(a^3 \div (2 \times b) = 4\).
Example 5
Each side of a cube is 5 units long. Find the surface area of the cube using the formula \(A = 6s^2\), where \(s\) is the side length of the cube.

Strategy Substitute the given side length of the cube in the formula. Then evaluate.

Step 1 Rewrite the formula, substituting 5 for \(s\).

\[
A = 6(5^2)
\]

Step 2 Evaluate the expression to the right of the equal sign.

\[
A = 6(25)
A = 150
\]

Solution The surface area of the cube is 150 square units.

Coached Example

The volume of a cube can be found with the formula \(V = s^3\), where \(s\) is the side length of the cube. Tamara has two cubes. The first cube has a side length of 4 centimeters. The second has a side length of 7 centimeters. What are the volumes of Tamara’s cubes?

Interpret the exponent in the formula.

\(s^3\) means to use \(s\) as a factor _____ times.

\[
V = s^3 = s \times _____ \times _____
\]

Find the volume of the first cube.
Substitute _____ for \(s\).

\[
V = _____ \times _____ \times _____ = _____
\]

Find the volume of the second cube.
Substitute _____ for \(s\).

\[
V = _____ \times _____ \times _____ = _____
\]

Tamara’s first cube has a volume of _____ cubic centimeters, and her second cube has a volume of _____ cubic centimeters.
Since variables represent numbers, number properties apply to variables as well. For example, the commutative property allows you to say both $8 + 2 = 2 + 8$ and $a + b = b + a$.

You can use number properties to write and identify **equivalent expressions**. Expressions are equivalent if they name the same number regardless of which value a variable stands for.

**Example 1**

Write an equivalent expression for $2(5 + n)$.

**Strategy** Use the distributive property of multiplication over addition.

**Step 1** Write the expression as the sum of two sets of factors.

\[ 2(5 + n) = (2 \times 5) + (2 \times n) \]

**Step 2** Multiply each set of factors.

\[ (2 \times 5) + (2 \times n) \]

\[ 10 + 2n \]

**Solution** An equivalent expression for $2(5 + n)$ is $10 + 2n$.

**Example 2**

Write an equivalent expression for $56x - 63$.

**Strategy** Use the distributive property.

**Step 1** Identify the greatest common factor (GCF) of the terms in the expression.

Factors of 56: 1, 2, 4, 7, 8, 14, 28, 56
Factors of 63: 1, 3, 7, 9, 21, 63
The GCF of 56 and 63 is 7.

**Step 2** Use the GCF and the distributive property to write an equivalent expression.

\[ 56x - 63 = 7(8x) - 7(9) = 7(8x - 9) \]

**Solution** An equivalent expression for $56x - 63$ is $7(8x - 9)$. 
Like terms are terms that have the same variable raised to the same power. For example, in the expression $x^2 + 3x + 2x + 4x^2$, $x^2$ and $4x^2$ are like terms, and $3x$ and $2x$ are like terms. You can simplify an expression by combining like terms.

**Example 3**

Write an equivalent expression for $a + a + a$.

**Strategy** Add like terms.

\[
a + a + a = 3a
\]

\[
3 \times a = 3a
\]

**Solution** An equivalent expression for $a + a + a$ is $3a$.

**Example 4**

Simplify $10x + 6y + 4x$.

**Strategy** Simplify using like terms.

**Step 1** Use the commutative property of addition to rewrite the expression.

\[
10x + 6y + 4x = 10x + 4x + 6y
\]

**Step 2** Use the associative property of addition to group like terms.

\[(10x + 4x) + 6y\]

**Step 3** Add like terms.

\[(10x + 4x) + 6y = 14x + 6y\]

**Solution** $10x + 6y + 4x$ can be simplified as $14x + 6y$. 
Coached Example

Are the expressions $9n - 27$ and $3(3n - 9)$ equivalent?

To simplify $3(3n - 9)$, use the __________________________ property.

$3(3n - 9) = 3(__) - (__) = __ - __$

Are the expressions equivalent? ______________

Check your work by substituting different values for $n$ into each expression.

Try $n = 4$ in both expressions first.

$9n - 27 = 9(__) - 27 = ___ - ___ = ___$  
$3(3n - 9) = 3(3 \times __) - 3(9) = 3(__) - ___ = ___ - ___ = ___$

Now try $n = 10$ in both expressions.

$9n - 27 = 9(__) - 27 = ___ - ___ = ___$  
$3(3n - 9) = 3(3 \times __) - 3(9) = 3(__) - ___ = ___ - ___ = ___$

Did both expressions have the same value for each given value of $n$? ______________

The expressions $9n - 27$ and $3(3n - 9) ______________ equivalent.
Domain 3 • Lesson 20

Equations

Getting the Idea

An equation is a mathematical statement that says two expressions are equal. An equation has an equal sign (=).

A variable can be used to represent an unknown number in an equation. Here are some examples of equations with variables.

\[ 6z = 36 \quad n = \frac{3}{4}p \quad a + 3 = 11 \quad d - 5 = 22 \quad \frac{c}{7} = 8 \]

You have already multiplied by placing a numeral before a variable. Another way to represent multiplication is by using the symbol \( \cdot \), which means the same as \( \times \). Since \( x \) is often used as a variable and looks like \( \times \), the symbol \( \cdot \) avoids any confusion.

\[ 5 \times n, \quad 5n, \quad 5(n), \quad (5)(n), \quad \text{and} \quad 5 \cdot n \text{ all mean to multiply 5 times a number } n. \]

Equations can be used to represent real-world situations. For example, imagine that Holly used 90 chocolate chips to make 15 cookies, and each cookie contained the same number of chips. You could use the equation \( 15x = 90 \) to represent this situation, with \( x \) representing the unknown number of chocolate chips in each cookie.

Example 1

Jerry scored a total of 15 points in his last basketball game. He made 3 free throws that are worth 1 point each and he made 4 field goals that are worth the same number of points. The equation \( 4x + 3 = 15 \) represents this situation. Determine whether Jerry made only 2-point field goals or only 3-point field goals.

Strategy

Use substitution to find the value of \( x \).

Step 1

Substitute 2 for \( x \).

\[ 4 \cdot 2 + 3 = 8 + 3 = 11 \]

The field goals were worth more than 2 points.

Step 2

Substitute 3 for \( x \).

\[ 4 \cdot 3 + 3 = 12 + 3 = 15 \]

The field goals were worth 3 points.

Solution

Jerry scored only 3-point field goals.
Example 2
What is the value of $x$ in the equation $x + 27 = 75$?

Strategy Use substitution to solve.

Step 1 Estimate a solution.

$50 + 25 = 75$, so $x$ will be less than, but close to, 50.

Step 2 Try the following values for $x$: 47, 48, and 49.

Substitute 47 for $x$.

\[
\begin{array}{c}
\text{Substitute 47 for } x. \\
x + 27 = 75 \\
47 + 27 \nleq 75 \\
74 \nneq 75
\end{array}
\]

Substitute 48 for $x$.

\[
\begin{array}{c}
\text{Substitute 48 for } x. \\
x + 27 = 75 \\
48 + 27 \nleq 75 \\
75 = 75 \checkmark
\end{array}
\]

Solution $x = 48$

To solve an equation, you need to isolate the variable. You can use inverse operations to isolate the variable. Inverse operations are related operations that undo each other, such as addition and subtraction, or multiplication and division.

Example 3
Solve for $d$.

\[
\frac{d}{4} = 16
\]

Strategy Use an inverse operation to isolate the variable.

Step 1 Identify the inverse operation.

\[
\frac{d}{4} \text{ is equivalent to } d \div 4.
\]

Multiplication is the inverse operation of division.

Step 2 Multiply both sides of the equation by 4.

\[
\begin{array}{c}
\frac{d}{4} = 16 \\
\frac{d}{4} \times 4 = 16 \times 4 \\
d = 64
\end{array}
\]

Solution $d = 64$
**Example 4**
What value of \( y \) makes this equation true?

\[
y - 19 = 24
\]

**Strategy**  
Use an inverse operation to isolate the variable.

**Step 1**  
Identify the inverse operation.  
Addition is the inverse operation of subtraction.

**Step 2**  
Add 19 to both sides of the equation.

\[
y - 19 = 24 \\
y - 19 + 19 = 24 + 19 \\
y = 43
\]

**Solution**  
\( y = 43 \)

Some equations may contain fractions. Remember that dividing by a fraction is the same as multiplying by its reciprocal.

**Example 5**
What is the value of \( z \) in the equation \( \frac{1}{3}z = 26 \)?

**Strategy**  
Use inverse operations to isolate the variable.

**Step 1**  
Divide both sides by \( \frac{1}{3} \) to isolate the variable.  
Multiply by the reciprocal of \( \frac{1}{3} \).  
The reciprocal of \( \frac{1}{3} \) is \( \frac{3}{1} \), or 3.

\[
\frac{1}{3}z \times 3 = 26 \times 3 \\
z = 78
\]

**Step 2**  
Check your answer.

Substitute 78 for \( z \) in the original equation.

\[
\frac{1}{3}(78) = 26 \\
26 = 26 \checkmark
\]

**Solution**  
\( z = 78 \)
Gavin worked 16 hours last week and earned $192. The equation $16 \cdot d = 192$ can be used to find $d$, the number of dollars he earns per hour. What is Gavin’s hourly wage? Use inverse operations to solve for $d$.

What operation is used in the term $16 \cdot d$? _________________

Write $16 \cdot d$ as an expression without an operation symbol. _________________

The inverse of multiplication is _________________.

To isolate the variable, _________________ both sides of the equation by ________.

Solve for $d$.

\[
16d = 192
\]

\[
16d \div \underline{\hspace{2cm}} = \underline{\hspace{2cm}} \div \underline{\hspace{2cm}}
\]

\[
d = \underline{\hspace{2cm}}
\]

Gavin’s hourly wage is $\underline{\hspace{2cm}}$. 
Dependent and Independent Variables

Getting the Idea

Two variables are often related in real-world situations. For example, sales of ice cream cones may be related to the temperature. There are different ways to show how two variables are related.

The equation \( y = x + 1 \) shows how the variables \( x \) and \( y \) are related. In the equation \( y = x + 1 \), \( x \) is called the independent variable and \( y \) is called the dependent variable, since the value of \( y \) depends on the value of \( x \).

You also can use a table or a graph to show how two variables are related.

Example 1
Create a table of values for the equation \( y = x + 1 \). Include at least four ordered pairs in the table.

Strategy
Make a table of values.

Step 1
Choose several values for \( x \).
Choose whole numbers, such as 2, 4, 6, and 8.

Step 2
Create a table of values.
Substitute each \( x \)-value into the equation to find its corresponding \( y \)-value.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = x + 1 )</th>
<th>( y )</th>
<th>( (x, y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>( y = 2 + 1 = 3 )</td>
<td>3</td>
<td>(2, 3)</td>
</tr>
<tr>
<td>4</td>
<td>( y = 4 + 1 = 5 )</td>
<td>5</td>
<td>(4, 5)</td>
</tr>
<tr>
<td>6</td>
<td>( y = 6 + 1 = 7 )</td>
<td>7</td>
<td>(6, 7)</td>
</tr>
<tr>
<td>8</td>
<td>( y = 8 + 1 = 9 )</td>
<td>9</td>
<td>(8, 9)</td>
</tr>
</tbody>
</table>

The last column of the table shows ordered pairs of \( x \)- and \( y \)-values for the equation.

Solution
The table of values in Step 2 above shows four ordered pairs for \( y = x + 1 \).
Example 2

Use the ordered pairs you found in Example 1 to graph $y = x + 1$.

**Strategy**  
Plot a point for each ordered pair, then connect them.

**Step 1**  
Plot points for each ordered pair on a coordinate grid.

**Step 2**  
Connect the points.

**Solution**  
The graph of $y = x + 1$ is shown in Step 2 above.
You can write an equation for the graph of a line.

**Example 3**

Write an equation to represent the graph of the line below.

![Graph of a line](image)

**Strategy** Identify the ordered pairs on the graph and write an equation to represent the relationship.

**Step 1** Identify the ordered pairs on the graph.
Points on the line are plotted at (0, 1), (1, 4), (2, 7), and (3, 10).

**Step 2** Find the difference in the value of \(y\) when the value of \(x\) is increased by 1.
Subtract the values of \(y\).
- \(4 - 1 = 3\)
- \(7 - 4 = 3\)
- \(10 - 7 = 3\)
The value of \(y\) increases by 3 when the value of \(x\) increases by 1.

\[y = 3 \cdot x\]

**Step 3** See if the equation works for the ordered pairs.
- \(0 \cdot 3 = 0\)
- \(1 \cdot 3 = 3\)
- \(2 \cdot 3 = 6\)
- \(3 \cdot 3 = 9\)
Each product is 1 less than the value of \(y\) in the ordered pairs.
Add 1 to the equation.

**Step 4** Write an equation.
The graph of the line can be represented by \(y = 3x + 1\).

**Solution** The equation \(y = 3x + 1\) represents the graph of the line.
Create a table of values to represent \( y = x + 2 \). Use it to find four ordered pairs that represent the equation. Then graph the equation.

Choose integers such as 1, 3, 5, and 7.

Then substitute each \( x \)-value into the equation to find its corresponding \( y \)-value.

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = x + 2 )</th>
<th>( y )</th>
<th>( (x, y) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( y = 1 + 2 = )</td>
<td>1</td>
<td>(1, ___)</td>
</tr>
<tr>
<td>3</td>
<td>( y = ) + 2 =</td>
<td>5</td>
<td>(3, ___)</td>
</tr>
<tr>
<td>5</td>
<td>( y = ) + 2 =</td>
<td>7</td>
<td>(5, ___)</td>
</tr>
<tr>
<td>7</td>
<td>( y = ) + 2 =</td>
<td>9</td>
<td>(7, ___)</td>
</tr>
</tbody>
</table>

Now plot the ordered pairs on the grid below. Draw a line to connect the points and graph the equation.

Four ordered pairs for \( y = x + 2 \) are (1, ___), (3, ___), (5, ___), and (7, ___).

The graph of \( y = x + 2 \) is shown above.
Use Equations to Solve Problems

Getting the Idea

You can write an equation to help you solve a word problem by choosing a variable to represent the unknown quantity.

Example 1
It cost $8 per hour to ice skate. Katy spent $24 skating. Write an equation you can use to find $h$, the number of hours Katy skated. Then solve the equation.

Strategy Write a separate expression for each side of the equation.

Step 1 Write what you know.

Katy spent $24 skating.

It cost $8 per hour to skate.

Step 2 Write an equation by writing two expressions that are equal.

One side of the equation is $8h$, the cost, in dollars, per hour times the number of hours.

The other side of the equation is 24, the total cost in dollars.

$8h = 24$

Step 3 Solve the equation for $h$.

Divide both sides of the equation by 8.

$\frac{8h}{8} = \frac{24}{8}$

$h = 3$

Solution The equation $8h = 24$ can be used to find the number of hours Katy skated. Katy skated for 3 hours.

Example 2
Maureen biked 82 kilometers the last two weeks. She biked 48 kilometers last week. Write an equation to find $k$, the number of kilometers Maureen biked this week. Then solve the equation.

Strategy Write a separate expression for each side of the equation.
Step 1  Write what you know.

Maureen biked 82 kilometers in 2 weeks.
She biked 48 kilometers last week.

Step 2  Write an equation by writing two expressions that are equal.

One side of the equation is $48 + k$, the number of kilometers Maureen biked each week.

The other side of the equation is 82, the total number of kilometers biked.

$48 + k = 82$

Step 3  Solve the equation for $k$.

Subtract 48 from both sides of the equation.

$48 + k - 48 = 82 - 48$

$k = 34$

Solution  The equation $48 + k = 82$ can be used to find the number of kilometers Maureen biked this week. Maureen biked 34 kilometers this week.

Example 3
Doug has two bins of recycling that weigh a total of $12\frac{1}{4}$ pounds. The blue bin has a weight of $6\frac{7}{8}$ pounds. The rest of the recycling is in the green bin. How many pounds of recycling are in the green bin?

Strategy  Write and solve an equation.

Step 1  Write what you know.

There are two bins that weigh a total of $12\frac{1}{4}$ pounds.

One bin weighs $6\frac{7}{8}$ pounds.

Step 2  Write an equation by writing two expressions that are equal.

One side of the equation is $6\frac{7}{8} + p$, the number of pounds of each bin.

The other side of the equation is $12\frac{1}{4}$, the total number of pounds of recycling.

$6\frac{7}{8} + p = 12\frac{1}{4}$
Step 3
Subtract $6\frac{7}{8}$ from both sides of the equation.

Rename $12\frac{1}{4}$, so the mixed numbers have common denominators.

\[
\frac{1}{4} \times \frac{2}{2} = \frac{2}{8}
\]

\[
12\frac{1}{4} = 12\frac{2}{8}
\]

\[
6\frac{7}{8} + p - 6\frac{7}{8} = 12\frac{2}{8} - 6\frac{7}{8}
\]

\[
p = 11\frac{10}{8} - 6\frac{7}{8}
\]

\[
p = 5\frac{3}{8}
\]

Solution  
Doug has $5\frac{3}{8}$ pounds of recycling in the green bin.

Example 4
Each class on media literacy lasted $\frac{3}{4}$ hour. There were 14 classes in the course. How many hours long was the course?

Strategy  
Write and solve an equation.

Step 1
Write what you know.

Each class lasted $\frac{3}{4}$ hour.

There were 14 classes.

Step 2
Write an equation by writing two expressions that are equal.

One side of the equation is $\frac{c}{14}$, the number of hours in the course divided by the number of classes.

The other side of the equation is $\frac{3}{4}$, the number of hours in each class.

\[
\frac{c}{14} = \frac{3}{4}
\]

Step 3
Solve the equation for c.

\[
\frac{c}{14} \cdot 14 = \frac{3}{4} \cdot 14
\]

\[
c = \frac{42}{4}
\]

\[
c = 10\frac{1}{2}
\]

Solution  
The course lasted $10\frac{1}{2}$ hours.
Example 5

The Red Trail is 13.2 kilometers. It took a group 2.4 hours to complete the Red Trail. What was the group’s speed in kilometers per hour?

Strategy  Write and solve an equation.

Step 1  Write what you know.

The trail was 13.2 kilometers.
It took 2.4 hours to complete the trail.

Step 2  Write an equation by writing two expressions that are equal.

One side of the equation is $2.4 \cdot k$, the number of hours times the number of kilometers hiked each hour.
The other side of the equation is 13.2, the length in kilometers of the trail.

\[ 2.4k = 13.2 \]

Step 3  Solve the equation for $k$.

\[
\begin{align*}
2.4k &= 13.2 \\
\frac{2.4k}{2.4} &= \frac{13.2}{2.4} \\
k &= 5.5
\end{align*}
\]

Solution  The group’s speed was 5.5 kilometers per hour.

Example 6

The Green Trail is 4.6 kilometers longer than the Blue Trail. The Blue Trail is 11.2 kilometers. What is the length of the Green Trail?

Strategy  Write and solve an equation.

Step 1  Write what you know.

The Green Trail is 4.6 kilometers longer.
The Blue Trail is 11.2 kilometers.

Step 2  Write an equation by writing two expressions that are equal.

One side of the equation is $g - 4.6$, the length of each trail.
The other side of the equation is 11.2, the length in kilometers of the Blue Trail.

\[ g - 4.6 = 11.2 \]
Step 3  Solve the equation for \( g \).
\[
g - 4.6 + 4.6 = 11.2 + 4.6
\]
\[
g = 15.8
\]

Solution  The Green Trail is 15.8 kilometers.

Coached Example

Each driving range golf ball costs $0.85. Mr. Costa spent $30.60 on driving range golf balls. Write an equation to find \( g \), the number of golf balls that Mr. Costa bought. Then solve the equation.

What is the cost per golf ball? 

What expression can you write to represent the cost of \( g \) golf balls?

What was the total cost to Mr. Costa?

Write an equation to represent how to find the cost of \( g \) golf balls.

Solve for \( g \).

\[
\text{_________ both sides of the equation by \text{_________}.}
\]
\[
g = \text{_________}
\]

The equation \text{_________} can be used to find the number of golf balls bought.

Mr. Costa bought \text{_________} golf balls.
An inequality is a mathematical statement that compares two expressions and includes an inequality symbol. The different inequality symbols are shown below.

- $>$ means “is greater than.”
- $\geq$ means “is greater than or equal to.”
- $<$ means “is less than.”
- $\leq$ means “is less than or equal to.”

**Example 1**
Write an inequality to represent “a number is less than 5.” Use the variable $n$.

**Strategy** Determine the inequality symbol.

**Step 1** Determine if 5 is part of the set.
The set of numbers less than 5 does not include 5.

**Step 2** Choose the inequality symbol and then write the inequality.
- $<$ means is less than.
- $n < 5$

**Solution** The inequality $n < 5$ represents the statement “a number is less than 5.”

**Example 2**
A pastry chef wants to spend no more than $100 on almonds. Almonds cost $4 per pound. Write an inequality to represent, $p$, the number of pounds of almonds the chef could buy?

**Strategy** Determine the inequality symbol.

**Step 1** Determine if 100 is part of the set.
The chef is willing to spend $100, so 100 is part of the set.

**Step 2** Write the inequality.
- $4p$ represents the cost of almonds per pound.
- $\leq$ means is less than or equal to.
- $4p \leq 100$

**Solution** The inequality $4p \leq 100$ represents the number of pounds of almonds the chef could buy.
Unlike an equation, which usually has only one solution, an inequality can have many solutions. These solutions are called the **solution set**. You can use substitution to determine if a number is in the solution set or not.

**Example 3**
Which of the following are possible solutions for the inequality below: 10.5, 20, 30?

\[ 4p \leq 100 \]

**Strategy**
Substitute each value for \( p \) in the inequality. Determine which value or values make the inequality true.

**Step 1**
Substitute 10.5 for \( p \).

\[ 4p \leq 100 \]
\[ 4(10.5) \leq 100 \]
\[ 42 \leq 100 \checkmark \]

10.5 is a solution for the inequality.

**Step 2**
Substitute 20 for \( p \).

\[ 4p \leq 100 \]
\[ 4(20) \leq 100 \]
\[ 80 \leq 100 \checkmark \]

Since the inequality uses the \( \leq \) symbol, any number less than or equal to 80 is a solution. So 20 is a solution to the inequality.

**Step 3**
Substitute 30 for \( p \).

\[ 4p \leq 100 \]
\[ 4(30) \leq 100 \]
\[ 120 \leq 100 \text{ is false, because } 120 > 100. \]

So, 30 is not a solution to the inequality.

**Solution**
The numbers 10.5 and 20 are solutions for the inequality \( 4p \leq 100 \).
The number 30 is not a solution.
Lesson 23: Inequalities

To solve an inequality, follow the same steps as you would to solve an equation. Remember, whatever you do to one side of the inequality must also be done to the other side.

Example 4
Solve for $p$: $4p \leq 100$

Strategy  Use inverse operations to isolate the variable.

Since $p$ is multiplied by 4, divide both sides by 4.

\[
\begin{align*}
4p & \leq 100 \\
\frac{4p}{4} & \leq \frac{100}{4} \\
p & \leq 25
\end{align*}
\]

Solution  The solution set for the inequality is all numbers less than or equal to 25, or $p \leq 25$.

Example 5
What is the solution set for $\frac{n}{3} \geq 12$?

Strategy  Use inverse operations to isolate the variable.

Step 1  Since $n$ is divided by 3, multiply both sides by 3.

\[
\begin{align*}
\frac{n}{3} & \geq 12 \\
\frac{n}{3} \times 3 & \geq 12 \times 3 \\
n & \geq 36
\end{align*}
\]
Step 2  Use substitution to check your answer.

Substitute a number that is less than 36, such as 33.
\[
\frac{33}{3} \geq 12
\]
\[
11 \geq 12 \text{ is false, because } 11 < 12.
\]

Now substitute 36.
\[
\frac{36}{3} \geq 12
\]
\[
12 \geq 12 \checkmark
\]

Substitute a number that is greater than 36, such as 39.
\[
\frac{39}{3} \geq 12
\]
\[
13 \geq 12 \checkmark
\]

Only the numbers that were greater than or equal to 36 made the inequality true.

Solution  The solution set for the inequality is \( n \geq 36 \).

You can graph the solution set of an inequality on a number line. Draw a circle and an arrow to show all the numbers that are part of the solution.

- An open circle indicates that the circled number is not a solution for the inequality. Use an open circle when the symbol is \( > \) or \( < \).

\[
\begin{align*}
8 & \quad 9 & \quad 10 & \quad 11 & \quad 12 & \quad 13 \\
\bullet & \quad & \quad & \quad & \quad & \checkmark \\
x > 10 & 
\end{align*}
\]

- A closed circle indicates that the circled number is a solution. Use a closed circle when the symbol is \( \geq \) or \( \leq \).

\[
\begin{align*}
8 & \quad 9 & \quad 10 & \quad 11 & \quad 12 & \quad 13 \\
\checkmark & \quad & \quad & \quad & \quad & \checkmark \\
x \geq 10 & 
\end{align*}
\]
Example 6
Solve and graph the inequality \( b + 13 > 17 \). Then identify 3 possible values for \( b \).

Strategy Use inverse operations to isolate the variable. Then graph the inequality.

Step 1 Isolate the variable.

\[
\begin{align*}
  b + 13 &> 17 \\
  b + 13 - 13 &> 17 - 13 \\
  b &> 4
\end{align*}
\]

Step 2 Draw a number line and make a circle at 4.

The symbol is \( > \), so draw an open circle at 4 to show that 4 is not a solution.

\[
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\end{array}
\]

Step 3 Draw an arrow on the number line to show the solution set.

Since all values greater than 4 are solutions, draw an arrow to the right.

\[
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\end{array}
\]

Step 4 Identify three solutions for the inequality.

5, \( 6\frac{1}{2} \), and 8 are all solutions because they are all part of the graph.

Solution The solution set to \( b + 13 > 17 \) is \( b > 4 \), which is graphed in Step 3 above. Three possible values for \( b \) are 5, \( 6\frac{1}{2} \), and 8.
Coached Example

Solve and graph the inequality \( z - 8 \leq 5 \).

Use inverse operations to isolate the variable.

Since \( ______ \) is subtracted from \( z \), add \( ______ \) to both sides.

\[
\begin{align*}
    z - 8 & \leq 5 \\
    z - 8 + ______ & \leq 5 + ______ \\
    z & \leq ______ \\
\end{align*}
\]

Graph the solution set on the number line below.

Since the symbol is \( \leq \), draw an \( ______ \) circle at \( _____ \).

This shows that \( _____ \) is a solution.

Since all values less than \( _____ \) are also solutions, draw an arrow to the \( ______ \).

The solution set for the inequality is \( z \leq _____ \). Its graph is shown above.
Area of Triangles

Getting the Idea

Area is a measure of the number of square units needed to cover a region. A square unit is a square with a side length of 1 of any particular unit. Square units can be square inches (in.²), square centimeters (cm²), or any other squared unit length.

The formula for the area of a triangle is \( A = \frac{1}{2}bh \), where \( b \) represents the base length and \( h \) represents the height of the triangle. Some examples of triangles, with their bases and heights labeled, are shown below.

Example 1

What is the area of this triangle?

Strategy

Use the formula for the area of a triangle.

Step 1

Substitute the values for the base and height into the formula.

The base, \( b \), measures 8 inches and the height, \( h \), measures 4 inches.

\[ A = \frac{1}{2}bh \]
\[ A = \frac{1}{2} \times 8 \times 4 \]

Step 2

Multiply.

\[ A = \frac{1}{2} \times 8 \times 4 \]
\[ A = 4 \times 4 \]
\[ A = 16 \]

Solution

The area of the triangle is 16 square inches, or 16 in.²
Example 2
What is the area of this triangle?

Strategy Use the formula for the area of a triangle.

Step 1 Substitute the values for the base and height into the formula.
The base, $b$, measures 9 centimeters and the height, $h$, measures 3 centimeters.

\[ A = \frac{1}{2}bh \]
\[ A = 0.5 \times 9 \times 3 \]

Step 2 Multiply.
\[ A = 0.5 \times 9 \times 3 \]
\[ A = 4.5 \times 3 \]
\[ A = 13.5 \]

Solution The area of the triangle is 13.5 square centimeters, or 13.5 cm$^2$.

In an obtuse triangle, you can extend a side to find the height.

Example 3
What is the area of this triangle?

Strategy Use the formula for the area of a triangle.
Step 1
Substitute the values for the base and height into the formula.

The base, \( b \), measures 12 feet and the height, \( h \), measures 5 feet.

\[
A = \frac{1}{2}bh
\]

\[
A = \frac{1}{2} \times 12 \times 5
\]

Step 2
Multiply.

\[
A = \frac{1}{2} \times 12 \times 5
\]

\[
A = 6 \times 5
\]

\[
A = 30
\]

Solution
The area of the triangle is 30 square feet, or 30 ft\(^2\).

Coached Example

The Clemente family built a triangular deck at the back of their house, as shown below. What is the area of the Clementes’ deck?

The deck is in the shape of a triangle.

The base of the triangle is _______ yards long and the height is _______ yards long.

The formula for the area of a triangle is \( A \) _______.

Substitute the values for the base and height into the formula.

\[
A = \frac{1}{2} \times \underline{_____} \times \underline{_____}
\]

\[
A = \underline{_____} \times \underline{_____}
\]

\[
A = \underline{______}
\]

What units should be used to express the area? _________________

The area of the Clementes’ deck is _________________.

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## Getting the Idea

A **quadrilateral** is a polygon that has 4 sides. You can use formulas to find the areas of quadrilaterals.

<table>
<thead>
<tr>
<th>Figure</th>
<th>Area Formula</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Parallelogram</strong></td>
<td>$A = bh$, where $b$ represents the base length and $h$ represents the height</td>
</tr>
<tr>
<td><img src="image" alt="Parallelogram" /></td>
<td></td>
</tr>
</tbody>
</table>
| **Rectangle** | $A = lw$, where $l$ represents the length and $w$ represents the height  
Or $A = bh$, where $b$ represents the base length and $h$ represents the height |
| ![Rectangle](image) | |
| **Square** | $A = s^2$, where $s$ represents the length of a side |
| ![Square](image) | |
| **Rhombus** | $A = bh$, where $b$ represents the base length and $h$ represents the height |
| ![Rhombus](image) | |
| **Trapezoid** | $A = \frac{1}{2}h(b_1 + b_2)$, where $h$ represents the height and $b_1$ and $b_2$ represent the lengths of the bases |
| ![Trapezoid](image) | |
A diagonal of a rectangle divides the rectangle into two right triangles that are equal in area.

The diagram above shows that the area of a rectangle is equal to the area of the two triangles formed by the diagonal. Remember that the formula for the area of a triangle is \( A = \frac{1}{2}bh \). So, the formula for the area of a rectangle is \( A = 2 \times \frac{1}{2}bh \), or simply \( A = bh \). You can substitute length for base and width for height to make the formula \( A = lw \).

**Example 1**
What is the area of this rectangle?

![Rectangle](image)

**Strategy**  
Use the formula for the area of a rectangle.

**Step 1**  
Substitute the values for the length and width into the formula.  
The length of the rectangle is 7 feet and the width is 5 feet.  
\[ A = lw \]  
\[ A = 7 \times 5 \]

**Step 2**  
Multiply.  
\[ A = 7 \times 5 = 35 \]

**Solution**  
The area of the rectangle is 35 square feet, or 35 ft\(^2\).
The formula for the area of a parallelogram, \( A = bh \), is the same as the formula for the area of a rectangle. The diagram below will help you see why the same formula works for both kinds of quadrilaterals.

Moving the right triangle from the left side to the right side of the parallelogram forms a rectangle. The rectangle has the same area as the original parallelogram.

**Example 2**
What is the area of this parallelogram?

**Strategy** Use the formula for the area of a parallelogram.

**Step 1** Substitute the values for the base and height into the formula.
- The side length of 7.5 centimeters is not the height.
- The base of the parallelogram is 15 cm and the height is 6 cm.

\[
A = bh \\
A = 15 \times 6
\]

**Step 2** Multiply.

\[
A = 15 \times 6 = 90
\]

**Solution** The area of the parallelogram is 90 square centimeters, or 90 cm\(^2\).
A rhombus is a parallelogram whose sides are all the same length. So, the formulas for the area of a rhombus and the area of a parallelogram are the same.

**Example 3**

What is the area of this rhombus?

![Rhombus Diagram]

**Strategy**  Use the formula for the area of a rhombus.

**Step 1**  Substitute the values for the base and height into the formula.

The base of the rhombus is 13 inches and the height is 12 inches.

\[ A = bh \]

\[ A = 13 \times 12 \]

**Step 2**  Multiply.

\[ A = 13 \times 12 = 156 \]

**Solution**  The area of the rhombus is 156 square inches, or 156 in.$^2$.

A diagonal of a trapezoid divides the trapezoid into two triangles, as shown below.

![Trapezoid Diagram]

The area of the trapezoid is the sum of the areas of the triangles. The height is the same for both triangles.

\[ A = \frac{1}{2}b_1h + \frac{1}{2}b_2h \]

You can use the distributive property to rewrite the formula. The common factors in each term are $\frac{1}{2}$ and $h$.

\[ A = \frac{1}{2}b_1h + \frac{1}{2}b_2h = \frac{1}{2}h(b_1 + b_2) \]
Example 4
What is the area of this trapezoid?

**Strategy**  Use the formula for the area of a trapezoid.

**Step 1**  Substitute the values for the bases and height into the formula.

The height of the trapezoid is 7 yards and the bases are 8 yards and 14 yards.

\[ A = \frac{1}{2}h(b_1 + b_2) = \frac{1}{2} \times 7(8 + 14) \]

**Step 2**  Use the order of operations and number properties.

\[ A = \frac{1}{2} \times 7(8 + 14) \]
\[ A = \frac{1}{2} \times 7 \times 22 = \frac{1}{2} \times 22 \times 7 \]
\[ A = 11 \times 7 = 77 \]

**Solution**  The area of the trapezoid is 77 square yards, or 77 yd².

---

**Coached Example**

What is the area of this square?

![Square Image]

The formula for the area of a square is \( A = _____ \).

Substitute the value of the length of a side of the square into the formula.

\[ A = (_____)^2 = _____ \times _____ = _____ \]

What units should be used to express the area? _______________

**The area of the square is ____________________**.
Getting the Idea

A composite polygon is a polygon that can be divided into simpler figures. To find the area of a composite polygon, find the sum of the areas of the simpler figures.

The formulas in the table below will help you find the area of many composite polygons.

<table>
<thead>
<tr>
<th>Figure</th>
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</tr>
</thead>
<tbody>
<tr>
<td>Rectangle</td>
<td>$A = lw$</td>
</tr>
<tr>
<td>Square</td>
<td>$A = s^2$</td>
</tr>
<tr>
<td>Triangle</td>
<td>$A = \frac{1}{2}bh$</td>
</tr>
</tbody>
</table>

Example 1

Eric drew a diagram of his backyard. What is the area of his backyard?

Strategy

Divide the figure into smaller figures. Add the areas of the smaller figures.

Step 1

Divide the figure into two rectangles.
Step 2 Subtract to find the width of the larger rectangle.
\[ 7 - 3 = 4 \]
The larger rectangle has a length of 8 meters and a width of 4 meters.

Step 3 Find the area of the larger rectangle.
\[ A = lw = 8 \times 4 = 32 \]
The area of the larger rectangle is 32 square meters.

Step 4 Find the area of the smaller rectangle.
\[ A = lw = 5 \times 3 = 15 \]
The area of the smaller rectangle is 15 square meters.

Step 5 Add the areas.
\[ 32 + 15 = 47 \]

Solution The area of Eric’s backyard is 47 square meters, or \(47 \text{ m}^2\).

Example 2
What is the area of this figure?

Strategy Divide the figure into smaller figures. Add the areas of the smaller figures.

Step 1 Divide the figure into a triangle and a rectangle.
Step 2  Find the area of the rectangle.

\[ A = lw = 10 \times 9 = 90 \]

Step 3  Find the area of the triangle.

The length of the base is 4 centimeters since \( 10 - 6 = 4 \).

\[ A = \frac{1}{2}bh = \frac{1}{2} \times 4 \times 3 = 6 \]

Step 4  Add the areas.

\[ 90 + 6 = 96 \]

Solution  The area of the figure is 96 square centimeters, or 96 cm\(^2\).

Coached Example

What is the area of this figure?

Divide the figure into a rectangle and a triangle.

The dimensions of the rectangle are ________ by ________.

The formula for the area of a rectangle is \( A = \) ________.

\[ A = \text{________} \times \text{________} = \text{________} \]

The area of the rectangle is ________ square millimeters.

In millimeters, the base of the triangle is 30 \(-\) ________ = ________.

In millimeters, the height of the triangle is 25 \(-\) ________ = ________.

The formula for the area of a triangle is \( A = \) ________.

\[ A = \text{________} \times \text{________} \times \text{________} = \text{________} \]

The area of the triangle is ________ square millimeters.

Add the areas. ________ + ________ = ________

The area of the figure is ________ square millimeters.
Polygons on the Coordinate Plane

Getting the Idea

You can connect points on a coordinate plane to form polygons.

Example 1

Plot the points (5, 10), (5, 5), (0, 5), and (0, 10) on a coordinate grid. Then connect the points. What figure is formed?

Strategy  Plot the points. Then connect the points to identify the figure formed.

Step 1  Plot each point on the grid and connect the points.

Step 2  Identify the geometric figure.

Each side is 5 units long.

The figure is a square.

Solution  The points form a square.
You can use given ordered pairs to complete polygons.

**Example 2**
Where should point $C$ be plotted so that when the points are connected it forms a parallelogram?

**Strategy**  
*Use the attributes of a parallelogram.*

**Step 1**  
Write the ordered pairs for the points that are already plotted.  
Point $D$ is at $(4, 2)$.  
Point $E$ is at $(3, -1)$.  
Point $F$ is at $(-3, -1)$.

**Step 2**  
Think about the attributes of a parallelogram.  
A parallelogram has 2 pairs of opposite parallel sides that have the same length.

**Step 3**  
Decide where to plot point $C$.  
It should be aligned horizontally with point $D$.  
Since point $D$ is 1 unit to the right of point $E$, point $C$ should be 1 unit to the right of point $F$.

**Solution**  
Point $C$ should be plotted at $(-2, 2)$. 
Example 3
Use parallelogram $CDEF$ from Example 2. What is the area of parallelogram $CDEF$?

**Strategy**  
Find the base and height and then use the formula for the area of a parallelogram.

**Step 1** Find the base.  
The base is the length of $\overline{CD}$.  
The distance from $-2$ to $0$ is $2$ units.  
The distance from $0$ to $4$ is $4$ units.  
The base is $6$ units.

**Step 2** Find the height.  
The height is the length of the vertical distance from $\overline{CD}$ to $\overline{FE}$.  
The distance from $2$ to $0$ is $2$ units.  
The distance from $0$ to $-1$ is $1$ unit.  
The height is $3$ units.

**Step 3** Use the formula for the area of a parallelogram.  
\[ A = bh \]  
\[ A = 6 \times 3 \]  
\[ A = 18 \text{ square units} \]

**Solution**  
The area of parallelogram $CDEF$ is $18$ square units.

Example 4
Two vertices of a right triangle are point $J$ at $(-4, -4)$ and point $K$ at $(4, 3)$. Name the points where point $L$ could be plotted to complete a right triangle.

**Strategy**  
Use the attributes of a right triangle.

**Step 1** Plot the two points on the coordinate plane.

**Step 2** Think about the attributes of a right triangle.  
A right triangle has exactly $1$ right angle.

**Step 3** Decide where to plot point $L$.  
Point $L$ must align vertically with one of the two vertices.  
Point $L$ must align horizontally with one of the two vertices.
Point \( L \) will share the \( x \)-coordinate with one vertex and the \( y \)-coordinate with the other vertex.

Point \( L \) can either be located at \((-4, 3)\) or at \((4, -4)\).

Solution  \( L \) can either be located at \((-4, 3)\) or at \((4, -4)\).

**Example 5**
Use triangle \( JKL \) from the left coordinate plane of Example 4. Find the area.

**Strategy**  Find the base and height and then use the formula for the area of a triangle.

**Step 1**  Find the base.
The base is the length of \( \overline{LK} \).
The distance from \(-4\) to \(0\) is \(4\) units.
The distance from \(0\) to \(4\) is \(4\) units.
The base is \(8\) units.

**Step 2**  Find the height.
The height is the length of \( \overline{LJ} \).
The distance from \(3\) to \(0\) is \(3\) units.
The distance from \(0\) to \(-4\) is \(4\) units.
The height is \(7\) units.

**Step 3**  Use the formula for the area of a triangle.
\[
A = \frac{1}{2}bh \\
A = \frac{1}{2} \times 8 \times 7 \\
A = 28 \text{ square units}
\]

Solution  The area of triangle \( JKL \) is \(28\) square units.
The figure below is the design for a new public swimming pool. What will be the area of the bottom of the pool?

Divide the figure into two rectangles. Draw a horizontal line segment from point ____________ to point ____________.

Label the top rectangle I and the bottom rectangle II.

The length of rectangle I is ______ + ______ = _____ units.
The width of rectangle I is _____ units.

Use the area formula to find the area of rectangle I.

\[ \text{Area of rectangle I} = _____ \times _____ = _____ \text{ square units} \]

The length of rectangle II is _____ + _____ = _____ units.
The width of rectangle II is _____ + _____ = _____ units.

Use the area formula to find the area of rectangle II.

\[ \text{Area of rectangle II} = _____ \times _____ = _____ \text{ square units} \]

Add the areas of the rectangles.

\[ \text{Area of rectangle I} + \text{area of rectangle II} = _____ + _____ = _____ \text{ square units} \]

Based on the scale, each unit equals 1 ____________.

The area of the bottom of the pool will be ______ square meters.
Getting the Idea
You can use the area formulas to solve problems.

Example 1
Nikki made a flag shaped like a triangle out of red cloth. The triangle has a base of 18 inches and a height of 12 inches. How many square inches of cloth did she use?

**Strategy** Use the formula for the area of a triangle.

**Step 1** Substitute the values for the base and height into the formula.
- The base of the triangle is 18 inches long and the height is 12 inches.
  \[ A = \frac{1}{2}bh \]
  \[ A = \frac{1}{2} \times 18 \times 12 \]

**Step 2** Multiply.
- \[ A = \frac{1}{2} \times 18 \times 12 = 9 \times 12 = 108 \]

**Solution** Nikki used 108 square inches of cloth.

Example 2
Alan’s kitchen is in the shape of a square that is 8 feet long on each side. He wants to cover the floor with 1-foot by 1-foot tiles. How many tiles does he need to cover the floor?

**Strategy** Use the formula for the area of a square.

**Step 1** Substitute the value for the length of a side into the formula.
- The length of a side of the square is 8 feet.
  \[ A = s^2 \]
  \[ A = 8^2 \]
Step 2 Evaluate.

\[ A = 8^2 = 8 \times 8 = 64 \]

Step 3 Determine how many tiles are needed.

The area of the floor is 64 square feet. The area of each tile is 1 \( \times \) 1, or 1 square foot. So, Alan will need 64 tiles.

Solution Alan needs 64 1-foot by 1-foot tiles to cover the floor.

Example 3

The Sanfords are getting new carpet for a bedroom and a hallway in their house. The bedroom is 12 feet long and 10 feet wide. The hallway is 20 feet long and 4 feet wide. How much more carpeting do the Sanfords need for the bedroom than for the hallway?

Strategy Find the areas of the bedroom and the hallway.

Step 1 Identify the values of length, \( l \), and width, \( w \), for the bedroom.

\[ l = 12 \text{ and } w = 10 \]

Step 2 Substitute the values into the formula for the area of a rectangle. Solve.

\[ A = lw \]

\[ A = 12 \times 10 = 120 \text{ square feet} \]

Step 3 Identify the values of length, \( l \), and width, \( w \), for the hallway.

\[ l = 20 \text{ and } w = 4 \]

Step 4 Substitute the values into the formula for the area of a rectangle. Solve.

\[ A = lw \]

\[ A = 20 \times 4 = 80 \text{ square feet} \]

Step 5 Find the difference of the areas.

\[ 120 - 80 = 40 \]

Solution The Sanfords need 40 square feet more carpeting for the bedroom than for the hallway.
Carrie is going to paint the ceiling of her rectangular family room, which is 28 feet long and 18 feet wide. Each quart of paint costs $12.50 and can cover 85 square feet. Carrie will only use one coat of paint. How much money will Carrie spend on paint?

Find the area of the ceiling: \[ A = \text{ } \times \text{ } \]

Multiply.

The area of the ceiling is ______ square feet.

To find the number of quarts of paint needed, ______ the area, in square feet, of the ceiling by the number of square feet a quart will cover.

Divide.

\[ \text{ } \div \text{ } = \text{ } \]

Interpret the remainder by ________.

Carrie needs to buy ________ quarts of paint.

To find the total cost, ________ the number of quarts times the cost per quart.

Multiply.

\[ \text{ } \times \text{ } = \text{ } \]

Carrie will spend ________ on paint.
Solid figures, also called **three-dimensional figures**, are figures that have length, width, and height.

Solid figures can be classified by the number of faces, edges, and vertices they have. A **face** is a flat surface of a solid figure. An **edge** is a line segment where two faces of a solid figure meet. A **vertex** is the point where three or more edges of a solid figure meet. The plural of vertex is vertices.

A **net** is a flat pattern that can be folded into a three-dimensional figure. A net shows each surface of the solid figure it forms.

A **prism** is a three-dimensional figure with a pair of parallel faces called **bases** that are congruent polygons. Its other faces are rectangles or parallelograms. The table below shows some common prisms and their nets.

<table>
<thead>
<tr>
<th>Cube</th>
<th>Rectangular Prism</th>
<th>Triangular Prism</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image" alt="Cube" /></td>
<td><img src="image" alt="Rectangular Prism" /></td>
<td><img src="image" alt="Triangular Prism" /></td>
</tr>
<tr>
<td>face</td>
<td>face</td>
<td>face</td>
</tr>
<tr>
<td>vertex</td>
<td>vertex</td>
<td>vertex</td>
</tr>
<tr>
<td>edge</td>
<td>edge</td>
<td>edge</td>
</tr>
<tr>
<td>6 faces</td>
<td>6 faces</td>
<td>5 faces</td>
</tr>
<tr>
<td>12 edges</td>
<td>12 edges</td>
<td>9 edges</td>
</tr>
<tr>
<td>8 vertices</td>
<td>8 vertices</td>
<td>6 vertices</td>
</tr>
</tbody>
</table>
A **pyramid** has a base that is a polygon. All its other faces are triangles. The table below shows some common pyramids and their nets.

<table>
<thead>
<tr>
<th>Rectangular Pyramid</th>
<th>Triangular Pyramid</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1.png" alt="Rectangular Pyramid Diagram" /></td>
<td><img src="image2.png" alt="Triangular Pyramid Diagram" /></td>
</tr>
<tr>
<td>5 faces</td>
<td>4 faces</td>
</tr>
<tr>
<td>8 edges</td>
<td>6 edges</td>
</tr>
<tr>
<td>5 vertices</td>
<td>4 vertices</td>
</tr>
</tbody>
</table>

**Example 1**

What kind of solid figure is shown?

**Strategy**  
Use the faces to identify the figure.

**Step 1**  
Count the number of faces.  
There are 5 faces.  
Triangular prisms and rectangular pyramids have 5 faces.

**Step 2**  
Identify the shapes of the faces.  
There are 3 rectangular faces and 2 triangular faces.  
A rectangular pyramid does not have more than 1 rectangular face.

**Solution**  
The figure is a **triangular prism**.
Example 2
How are a rectangular prism and a rectangular pyramid alike? How are they different?

Strategy Describe the properties of a rectangular prism and a rectangular pyramid.

Step 1 Describe the number of faces in each figure.
A rectangular prism has 6 faces.
A rectangular pyramid has 5 faces.

Step 2 Describe the bases of each figure.
A rectangular prism has two parallel bases that are congruent rectangles.
A rectangular pyramid has one base that is a rectangle.

Step 3 Describe the faces of each figure.
The faces of a rectangular prism are rectangles.
The faces of a rectangular pyramid are triangles.

Step 4 List the similarities of the figures.
The rectangular prism and the rectangular pyramid each have a rectangular base.

Step 5 List the differences in the figures.
The rectangular prism and the rectangular pyramid have different numbers of faces.
All faces of the rectangular prism are rectangles. Only one face of the rectangular pyramid is a rectangle. All its other faces are triangles.

Solution The similarities and differences in a rectangular prism and a rectangular pyramid are listed in Steps 4 and 5.
Coached Example

How are a triangular prism and a triangular pyramid alike? How are they different?

A triangular prism has ____ faces.
A triangular pyramid has ____ faces.
A triangular prism has two bases that are ___________________.
A triangular pyramid has one base that is a _________________.
The faces of a triangular prism are _________________ and _________________.
The faces of a triangular pyramid are all _________________.

List the similarities in the figures.
A triangular prism and a triangular pyramid each have a _________________ base.

List the differences in the figures.
A triangular prism has ____ faces, but a triangular pyramid has ____ faces.
The faces of a triangular prism are _________________ and _________________, but
the faces of a triangular pyramid are all _________________.

Surface Area

Getting the Idea

Surface area is the total area of the surfaces of a solid figure. Surface area is measured in square units. You can use a net to help you find the surface area of a solid figure.

Example 1

What is the surface area of this rectangular prism?

Strategy Use a net.

Step 1

Draw the net showing the dimensions.

Find the area of each face.

\[ A = 7 \times 5 = 35 \]
\[ A = 2 \times 5 = 10 \]
\[ A = 7 \times 2 = 14 \]
\[ A = 2 \times 5 = 10 \]
\[ A = 7 \times 5 = 35 \]
\[ A = 7 \times 2 = 14 \]

Step 2

Add the areas to find the total surface area.

\[ 35 + 10 + 14 + 10 + 35 + 14 = 118 \]

Solution The surface area of the rectangular prism is 118 cm\(^2\).

A formula can be used to find the surface area (SA) of a rectangular prism:

\[ SA = 2lw + 2lh + 2wh, \text{ where } l \text{ is the length, } w \text{ is the width, and } h \text{ is the height} \]
Example 2
What is the surface area of this rectangular prism?

![Rectangular Prism Diagram]

Strategy  Use the formula for the surface area of a rectangular prism.

Step 1  Substitute the values into the formula.
\[
SA = 2lw + 2lh + 2wh
\]
\[
SA = (2 \times 9 \times 8) + (2 \times 9 \times 6) + (2 \times 8 \times 6)
\]

Step 2  Multiply.
\[
SA = (2 \times 9 \times 8) + (2 \times 9 \times 6) + (2 \times 8 \times 6)
\]
\[
SA = (18 \times 8) + (18 \times 6) + (16 \times 6) = 144 + 108 + 96
\]

Step 3  Add.
\[
SA = 144 + 108 + 96 = 348
\]

Solution  The surface area of the rectangular prism is 348 in.²

Example 3
What is the surface area of this cube?

![Cube Diagram]

Strategy  Use the formula for the surface area of a cube.

Step 1  Substitute the values into the formula.
\[
SA = 6s^2
\]
\[
SA = 6 \times 8^2
\]
Step 2  Multiply.

\[
SA = 6 \times 8^2 = 6 \times 64 = 384
\]

Solution  The surface area of the cube is 384 in.\(^2\).

To find the surface area of a triangular prism, find the area of each of the faces. Then add the areas. Use a net to help you.

Example 4

What is the surface area of this triangular prism?

Strategy  Use a net. Add the areas of the faces.

Step 1  Draw the net showing the dimensions.

Step 2  Find the area of each face.

\[
\begin{align*}
A &= \frac{1}{2}bh \\
A &= \frac{1}{2} \times 12 \times 8 = 48 \\
A &= 15 \times 12 = 180 \\
A &= \frac{1}{2} \times 12 \times 8 = 48 \\
A &= 15 \times 10 = 150
\end{align*}
\]

Step 3  Add the areas.

\[
150 + 48 + 180 + 48 + 150 = 576
\]

Solution  The surface area of the triangular prism is 576 cm\(^2\).
A pyramid has one base, which can be any polygon. All of the other faces of the pyramid, known as **lateral faces**, are triangles.

The surface area of a pyramid is the sum of the areas of its base and lateral faces. To find the area of a lateral face, use the formula for the area of a triangle: \( A = \frac{1}{2}bh \). Use the **slant height** of the lateral faces.

### Example 5
What is the surface area of this square pyramid?

**Strategy** Use a net. Add the areas of the faces.

**Step 1** Draw the net showing the dimensions.

**Step 2** Find the area of the square base.
\[
A = s^2 = 3 \times 3 = 9
\]

**Step 3** Find the total area of the four triangular faces.
Each triangular face has the same base and slant height.
Find the area of one triangular face.
\[
A = \frac{1}{2}bh = \frac{1}{2} \times 3 \times 5 = 7.5
\]
Multiply the area by 4.
\[
4 \times 7.5 = 30
\]

**Step 4** Add the areas of the base and the faces.
\[
9 + 30 = 39
\]

**Solution** The surface area of the pyramid is 39 \( \text{cm}^2 \). 
Coached Example

Akira is wrapping a box that is shaped like a cube. The box has a side length of 5 inches. What is the least amount of wrapping paper that Akira needs?

What is the formula for the surface area of a cube?

\[ SA = \]

Substitute the known value into the formula.

\[ SA = \]

What is the surface area of the box? ____________

Akira will need at least _______________ of wrapping paper to wrap the box.
Volume (V) is a measure of the number of cubic units that fit inside a solid figure. A cubic unit can be any unit such as a cubic inch (in.\(^3\)) or a cubic centimeter (cm\(^3\)), both shown below.

Sometimes the edge lengths of a prism include fractions or decimals. For example, a cube with a volume of \(\frac{1}{8}\) cubic inch has an edge length of \(\frac{1}{2}\) inch and a cube with a volume of 0.125 cubic centimeters has an edge length of 0.5 centimeter.

**Example 1**

Each cube has edges of \(\frac{1}{2}\) inch. What is the volume of the rectangular prism?

**Strategy**

Find how many cubes fit inside the prism. Then multiply by the volume of 1 cube.

**Step 1**

Find the number of cubes on the bottom layer.
- There are 3 rows of 6 cubes each.
- There are 18 cubes on the bottom layer.
Step 2  Find the number of cubes in the prism.

Multiply the number of cubes in the bottom layer by the number of layers.

\[ 3 \times 18 = 54 \]

There are 54 cubes inside the prism.

Step 3  Find the volume of one cube.

\[ \left( \frac{1}{2} \right)^3 = \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{8} \]

Each cube has a volume of \( \frac{1}{8} \) cubic inches

Step 4  Find the volume of the rectangular prism.

Multiply the number of cubes by the volume of each cube.

\[ 54 \times \frac{1}{8} = \frac{54}{8} = 6\frac{6}{8} = 6\frac{3}{4} \]

Solution  The volume of the rectangular prism is \( 6\frac{3}{4} \text{ in.}^3 \)

You can use cubes like the ones above to find the volume of a rectangular prism.

Example 2

Each cube inside the rectangular prism has an edge length of \( \frac{1}{2} \) inch and a volume of \( \frac{1}{8} \) cubic inches. What is the volume of the rectangular prism below?

Strategy  Find how many cubes fit inside the prism. Multiply to find the volume.

Step 1  Find the number of cubes in the bottom layer.
There are 5 rows of 9 cubes each.
So, there are $5 \times 9 = 45$ cubes in the bottom layer.

**Step 2**
Find the total number of cubes in the prism.
Multiply the number of cubes in the bottom layer by the number of layers.
There are 4 layers.
$4 \times 45 = 180$
There are 180 cubes, each with an edge length of $\frac{1}{2}$ inch, in the prism.

**Step 3**
Multiply to find the volume of the prism.
Each cube has a volume of $\frac{1}{8}$ in.$^3$, so the volume of the prism is:

$$180 \times \frac{1}{8} = \frac{180}{1} \times \frac{1}{8} = \frac{45}{2} = 22\frac{1}{2}$$

**Solution**
The volume of the rectangular prism is $22\frac{1}{2}$ in.$^3$

You could also have used multiplication to find the volumes of the prisms in Examples 1 and 2. Each prism is equal to length × width × height.

- In Example 1: volume $= 6 \times 3 \times 3 = 54$ cubic inches
- In Example 2: volume $= 4\frac{1}{2} \times 2\frac{1}{2} \times 2 = 22\frac{1}{2}$ cubic inches

You can use formulas to find the volumes of rectangular prisms or cubes.

<table>
<thead>
<tr>
<th>Formula for Volume, $V$</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Rectangular Prism</strong></td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td>$V = lwh$, where $l$ represents the length, $w$ represents the width, and $h$ represents the height.</td>
<td></td>
</tr>
<tr>
<td>This formula is also written as $V = Bh$, where $B$ represents the area of the base (length × width) and $h$ represents the height.</td>
<td></td>
</tr>
<tr>
<td><strong>Cube</strong></td>
<td><img src="image" alt="Diagram" /></td>
</tr>
<tr>
<td>$V = s^3$ where $s$ represents the side length.</td>
<td></td>
</tr>
</tbody>
</table>
Example 3
What is the volume of this fish tank?

Strategy  Use the formula for the volume of a rectangular prism.

Step 1  Substitute known values into the formula, \( V = lwh \).
\[
V = 27 \times 11\frac{1}{4} \times 13\frac{1}{2}
\]

Step 2  Rename the dimensions as improper fractions. Multiply.
\[
V = \frac{27}{1} \times \frac{45}{4} \times \frac{27}{2} = \frac{32,805}{8} = 4,100\frac{5}{8}
\]

Solution  The volume of the fish tank is \( 4,100\frac{5}{8} \) in.\(^3\)

You can also use the formula \( V = Bh \) to find the volume of the fish tank in Example 3.
\[
V = Bh
= \left( 27 \times 11\frac{1}{4} \right) \times 13\frac{1}{2}
= \left( \frac{27}{1} \times \frac{45}{4} \right) \times \frac{27}{2}
= \frac{1,215}{4} \times \frac{27}{2}
= \frac{32,805}{8} = 4,100\frac{5}{8}
\]
Example 4
What is the volume of this cube?

Strategy Use the formula for the volume of a cube.
Each side, \( s \), measures \( 1 \frac{1}{2} \) feet or \( \frac{3}{2} \) feet.

\[
V = \frac{3}{2} \times \frac{3}{2} \times \frac{3}{2} = \frac{27}{8} = 3 \frac{3}{8}
\]

Solution The volume of the cube is \( 3 \frac{3}{8} \text{ ft}^3 \).

Coached Example
What is the volume of a rectangular prism with the dimensions shown below?

The formula for the volume of a rectangular prism is \( V = \text{______} \).

Substitute known values into the formula.

Length (\( l \)) = \( \text{______} \) Width (\( w \)) = \( \text{______} \) Height (\( h \)) = \( \text{______} \)

\[
V = \text{______} \times \text{______} \times \text{______}
\]

\[
V = \text{______}
\]

The volume of the prism shown above is \( \text{______} \) cubic centimeters.
Measures of Center

Getting the Idea

If you ask 1 friend “How much time did you spend watching TV yesterday?” you will get just 1 answer. But if you ask 50 people the same question, the answers you get will vary. A statistical question is any question for which you expect to get a variety of answers.

Example 1

Which of the following questions is a statistical question?

1. How tall is the town’s mayor?
2. What are the heights of the players on the school basketball team?

Strategy Determine whether answers to the question will vary.

Step 1 Analyze question 1.
The question is “How tall is the town’s mayor?”
A town has only one mayor. There is only one answer to the question: the height of the person who is mayor.

Step 2 Analyze question 2.
The question is “What are the heights of the players on the school basketball team?”
There are many players on the basketball team. Those players are probably not all the same height.
You expect to get many different answers to this question.

Solution Question 1 is not a statistical question. Question 2 is a statistical question.

A measure of center is a single number that you can use to describe all of the values in a data set. You can think of a measure of center as a number that tells you roughly what the middle or average value in a data set is. Mean, median, and mode are all measures of center.

The mean is equal to the sum of the terms in a data set divided by the number of terms in the set.

The median is the middle term in a data set ordered from least to greatest. If there is an even number of terms in a set, the median is the mean of the two middle numbers.
The **mode** is the term or terms that appear most frequently in the data set. A set may have no mode, one mode, or more than one mode.

**Example 2**

Frank asked 5 of his relatives how many miles they drive to work. Their answers are 20 miles, 24 miles, 12 miles, 3 miles, and 16 miles. Find the mean and the median of the data.

**Strategy**  
Use the definitions of mean and median.

**Step 1** Find the mean of the data.  
The mean is the quotient of the sum of the terms divided by the number of terms.  
Add the terms.  
\[3 + 12 + 16 + 20 + 24 = 75\]  
Divide that sum by the number of terms. There are 5 terms.  
\[75 \div 5 = 15\]  
The mean is 15 miles.

**Step 2** Find the median of the data.  
The median is the middle term in a data set ordered from least to greatest.  
Order the data from least to greatest.  
3, 12, 16, 20, 24  
The middle term is 16.  
The median is 16 miles.

**Solution**  
The mean is 15 miles and the median is 16 miles.

Notice that both 15 miles and 16 miles are roughly in the middle of the values in the data set. You could use either the mean or the median as a good way to describe the typical distance driven to work by Frank’s relatives.

**Example 3**

The list below shows the weights, in pounds, of six dogs at a dog show:

52, 52, 55, 55, 54, 47

What are the median and the mode of the data?

**Strategy**  
Use the definitions of median and mode.

**Step 1** Order the weights from least to greatest.  
47, 52, 52, 54, 55, 55
Step 2  Find the median.

There is an even number of terms, so underline the two middle numbers.
47, 52, 52, 54, 55

The median is the mean of those two middle numbers.
52 + 54 = 106 and 106 ÷ 2 = 53

The median weight is 53 pounds.

Step 3  Find the mode.

The mode is the number or numbers that appear most often in a data set.
Two numbers appear twice in the data set: 52 and 55.
This data set has two modes: 52 pounds and 55 pounds.

Solution  The median weight is 53 pounds. There are 2 mode weights: 52 pounds and 55 pounds.

The mode of 52 pounds is better than the mode of 55 pounds to describe the center of the data. The mode of 52 pounds is close to the median of 53 pounds, while the mode of 55 pounds represents the greatest value of the data.

Example 4

The table shows Javier’s scores for the last 4 games he bowled.

<table>
<thead>
<tr>
<th>Javiers Bowling Scores</th>
</tr>
</thead>
<tbody>
<tr>
<td>Game</td>
</tr>
<tr>
<td>Score</td>
</tr>
</tbody>
</table>

What score does Javier need in his fifth game to have a mean bowling score of exactly 100 for all 5 games?

Strategy  Use what you know about the mean to solve the problem.

Step 1  Find the sum of the scores of the first 4 games.
90 + 93 + 101 + 104 = 388

Step 2  Find the total score Javier needs to have a mean score of 100 for all 5 games.
5 × 100 = 500    Javier needs a total score of 500.

Step 3  Find the number you need to add to 388 to get a sum of 500.
500 − 388 = 112    Javier needs to bowl 112 in game 5.

Solution  Javier needs to score 112 points in game 5 to have a mean score of exactly 100 for all 5 games.
If one data value is much greater or much less than the rest of the data values, it is called an **outlier**. An outlier may affect a measure of center.

**Example 5**

The low temperatures for five days last week were 20°F, 24°F, 18°F, 5°F, and 23°F. Which better describes the data, the median or the mean?

**Strategy** Determine how the outlier affects the median and mean.

**Step 1** Identify the outlier.

The outlier is 5°F.

**Step 2** Find the median.

Order the data from least to greatest: 5, 18, 20, 23, 24

The median is 20.

**Step 3** Determine how the outlier affects the median.

If 5 is taken out of the data, the median becomes 21.5, which is close to 20.

**Step 4** Find the mean.

Add: 20 + 24 + 18 + 5 + 23 = 90

Divide: 90 ÷ 5 = 18

The mean is 18.

**Step 5** Determine how the outlier affects the mean.

Find the mean of the data without the outlier.

Add: 20 + 24 + 18 + 23 = 85

Divide: 85 ÷ 4 = 21.25

Without the outlier, the mean is 21.25.

**Solution** The median describes the data better than the mean because the mean is greater than or equal to every value except the outlier.

An outlier will affect the mean more than it will affect the median. If you have a data set with an outlier, the median usually will be a better choice as a measure of center.
Edie has taken 5 math quizzes. Her scores are: 89, 96, 92, 84, 94. What are Edie’s mean and median scores?

The mean is equal to the _________ of the data items divided by the number of _________.

Find the sum of Edie’s scores.

   ______ + ______ + ______ + ______ + ______ = ______

Divide the sum by ______ to find the mean score.

   ______ ÷ ______ = ______

The median is the ___________ value in a data set ordered from _________ to _____________________.

Order Edie’s scores from least to greatest.

   ______, ______, ______, ______, ______

What is the middle value? _________

Edie’s mean score is __________. Edie’s median score is __________.
Measures of Variability

Getting the Idea

Like a measure of center, a **measure of variability** is a single number that can be used to describe an entire data set. The difference is that a measure of variability describes how spread out the data is, instead of describing the middle or the average of the data.

A common measure of the variability in a data set is the range. The **range** of a data set is the difference of the greatest value and the least value in the data set.

Example 1

Griffin’s scores on his first seven math quizzes are shown below.

94, 86, 95, 86, 82, 90, 95

What is the range of Griffin’s math quiz scores?

**Strategy**  
Order the numbers from least to greatest. Then find the range.

**Step 1**  
Order the numbers from least to greatest.  
82, 86, 86, 90, 94, 95, 95

**Step 2**  
Identify the least and greatest values in the data.  
The least value is 82.  
The greatest value is 95.

**Step 3**  
Find the difference of the greatest and least values.  
95 - 82 = 13

**Solution**  
The range of Griffin’s quiz scores is 13.

The median divides a data set into two halves. The median of the lower half of the data is called the **first quartile**. The median of the upper half of the data is called the **third quartile**. As shown in the diagram on the next page, the first quartile, median, and third quartile divide the full data set into four smaller data sets.

The **interquartile range (IQR)** is the difference of the third quartile and the first quartile. The IQR is the range of the middle half of the data which can be used to measure the variability of a data set.
**Example 2**  
The ages of Abe’s grandchildren are: 20, 15, 23, 8, 20, 10, 15, 25, 16, and 18.  
What are the first quartile, third quartile, and interquartile range of the grandchildren’s ages?

**Strategy**  
Order the numbers. Then find the quartiles.

**Step 1**  
Order the numbers from least to greatest.  
8, 10, 15, 15, 16, 18, 20, 20, 23, 25

**Step 2**  
Find the median.  
There are 10 data items, so the median is the mean of the two middle numbers.  
8, 10, 15, 15, 16, 18, 20, 20, 23, 25  
16 + 18 = 34  
34 ÷ 2 = 17  
The median is 17.

**Step 3**  
Find the first quartile.  
The first quartile is the median of the lower half of the data:  
8, 10, 15, 15, 16.  
The median of the lower half of the data is 15, so the first quartile is 15.

**Step 4**  
Find the third quartile.  
The third quartile is the median of the upper half of the data:  
18, 20, 20, 23, 25.  
The median of the upper half of the data is 20, so the third quartile is 20.

**Step 5**  
Find the interquartile range.  
Subtract the first quartile from the third quartile: 20 − 15 = 5

**Solution**  
The first quartile is 15, the third quartile is 20, and the interquartile range is 5.
Some data sets will include one or more outliers. In general, the interquartile range is a better measure of variation than the range because an outlier will affect the range more than the IQR.

**Mean absolute deviation (MAD)** is another measure of the variability in a data set. The mean absolute deviation tells you by how much, on average, each item in the data set differs from the mean.

To find the mean absolute deviation of a data set:
1. Find the mean of the data set.
2. Find the absolute value of the difference of each data item and the mean.
3. Find the sum of the absolute values.
4. Divide the sum by the total number of items in the data set.

### Example 3
The ages of Abe’s grandchildren are:

20, 15, 23, 8, 20, 10, 15, 25, 16, and 18.

What is the mean absolute deviation of the ages of Abe’s grandchildren?

**Strategy**  
Find the mean. Subtract each value from the mean. Then find the mean of the absolute values of the deviations from the mean.

**Step 1**  
Find the mean.

\[
20 + 15 + 23 + 8 + 20 + 10 + 15 + 25 + 16 + 18 = 170
\]

\[
170 \div 10 = 17
\]

**Step 2**  
Find the absolute values of the deviations.

Make a table with the data items ordered from least to greatest.

Find the deviations from the mean. Then find their absolute values.
Since you are finding the absolute values of the differences, subtract the lesser number from the greater number.

<table>
<thead>
<tr>
<th>Data Item</th>
<th>Absolute Value of the Difference from the Mean</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>17 – 8 = 9</td>
</tr>
<tr>
<td>10</td>
<td>17 – 10 = 7</td>
</tr>
<tr>
<td>15</td>
<td>17 – 15 = 2</td>
</tr>
<tr>
<td>15</td>
<td>17 – 15 = 2</td>
</tr>
<tr>
<td>16</td>
<td>17 – 16 = 1</td>
</tr>
<tr>
<td>18</td>
<td>18 – 17 = 1</td>
</tr>
<tr>
<td>20</td>
<td>20 – 17 = 3</td>
</tr>
<tr>
<td>20</td>
<td>20 – 17 = 3</td>
</tr>
<tr>
<td>23</td>
<td>23 – 17 = 6</td>
</tr>
<tr>
<td>25</td>
<td>25 – 17 = 8</td>
</tr>
</tbody>
</table>

**Step 3** Find the mean absolute deviation.

Add the absolute values.

\[
9 + 7 + 2 + 2 + 1 + 1 + 3 + 3 + 6 + 8 = 42
\]

Divide by the number of items in the data set.

\[
42 \div 10 = 4.2
\]

The mean absolute deviation is 4.2.

**Solution** The mean absolute deviation of the ages of Abe’s grandchildren is 4.2.

When a measure of variability for a data set is small, it means the data is clustered together. If the measure of variability is large, the data is more spread out.

**Example 4**

What do the interquartile range (IQR) and the mean absolute deviation (MAD) from Examples 2 and 3 tell you about the ages of Abe’s grandchildren?

**Strategy** Use the definitions of interquartile range and mean absolute deviation.

**Step 1** What does the IQR tell you about the ages of the grandchildren?

The IQR is 5.

That means that the middle half of the data values are within 5 of each other, from the ages of 15 to 20.

The IQR is small, so the ages of Abe’s grandchildren are clustered together.
Lesson 33: Measures of Variability

Step 2
What does the MAD tell you about the ages of the grandchildren?

The MAD is 4.2.
The MAD is small, so the ages of Abe’s grandchildren are clustered together.

Solution
The interquartile range and the mean absolute deviation both show that the ages of Abe’s grandchildren are close in value to each other.

Coached Example

The number of text messages that Aimee sent on each of the past 7 days is shown below.

24, 19, 11, 15, 11, 28, 20

What are the first quartile, third quartile, and interquartile range of this data set?

Order the numbers from least to greatest.

____, ____ , _____, _____, _____, _____, _____

Find the median. The median is the ________________ value of an ordered data set.

Median: ______

Find the first quartile. The first quartile is the median of the ________________ half of a data set.

First quartile: ______

Find the third quartile. The third quartile is the median of the ________________ half of a data set.

Third quartile: ______

The interquartile range is the ________________ of the ____________ quartile and the ____________ quartile.

Interquartile range: _____ – _____ = ______

Interquartile range: __________

The first quartile is ______, the third quartile is ______, and the interquartile range is ______.
Dot Plots

Getting the Idea

A dot plot uses a number line and dots to display numerical data. The number of dots above each value on the number line tells how many times that value occurs in a data set. Since a dot plot uses a number line, dot plots allow you to see the variation in a data set.

In a dot plot, a cluster shows where a group of data points fall. A gap is an interval where there are no data items.

Example 1

In a science class, the students weighed some samples to the nearest \( \frac{1}{8} \) pound, which are given below.

\[
\frac{1}{8}, \frac{3}{8}, \frac{3}{4}, \frac{1}{4}, \frac{1}{8}, \frac{1}{4}, \frac{7}{8}, \frac{1}{4}, \frac{3}{8}, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, \frac{1}{8}
\]

Make a dot plot for the data. Then analyze the data.

Strategy

Create a dot plot to display the data.

Step 1

Draw a number line.

The least value in the data is \( \frac{1}{8} \) and the greatest value is \( \frac{7}{8} \).

Draw a number line from 0 to 1, divided into eighths.

Step 2

Order the data values from least to greatest.

\[
\frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{8}, \frac{1}{4}, \frac{1}{4}, \frac{1}{4}, \frac{3}{8}, \frac{3}{8}, \frac{3}{8}, \frac{3}{8}, \frac{3}{4}, \frac{3}{4}, \frac{7}{8}, \frac{1}{8}, \frac{1}{2}, \frac{3}{4}, \frac{1}{8}
\]

Step 3

Draw a dot for each value in the data set.
Step 4  Write a title for the dot plot.
   The data is about the size of samples to the nearest \(\frac{1}{8}\) pound.

![Size of Samples to Nearest \(\frac{1}{8}\) Pound](image)

Step 5  Analyze the data.
   There are a few values greater than or equal to \(\frac{1}{2}\), but most of the values cluster from \(\frac{1}{8}\) to \(\frac{3}{8}\).

Solution  The dot plot for the weight data is shown in Step 3. The values cluster from \(\frac{1}{8}\) pound to \(\frac{3}{8}\) pound.

Example 2
The dot plot below shows the ages of people in Tori’s pottery class.

![Ages (in years)](image)

What are the mean, median, and mode ages of the people in the class?

Strategy  Analyze the dot plot.

Step 1  Find the mean of the data.
   Count the number of people in the class. There are 15 dots, so there are 15 people in the class.
   Find the sum of the ages. Multiply each age by the number of dots above it, then add the products.
   \[
   (2 \times 12) + (2 \times 13) + (2 \times 14) + (4 \times 15) + (0 \times 16) + (3 \times 17) + (2 \times 18) \\
   24 + 26 + 28 + 60 + 0 + 51 + 36 = 225
   \]
   Divide the sum by the number of people in the class.
   \[
   225 \div 15 = 15
   \]
   The mean age is 15 years.
**Step 2**  
Find the median of the data.  
Divide the total number of dots by 2.  
\[ 15 \div 2 = 7.5 \]  
Round up to the next whole number. 7.5 rounds to 8.  
Find the value of the eighth dot.  
The eighth dot is above 15.  
The median age is 15 years.

**Step 3**  
Find the mode of the data.  
The greatest number of dots is above 15.  
The mode age is 15 years.

**Solution**  
The mean, median, and mode ages are 15 years.

**Example 3**  
The dot plot shows the distance that the students in Miss Hall’s class live from school.

What is the range of the distances that the students live from school?

**Strategy**  
Use the dot plot.

**Step 1**  
Find the greatest data value.  
The greatest number of dots is above 4.

**Step 2**  
Find the least data value.  
The least number of dots is above \( \frac{1}{2} \).

**Step 3**  
Find the difference between the greatest and least values.  
\[ 4 - \frac{1}{2} = 3 \frac{1}{2} \]

**Solution**  
The range is \( 3 \frac{1}{2} \) miles.
Jeffrey surveyed all the students in his class. He asked them how many children were in their families. He recorded the results in this dot plot.

How many students are in families with more than 3 children?

More than 3 children means ______ or more children.

How many students are in families with 4 children? ______
How many students are in families with 5 children? ______
How many students are in families with 6 children? ______

Add those numbers to find the number of students with more than 3 children in their families.

_______ + _______ + _______ = _______

A total of _______ students have families with more than 3 children.
Choose the Best Measure

Getting the Idea

The shape of the data in a data set can help you choose whether to use the median or the mean as a measure of center.

Example 1

Mr. Spector separated his class into groups to do a science project. The dot plot below shows the number of hours that each group worked on its science project.

Which measure of center, the median or the mean, is the better measure for describing the data?

Strategy

Find the median and the mean of the data in the line plot.

Step 1

Find the median.

Count the number of dots on the line plot. There are 15 dots, so there were 15 groups in the class.

\[ 15 \div 2 = 7.5 \]

The median will be the value of the eighth dot. Starting at the left, count dots until you get to the eighth dot. The median is 6.

Step 2

Find the mean.

Add: \[ 1 + 1 + 2 + 2 + 2 + 4 + 6 + 6 + 6 + 6 + 7 + 8 + 8 + 8 + 8 = 75 \]

Divide by 15: \[ 75 \div 15 = 5 \]

The mean is 5.

Step 3

Look at the shape of the data.

Most of the data is greater than or equal to 6, the median.

Solution

The median is a better measure for describing the data.
The shape of the data in a data set can also affect whether you use the interquartile range or the mean absolute deviation as a measure of variability.

**Example 2**
The booster club is selling sweatshirts for a fundraiser. The dot plot shows how many sweatshirts were sold by each member of the booster club on the first day.

![Dot plot](image)

The mean absolute deviation for the data is 2.76. Find the interquartile range for the data. Which better describes the data, the interquartile range or the mean absolute deviation?

**Strategy**  
Find the interquartile range of the data.

**Step 1**  
Find the median of the data.
Order the data values from least to greatest.
2, 2, 2, 2, 2, 3, 3, 3, 11, 11
The two middle values are 2 and 3, so the median is 2.5.

**Step 2**  
Find the first and third quartiles.
The first quartile is the median of the lower half of the data: 2, 2, 2, 2, 2.
The middle number is 2, so the first quartile is 2.
The third quartile is the median of the upper half of the data: 3, 3, 3, 11, 11.
The middle number is 3, so the third quartile is 3.

**Step 3**  
Find the interquartile range.
The interquartile range is the difference of the first and third quartiles.
3 – 2 = 1
The interquartile range of the data is 1.

**Step 4**  
Look at the shape of the data.
Most of the data is clustered around 2 and 3. The difference of 3 and 2 is 1, which is the same as the interquartile range.
The data does not show as much variability as the mean absolute deviation.

**Solution**  
The interquartile range of the data is 1. The interquartile range describes the data better than the mean absolute deviation does.
The dot plot shows the number of pairs of sneakers of each size sold at a shoe store in one day.

Size of Sneakers Sold

Which measure of center, the median or mean, better describes the data? Explain your answer.

Find the median.
Count the number of dots on the line plot. There are ______ dots.
12 ÷ 2 = ______
The median will be the mean of the sixth and seventh dots.
The value of both the sixth and seventh dots is ______. The median is ______.

Find the mean. Add the values.

____ + _____ + _____ + _____ + _____ + _____ + _____ + 
_____ + _____ + _____ + _____ + _____ = ______
Divide by _______. Write the answer as a mixed number.
_______ ÷ _________ = _________
The mean is ________.
Do shoe sizes come in the same size as the mean? ________

The _________________ is a better measure for describing the data.
Box Plots

Getting the Idea

A box plot can help you see the variation in a data set. You can see some measures of variation easily on a box plot, as shown below.

Example 1

Make a box plot of the following data.

250, 175, 215, 350, 320, 235, 250, 280

Strategy

Find the least and greatest values, the quartiles, and the median.

Step 1
Order the numbers from least to greatest.

175, 215, 235, 250, 250, 280, 320, 350

The least value is 175. The greatest value is 350.

Step 2
Find the median.

175, 215, 235, 250, 250, 280, 320, 350

The median is 250.

Step 3
Find the first quartile.

The first quartile is the median of the lower half of the data.

175, 215, 235, 250

215 + 235 = 450 and 450 ÷ 2 = 225.

The first quartile is 225.

Step 4
Find the third quartile.

The third quartile is the median of the upper half of the data.

250, 280, 320, 350

280 + 320 = 600 and 600 ÷ 2 = 300.

The upper quartile is 300.
Step 5  Make the box plot.
Pick an interval that makes the box plot easy to read.
All of the values are multiples of 25, so use intervals of 25.
Use dots for the least and greatest values.
Use line segments for the quartiles and the median.

Solution  A box plot of the data is shown in Step 5 above.

Example 2
Isabella drew the box plot below to show the heights, in inches, of her tomato plants.

What are the median plant height and the interquartile range, in both inches and feet?

Strategy  Find the median and interquartile range in inches. Then convert to feet.

Step 1  Identify the median.
The line segment inside the box represents the median.
The median is 24 inches.

Step 2  Convert the height of the median to feet.
24 inches = 2 feet
The median plant height is 2 feet.

Step 3  Find the interquartile range.
The interquartile range is the difference of the third quartile and the first quartile.
The third quartile is 32. The first quartile is 14.
32 − 14 = 18
The interquartile range is 18 inches.
Step 4  Convert the interquartile range to feet.
18 inches = 1 foot 6 inches or \(1 \frac{1}{2}\) feet
The interquartile range is \(1 \frac{1}{2}\) feet.

Solution  The median plant height is 24 inches or 2 feet. The interquartile range is 18 inches or \(1 \frac{1}{2}\) feet.

Coached Example

Molly recorded the heights, in inches, of her cousins. Make a box plot of the data.

47, 56, 34, 52, 44, 28, 38

To make a box plot, you need to find five pieces of data: ___________________ value, _______________ quartile, median, _______________ quartile, and _______________ value.

Order the numbers from least to greatest:

______, ______, ______, ______, ______, ______, ______

Least value: __________
Greatest value: __________
The median is the ________________ number in the ordered list.
Median: __________
The first quartile is the median of the numbers in the ________________ half of the data.
First quartile: __________
The third quartile is the median of the numbers in the ________________ half of the data.
Third quartile: __________

Complete the number line below. Then make your box plot.

Heights (in inches)

A box plot for the data is shown above.
Getting the Idea

Frequency is a measure of how many times an event occurs. A frequency table can be used to record and display data. A frequency table may use tally marks. Each | represents 1 and each |||| represents 5.

You can display data from a frequency table in a histogram. A histogram is a bar graph that shows the frequency of data within equal intervals. A histogram does not have gaps between the bars, unless the frequency for an interval is 0.

Example 1

Rex and Jai performed a science experiment in which they recorded the outdoor temperature for 30 consecutive mornings. They displayed the results in the frequency table below.

<table>
<thead>
<tr>
<th>Temperature (in °F)</th>
<th>Tally</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>15–20</td>
<td></td>
<td></td>
</tr>
<tr>
<td>20–25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>25–30</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30–35</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Display the data in a histogram.

Strategy

Use the data in the frequency table to make the histogram.

Step 1

Decide how to label the axes.

Use the horizontal axis for the temperature intervals and the vertical axis for the frequency.

Step 2

Draw bars indicating the frequency for each of the intervals.
Step 3  Give the histogram a title.

Solution  A histogram for the temperature data is shown in Step 3 above.

Example 2
The histogram shows the number of phone calls that Sherry made each day in April.

On how many days did Sherry make from 1 to 6 calls?

Strategy  Find the frequencies for the intervals 1–3 and 4–6.

Step 1  Find the frequency for 1–3 calls.
The bar for 1–3 calls stops at 8.
Sherry made 1–3 calls on 8 different days.

Step 2  Find the frequency for 4–6 calls.
The bar for 4–6 calls stops halfway between 12 and 14, or at 13.
Sherry made 4–6 calls on 13 different days.
Step 3  Add the frequencies for the two intervals.

\[8 + 13 = 21\]

Solution  Sherry made from 1 to 6 calls on 21 days in April.

Example 3
The histogram below shows the times, in seconds, of the students on the track team in the 200-meter dash.

What time interval contains the median time?

Strategy  Find the total number of data items. Then find the interval that contains the median.

Step 1  Find the number of students who ran the 200-meter dash.

Each bar in the histogram tells you how many students had a 200-meter dash time that fell in that interval.

Add the frequencies for the intervals.

\[2 + 5 + 7 + 5 + 2 = 21\]

Twenty-one students ran the 200-meter dash.

Step 2  Find the interval that contains the median.

There are 21 data items.

\[21 \div 2 = 10.5\], so the median is the 11th data item in an ordered list.

The median is in the third interval, the times from 30 to 32 seconds.

Solution  The median time is in the interval for 30 to 32 seconds.
The histogram below shows the number of voters by age group in an election for mayor.

![Histogram of Voter Age Groups](image)

How many more voters were in the age group 51–60 than in the age group 21–30?

Find the number of voters in the age group 51–60.

The bar for the age group 51–60 is halfway between _____________ and _____________.

There are _____________ voters in the age group 51–60.

Find the number of voters in the age group 21–30.

The bar for the age group 21–30 ends on _____________.

Subtract. _____________ − _____________ = _____________

There were _____________ more voters in the age group 51–60 than in the age group 21–30.
Lesson 1
Coached Example
If the classes are divided into equal groups, the number of students in each group must be a factor of 24 and 27.
List the factors of 24: 1, 2, 3, 4, 6, 8, 12, 24
List the factors of 27: 1, 3, 9, 27
What are the common factors of 24 and 27? 1 and 3
What is the greatest common factor of 24 and 27? 3
If all the groups have the same number of students, there will be 3 students in each group.

Lesson 2
Coached Example
Multiply: 2 × 32 = 64
Multiply: 8 × 32 = 256
Multiply: 0 × 32 = 0
Multiply: 7 × 32 = 224

Lesson 3
Coached Example
What integer represents a play in which the team neither gains yards nor loses yards? 0
A play that gains yards would be represented by a positive integer.
A play that loses yards would be represented by a negative integer.
On the first play, the team gained 6 yards.
A gain of 6 yards is represented by the integer 6.
On the second play, the team lost 2 yards.
A loss of 2 yards is represented by the integer −2.
On the third play, the team gained 7 yards.

Lesson 4
Coached Example
The price change with the greatest absolute value is the greatest change.
|−12| = 12
|9| = 9
Which number has the greater absolute value, −12 or 9? −12
The stock with the price change of −12 dollars changed by the greatest amount.
That stock was Stock A.

Lesson 5
Coached Example
There are 5 spaces between 0 and 1.
The number line is divided into fifths. Each mark stands for \(\frac{1}{5}\).
−\(\frac{1}{5}\) is between −1 and 0. It is located at point B.
\(\frac{4}{5}\) is between 0 and 1. It is located at point C.
\(\frac{2}{5}\) is between 2 and 3. It is located at point D.
−\(\frac{3}{5}\) is between −2 and −1. It is located at point A.
Point B represents −\(\frac{1}{5}\), point C represents \(\frac{4}{5}\), point D represents \(\frac{2}{5}\), and point A represents −\(\frac{3}{5}\).

Lesson 6
Coached Example
The positive numbers are 2.6, 3, and 2\(\frac{3}{10}\).
2.6 = \(\frac{26}{10}\)
The greatest positive number is 3.
Compare the remaining two positive numbers.
\(\frac{26}{10} > \frac{3}{10}\)
From greatest to least, the positive numbers are 3, 2.6, and 2\(\frac{3}{10}\).
The negative numbers are $-3$ and $-3\frac{1}{4}$.
Which negative number is greater? $-3$
From greatest to least, the numbers are $3, 2.6, 2\frac{3}{10}, -3, \text{ and } -3\frac{1}{4}$.

Lesson 7
Coached Example
The jackets cost $86.56.
The hats cost $45.38.
Add $86.56 + 45.38 = 131.94$
Subtract $225.00 - 131.94 = 93.06$
Melanie has $93.06 left in the budget.

Lesson 8
Coached Example
To rename the divisor as a whole number, multiply the dividend and divisor by $10$.
$7.5 \times 10 = 75$
$\$142.50 \times 10 = \$1,425$
Divide: $1,425 \div 75$
\[
\begin{array}{c}
75 \div 1425 \\
= 75 \\
= 675 \\
= 0 \\
\end{array}
\]
The quotient is $19$.
Each pound of tea leaves costs $19$.

Lesson 9
Coached Example
\[
\frac{7}{8} + \frac{1}{16} = s
\]
\[
\frac{1}{16} \times \frac{16}{1} = 1, \text{ so the reciprocal of } \frac{1}{16} \text{ is } \frac{16}{1}.
\]
\[
\frac{7}{8} \times \frac{1}{16} = \frac{7 \times 1}{8 \times 1} = \frac{112}{8} = 14
\]
Mrs. Castillo can offer 14 servings of lemonade.

Lesson 10
Coached Example
The $x$-coordinate is negative and the $y$-coordinate is positive.
Start at the origin, which is the point $(0, 0)$.
Move 6 units to the left of the origin.

Lesson 11
Coached Example
The $y$-coordinates of the ordered pairs have different signs too.
Since the signs of both coordinates are different, point $J$ would need to be reflected across both axes.
The coordinates of that reflected point are $(4, -3)$.
The coordinates of that reflected point are $(-4, -3)$.
Point $L$ is at $(-4, -3)$. Both of its coordinates have different signs than the coordinates of point $J$. Point $L$ is a reflection of point $J$ across both axes.
Lesson 12
Coached Example
The ratio compares the total number of comedies and dramas to the total number of DVDs.
There are 8 comedies and 6 dramas.
Add: $8 + 6 = 14$
There are $8 + 6 + 3 + 7 = 24$ DVDs in all.
Write the ratio. $14$ to $24$
$$\frac{14}{24} = \frac{14 \div 2}{24 \div 2} = \frac{7}{12}$$
So, $14$ to $24$ can be simplified as $7$ to $12$.
The ratio of comedies and dramas to total DVDs is $7$ to $12$.

Lesson 13
Coached Example
$$\begin{array}{c}
\text{10 green tennis balls} \\
\hline
\text{5 yellow tennis balls} \\
\hline
\end{array}$$
Of $10$ green tennis balls
$$\frac{10}{5} = \frac{2}{x}$$
$$10 \times x = 5 \times 2$$
$$10x = 10$$
$$\frac{10}{10} = \frac{x}{1}$$
$$x = 1$$
Each can contains 2 green tennis balls and 1 yellow tennis ball(s).

Lesson 14
Coached Example
The speed formula is $r = \frac{d}{t}$.
The distance is 15 laps.
The time is 30 minutes.
$$r = \frac{15}{30}$$
$$r = \frac{1}{2}$$
Tanya’s average speed was $\frac{1}{2}$ lap per minute.

Lesson 15
Coached Example
$80\% = \frac{80}{100} = 0.8$
whole $= 52 \div 0.8 = 65$
The store has 65 fish in all.

Lesson 16
Coached Example

inches $= \frac{12}{1}$
inches $= \frac{78}{f}$

$\frac{12}{1} = \frac{78}{f}$
$12f = 78$

$f = 6 \frac{1}{2}$
The remainder is the additional number of inches.
The remainder is 6, so there are 6 additional inches.
1 foot = 12 inches, so 6 inches = $\frac{1}{2}$ foot.
The basketball player is $6 \frac{1}{2}$ feet tall.

Lesson 17
Coached Example
An expression that means “twice a number $n$” is $2n$.
“Increased by 11” means to add 11.
Write an expression combining the two parts.

$$2n + 11$$
An expression that represents “twice a number $n$ increased by 11” is $2n + 11$.

Lesson 18
Coached Example
$s^3$ means to use $s$ as a factor 3 times.
$$V = s^3 = s \times s \times s$$
Substitute 4 for $s$.
$$V = 4 \times 4 \times 4 = 64$$
Substitute 7 for $s$.
$$V = 7 \times 7 \times 7 = 343$$
Tamara’s first cube has a volume of 64 cubic centimeters, and her second cube has a volume of 343 cubic centimeters.

Lesson 19
Coached Example
To simplify $3(3n - 9)$, use the distributive property.

$3(3n - 9) = 3(3n) - 3(9) = 9n - 27$
Are the expressions equivalent? yes

$9n - 27 = 9(4) - 27 = 36 - 27 = 9$
$3(3n - 9) = 3(3 \times 4) - 3(9) = 3(12) - 27 = 36 - 27 = 9$
$9n - 27 = 9(10) - 27 = 90 - 27 = 63$
3(3n - 9) = 3(3 \times 10) - 3(9) = 3(30) - 27 = 90 - 27 = 63
Did both expressions have the same value for each given value of n? yes
The expressions 9n - 27 and 3(3n - 9) are equivalent.

Lesson 20
Coached Example
What operation is used in the term 16 \cdot d? multiplication
Write 16 \cdot d as an expression without an operation symbol. 16d
The inverse of multiplication is division.
To isolate the variable, divide both sides of the equation by 16.
16d \div 16 = 192 \div 16
d = 12
Gavin’s hourly wage is $12.

Lesson 21
Coached Example

<table>
<thead>
<tr>
<th>x</th>
<th>y = x + 2</th>
<th>y</th>
<th>(x, y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>y = 1 + 2 = 3</td>
<td>3</td>
<td>(1, 3)</td>
</tr>
<tr>
<td>3</td>
<td>y = 3 + 2 = 5</td>
<td>5</td>
<td>(3, 5)</td>
</tr>
<tr>
<td>5</td>
<td>y = 5 + 2 = 7</td>
<td>7</td>
<td>(5, 7)</td>
</tr>
<tr>
<td>7</td>
<td>y = 7 + 2 = 9</td>
<td>9</td>
<td>(7, 9)</td>
</tr>
</tbody>
</table>

Four ordered pairs for y = 3x + 2 are (1, 3), (3, 5), (5, 7), and (7, 9).

Lesson 22
Coached Example
What is the cost per golf ball? $0.85
What expression can you write to represent the cost of g golf balls? 0.85g
What was the total cost to Mr. Costa? $30.60
Write an equation to represent how to find the cost of g golf balls. 0.85g = 30.6
Divide both sides of the equation by 0.85.
g = 36
The equation 0.85g = 30.6 can be used to find the number of golf balls bought.
Mr. Costa bought 36 golf balls.

Lesson 23
Coached Example
Since 8 is subtracted from z, add 8 to both sides.
z - 8 + 8 \leq 5 + 8
z \leq 13
Since the symbol is \leq, draw an open circle at 13.
This shows that 13 is a solution.
Since all values less than 13 are also solutions, draw an arrow to the left.
The solution set for the inequality is z \leq 13. Its graph is shown above.

Lesson 24
Coached Example
The base of the triangle is 9 yards long and the height is 7 yards long.
The formula for the area of a triangle is \( A = \frac{1}{2}bh \).
\[ A = \frac{1}{2} \times 9 \times 7 \]
\[ A = 41 \times 7 \]
\[ A = 311 \times 2 \]
What units should be used to express the area? square yards
The area of the Clementes’ deck is 311 \times \frac{1}{2} \text{ yd}^2.
Lesson 25
Coached Example
The formula for the area of a square is $A = s^2$.
$A = \left(1 \frac{1}{2}\right)^2 = 1 \frac{1}{2} \times 1 \frac{1}{2} = 2 \frac{1}{4}$
What units should be used to express the area? square inches
The area of the square is $2 \frac{1}{4}$ in.$^2$.

Lesson 26
Coached Example
The dimensions of the rectangle are 30 mm by 15 mm.
The formula for the area of a rectangle is $A = lw$.
$A = 30 \times 15 = 450$
The area of the rectangle is 450 square millimeters.
In millimeters, the base of the triangle is $30 - 15 = 15$.
In millimeters, the height of the triangle is $25 - 15 = 10$.
The formula for the area of a triangle is $A = \frac{1}{2}bh$.
$A = \frac{1}{2} \times 15 \times 10 = 75$
The area of the triangle is 75 square millimeters.
Add the areas. $450 + 75 = 525$
The area of the figure is 525 square millimeters.

Lesson 27
Coached Example
Divide the figure into two rectangles. Draw a horizontal line segment from point $(-1, 2)$ to point $(2, 2)$.
The length of rectangle I is 4 + 5 = 9 units.
The width of rectangle I is 3 units.
Area of rectangle I = $9 \times 3 = 27$ square units
The length of rectangle II is 2 + 6 = 8 units.
The width of rectangle II is 1 + 2 = 3 units.
Area of rectangle II = $8 \times 3 = 24$ square units
Area of rectangle I + area of rectangle II = $27 + 24 = 51$ square units
Based on the key, each unit equals 1 meter.
The area of the bottom of the pool will be 51 square meters.

Lesson 28
Coached Example
Find the area of the ceiling: $A = 28 \times 18$
The area of the ceiling is 504 square feet.
To find the number of quarts of paint needed, divide the area, in square feet, of the ceiling by the number of square feet a quart will cover.
$504 \div 85 = 5 R79$
Interpret the remainder by adding 1 to the quotient.
Carrie needs to buy 6 quarts of paint.
To find the total cost, multiply the number of quarts times the cost per quart.
$12.50 \times 6 = 75$
Carrie will spend $75 on paint.

Lesson 29
Coached Example
A triangular prism has 5 faces.
A triangular pyramid has 4 faces.
A triangular prism has two bases that are triangles.
A triangular pyramid has one base that is a triangle.
The faces of a triangular prism are triangles and rectangles.
The faces of a triangular pyramid are all triangles.
A triangular prism and a triangular pyramid each have a triangular base.
A triangular prism has 5 faces, but a triangular pyramid has 4 faces.
The faces of a triangular prism are triangles and rectangles, but the faces of a triangular pyramid are all triangles.

Lesson 30
Coached Example
What is the formula for the surface area of a cube?
$SA = 6s^2$
Substitute the known value into the formula.
$SA = 6 \times 5^2$
What is the surface area of the box? 150 in.$^2$
Akira will need at least 150 in.$^2$ of wrapping paper to wrap the box.
Lesson 31
Coached Example
The formula for the volume of a rectangular prism is \( V = lwh \).
Length \( (l) = 3.5 \)
Width \( (w) = 2.5 \)
Height \( (h) = 3 \)
\( V = 3.5 \times 2.5 \times 3 \)
\( V = 26.25 \)
The volume of the prism shown above is 26.25 cubic centimeters.

Lesson 32
Coached Example
The mean is equal to the sum of the data items divided by the number of data items.
\[ 89 + 96 + 92 + 84 + 94 = 455 \]
Divide the sum by 5 to find the mean score.
\[ 455 \div 5 = 91 \]
The median is the middle value in a data set ordered from least to greatest.
84, 89, 92, 94, 96
What is the middle value? 92
Edie’s mean score is 91. Edie’s median score is 92.

Lesson 33
Coached Example
11, 11, 15, 19, 20, 24, 28
The median is the middle value of an ordered data set.
Median: 19
The first quartile is the median of the lower half of a data set.
First quartile: 11
The third quartile is the median of the upper half of a data set.
Third quartile: 24
The interquartile range is the difference of the first quartile and the third quartile.
\[ 24 - 11 = 13 \]
Interquartile range: 13
The first quartile is 11, the third quartile is 24, and the interquartile range is 13.

Lesson 34
Coached Example
More than 3 children means 4 or more children.
How many students are in families with 4 children? 3
How many students are in families with 5 children? 1
How many students are in families with 6 children? 1
Add those numbers to find the number of students with more than 3 children in their families.
\[ 3 + 1 + 1 = 5 \]
A total of 5 students have families with more than 3 children.

Lesson 35
Coached Example
There are 12 dots.
\[ 12 \div 2 = 6 \]
The value of both the sixth and seventh dots is \( 7 \frac{1}{2} \).
The median is \( 7 \frac{1}{2} \).
\[ 6 + 6 + 6 \frac{1}{2} + 7 + 7 + 7 \frac{1}{2} + 8 + 8 \frac{1}{2} + 9 + 9 + 10 = 92 \]
Divide by 12.
\[ 92 \div 12 = 7 \frac{2}{3} \]
The mean is \( 7 \frac{2}{3} \).
Do shoe sizes come in the same size as the mean? no
The median is a better measure for describing the data.

Lesson 36
Coached Example
To make a box plot, you need to find five pieces of data: least value, first quartile, median, third quartile, and greatest value.
Order the numbers from least to greatest:
28, 34, 38, 44, 47, 52, 56
Least value: 28
Greatest value: 56
The median is the middle number in the ordered list.
Median: 44
The first quartile is the median of the numbers in the lower half of the data.
First quartile: 34
The third quartile is the median of the numbers in the upper half of the data.
Third quartile: 52

Heights (in inches)

Lesson 37
Coached Example
The bar for the age group 51–60 is halfway between 70 and 80.
There are 75 voters in the age group 51–60.
The bar for the age group 21–30 ends on 40.
Subtract. \(75 - 40 = 35\)
There were 35 more voters in the age group 51–60 than in the age group 21–30.