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Integers include the set of whole numbers \((0, 1, 2, 3, \ldots)\) and their opposites \((-1, -2, -3, \ldots)\). The number line below shows integers from \(-5\) to \(5\). Positive numbers are located to the right of zero, and negative numbers are located to the left of zero.

A rational number is any real number that can be expressed as the ratio of two integers \(\frac{a}{b}\), where \(b\) is not equal to zero. Some examples of rational numbers are shown below.

\[-6, \ -\frac{3}{5}, \ \frac{27}{9}, \ 16\%, \ \sqrt{\frac{4}{9}}, \ 0.7, \ 0.79\]

A rational number can be expanded to form a terminating decimal or a repeating decimal. To convert a fraction to a decimal, you can divide the numerator by the denominator.

**Example 1**

Is \(\frac{5}{11}\) a rational number? If so, write it as a decimal.

**Strategy** Decide if \(\frac{5}{11}\) is rational. Then divide to write it as a decimal.

**Step 1** Is \(\frac{5}{11}\) a rational number?

5 and 11 are both integers.

So, \(\frac{5}{11}\) shows the ratio of two integers. It is rational.

**Step 2** Divide the numerator, 5, by the denominator, 11.

Insert zeros after the decimal point in 5 as needed.

\[
\begin{align*}
0.45 \\
11)5.00 \\
- 4.4 \\
- 0.60 \\
- 0.55 \\
- 0.5
\end{align*}
\]

**Step 3** The decimal repeats. Write a bar to show the repeating digits.

\(0.4\overline{5}\)

**Solution** \(\frac{5}{11}\) is rational. It can be expressed as the repeating decimal \(0.4\overline{5}\).
Example 2
Is $-2\frac{3}{5}$ a rational number? If so, write it as a decimal.

**Strategy**
Decide if $-2\frac{3}{5}$ is rational. Then divide to write it as a decimal.

**Step 1**
Is $-2\frac{3}{5}$ a rational number?
Convert $-2\frac{3}{5}$ to an improper fraction.

$$-2\frac{3}{5} = \frac{(2 \cdot 5) + 3}{5} = \frac{10 + 3}{5} = \frac{13}{5}$$

Since $-\frac{13}{5}$ is the ratio of two integers, $-2\frac{3}{5}$ is rational.

**Step 2**
Divide the numerator, $-13$, by the denominator, 5.
Since the signs are different, the quotient will be negative.
For now, drop the negative sign.

$$\begin{array}{c}
2.6 \\
\hline
5)13.0 \\
-10 \\
\hline
30 \\
-30 \\
\hline
0
\end{array}$$

The actual quotient is $-2.6$.

**Solution**
$-2\frac{3}{5}$ is rational. It can be expressed as $-2.6$. 
All rational numbers can be represented on a number line. To plot rational numbers on a number line, it is helpful to convert them to the same form.

You can convert a percent to a decimal by dividing the percent by 100 and dropping the percent sign. This is the same as moving the decimal point in the percent two places to the left.

Some square roots are also rational. Any number that has a whole-number square root is a perfect square.

If the number under a radical symbol ($\sqrt{\text{ }}$) is a perfect square, its value is an integer and a rational number. For example, $\sqrt{9}$ is rational because it is equal to 3.

**Example 3**

Plot and label a point for each rational number below on a number line.

$-\frac{2}{5}, 1\frac{1}{2}, -0.25$, $72.5\%$, $\sqrt{4}$

**Strategy** Write the numbers in an equivalent form.

**Step 1** Rewrite each number as a decimal or integer.

$-\frac{2}{5} = -1$

$\frac{1}{2} = 1 \div 2 = 0.5$, so $1\frac{1}{2} = 1.5$.

$-0.25$ is already in decimal form.

$72.5\% = 72.5\% \div 100\% = 0.725$

$\sqrt{4} = 2$, because $2^2 = 4$.

**Step 2** Plot and label each number on a number line.

Draw a number line from $-1$ to $2$ and divide it into tenths.

**Solution** The number line with the rational numbers labeled is shown in Step 2.
Coached Example

What decimal is represented by point $P$ on the number line below?

The number line is divided into sixths.

Starting at the tick mark for 1, you count ______ tick marks from 1 to point $P$.

So, point $P$ represents the mixed number ______.

Instead of converting the mixed number to an improper fraction, just convert the fractional part, _______, to a decimal.

Divide the numerator, _______, by the denominator, _______.

$\frac{6}{5}$

If the decimal repeats, write it with a bar over the repeating digit: _______

Add 1 to the decimal: $1\frac{5}{6} = 1 + _____ = _____$

The decimal represented by point $P$ is _______.
Domain 1 • Lesson 2

Irrational Numbers

Getting the Idea

The set of **real numbers** includes both rational numbers and **irrational numbers**. Any real number that cannot be written as the ratio of two integers \( \frac{a}{b} \), where \( b \) is not equal to zero, is irrational. An irrational number can be expressed as a non-terminating, non-repeating decimal.

Example 1

The value of the number \( \pi \) (pi) is shown below.

\[
\pi = 3.1459265358\ldots
\]

Explain why \( \pi \) is an irrational number.

**Strategy**   **Use the definition of an irrational number.**

**Step 1**   Recall the characteristics of irrational numbers.

The value of an irrational number is a non-terminating, non-repeating decimal.

**Step 2**   Describe the decimal value of pi.

The ellipsis (…) shows that the digits continue on forever and do not repeat.

So, \( \pi \) is a non-terminating non-repeating decimal.

**Solution**   \( \pi \) (pi) is an irrational number because it is a non-terminating, non-repeating decimal.

\( \pi \) is not the only irrational number. The square roots of positive numbers that are not perfect squares are also irrational. You can estimate the value of a square root by determining which two perfect squares it lies between and then using guess and check to approximate its value more precisely. The symbol \( \approx \) means approximately.
Example 2

Approximate the value of $\sqrt{2}$.

**Strategy**

Use the definition of an irrational number. Then use guess and check to approximate the value of $\sqrt{2}$.

**Step 1**

Find which perfect squares the number 2 lies between.

You can find perfect squares by squaring consecutive whole numbers.

$1^2 = 1 \cdot 1 = 1$

$2^2 = 2 \cdot 2 = 4$

$\sqrt{2}$ is between 1 and 2.

**Step 2**

Determine if the value is closer to 1 or 2.

2 is closer to 1 than to 4.

So, $\sqrt{2}$ is closer to 1 than to 2.

**Step 3**

Use guess and check to estimate $\sqrt{2}$ to the nearest tenth.

Try 1.4.

$1.4^2 = 1.4 \cdot 1.4 = 1.96$  close, but slightly less than 2.

Try 1.5.

$1.5^2 = 1.5 \cdot 1.5 = 2.25$  close, but not as close as 1.96.

Since 2 is closer to $1.4^2$ than to $1.5^2$, $\sqrt{2} \approx 1.4$.

**Solution**

The number $\sqrt{2}$ has a value close to 1.4.
Example 3

Graph the approximate location of $\sqrt{34}$ on a number line.

**Strategy** Use guess and check to approximate the value of $\sqrt{34}$ to the nearest tenth. Then graph the decimal on a number line.

**Step 1** Find which perfect squares 34 lies between.

\[5^2 = 5 \cdot 5 = 25\]
\[6^2 = 6 \cdot 6 = 36\]

34 is between the perfect squares 25 and 36, so $\sqrt{34}$ is between 5 and 6.

**Step 2** Find which whole number $\sqrt{34}$ is closer to.

34 is closer to 36 than to 25, so $\sqrt{34}$ is closer to 6 than to 5.

**Step 3** Use guess and check to estimate $\sqrt{34}$ to the nearest tenth.

Try 5.9.

\[5.9^2 = 5.9 \cdot 5.9 = 34.81 \quad \rightarrow \quad \text{close, but more than 34.}\]

Try 5.8.

\[5.8^2 = 5.8 \cdot 5.8 = 33.64 \quad \rightarrow \quad \text{close, and less than 34.}\]

Since 34 is closer to $5.8^2$ than to $5.9^2$, $\sqrt{34} \approx 5.8$.

**Step 4** Graph $\sqrt{34}$ on a number line divided into tenths.

Plot $\sqrt{34}$ between 5.8 and 5.9, but closer to 5.8.

![Number line with $\sqrt{34}$ marked between 5.8 and 5.9]

**Solution** $\sqrt{34}$ is graphed on the number line in Step 4.
Coached Example

The area of a square is 67 square meters. Find the exact length, in meters, of one side of the square. Then graph that approximate value on a number line.

The area, \( A \), of a square is found using the formula \( A = s^2 \), where \( s \) shows the length of one side.

So, the length of one side, \( s \), can be found by taking the square root of \( \sqrt{\square} \).

The exact length of each side of the square is \( \sqrt{\square} \) meters.

To graph that number on a number line, first approximate its value as a decimal.

67 lies between the perfect squares 64 and \( \square \).

\[ \sqrt{64} = \square, \text{ and the square root of the other perfect square is } \square. \]

So, \( \sqrt{67} \) lies between the whole numbers \( \square \) and \( \square \), but is closer to \( \square \).

Use guess and check to approximate its value to the nearest tenth.

Try 8.1:

\[ 8.1^2 = 8.1 \cdot 8.1 = \square \rightarrow \text{ close, but } \square \text{ than 67.} \]

Try 8.2:

\[ 8.2^2 = 8.2 \cdot 8.2 = \square \rightarrow \text{ close, and } \square \text{ than 67.} \]

Which is closer to 67: 8.1 or 8.2? \( \square \)

So, \( \sqrt{67} \) is between 8.1 and 8.2, but is closer to \( \square \).

Graph \( \sqrt{67} \) on the number line below.

The exact length of one side of the square is \( \square \) meters.

The number line above shows the approximate decimal value.
Getting the Idea

To compare and order numbers, use the following symbols:

- \( \geq \) (is greater than)
- \( \leq \) (is less than)
- \( = \) (is equal to)

To compare an irrational number to another number, approximate its decimal value. Convert the other number to a decimal also. Then compare the digits to determine which decimal is greater.

Example 1

Which symbol makes this sentence true? Use \( \geq \), \( \leq \), or \( = \).

\[ 7.745966... \; \bigcirc \; \sqrt{59} \]

**Strategy**  
Estimate the values of the irrational numbers.

**Step 1**  
Round \( 7.745966... \) to the nearest hundredth.  
\[ 7.745966... \approx 7.75 \]

**Step 2**  
Approximate \( \sqrt{59} \) to the nearest whole number.  
\[ \sqrt{49} < \sqrt{59} < \sqrt{64}, \text{ so:} \]
\[ 7 < \sqrt{59} < 8. \]

59 is closer to 64 than to 49, so \( \sqrt{59} \) is closer to 8.

**Step 3**  
Continue estimating and compare.  
\[ 7.7^2 = 7.7 \cdot 7.7 = 59.29 \quad \text{close, but more than 59} \]

Since 59 is less than \( 7.7^2 \), \( \sqrt{59} \) is less than 7.7.  
\[ 7.75 > 7.7, \text{ so } 7.745966... > \sqrt{59}. \]

**Solution**  
The symbol \( \geq \) makes the sentence true.  

\[ 7.745966... \geq \sqrt{59} \]
When ordering a set of numbers with different signs, know that a positive number is always greater than a negative number.

**Example 2**

Order the numbers below from least to greatest.

28%, $-2\frac{1}{2}$, $\frac{2}{7}$, $-\sqrt{9}$

**Strategy**

Separate the negative numbers from the positive numbers. Then convert each group of numbers to the same form.

**Step 1**

Write the negative numbers as decimals and compare.

$-2\frac{1}{2} = -2.5$

$-\sqrt{9} = -3$

$-\sqrt{9} < -2\frac{1}{2}$

**Step 2**

Write the positive numbers as decimals and compare.

$28\% = 0.28$

$\frac{2}{7} = 2 \div 7 = 0.285714 \approx 0.29$

$0.28 < 0.29$, so $28\% < \frac{2}{7}$.

**Step 3**

Order all four numbers.

$-\sqrt{9} < -2\frac{1}{2} < 28\% < \frac{2}{7}$

**Solution**

From least to greatest, the numbers are $-\sqrt{9}$, $-2\frac{1}{2}$, $28\%$, $\frac{2}{7}$.

**Example 3**

Order these numbers from greatest to least.

$\pi$, $3\frac{1}{3}$, $\sqrt{14}$

**Strategy**

Approximate the value of each number.

**Step 1**

Approximate the value of $\pi$.

$\pi \approx 3.14$

**Step 2**

Write the value of $3\frac{1}{3}$.

$3\frac{1}{3} = 3.3$
Step 3  Estimate $\sqrt{14}$ to the nearest tenth.
\[
\sqrt{9} < \sqrt{14} < \sqrt{16}, \text{ so:} \\
3 < \sqrt{14} < 4.
\]
14 is closer to 16 than to 9, so $\sqrt{14}$ is closer to 4.
\[
3.7^2 = 3.7 \cdot 3.7 = 13.69 \quad \text{close}
\]
\[
3.8^2 = 3.8 \cdot 3.8 = 14.44 \quad \text{not as close as } 3.7^2
\]
$\sqrt{14} \approx 3.7$

Step 4  Order the decimals and then the numbers.
\[
3.7 > 3.3 > 3.14
\]
\[
\sqrt{14} > 3\frac{1}{3} > \pi
\]

Solution  From greatest to least, the order of the numbers is: $\sqrt{14}$, $3\frac{1}{3}$, $\pi$.

Coached Example

Order the following numbers from least to greatest.

$-\frac{1}{9}$, $\sqrt{5}$, $.8$, $3.5$

Separate the negative numbers from the positive numbers.

The negative numbers are: $-\frac{1}{9}$ and ________.

Write $-\frac{1}{9}$ as a decimal:
\[
-\frac{1}{9} = -1 \div 9 = \text{______________}
\]

The other negative number is a decimal. Compare the decimals.

On a number line, ______ is farther to the left than ________.

So, ______ < ________.

The positive numbers are: $\sqrt{5}$ and ________.

Approximate $\sqrt{5}$ to the nearest tenth.
\[
\sqrt{4} < \sqrt{5} < \sqrt{9}, \text{ so:} \\
2 < \sqrt{5} < _____.
\]

Since the value of $\sqrt{5}$ is less than 3, 3.5 must be ______________ than $\sqrt{5}$.

From least to greatest, the order is: ________, ________, ________, ________.
Domain 1 • Lesson 4

Estimate the Value of Expressions

Getting the Idea

Sometimes, you may need to estimate the value of an expression that includes an irrational number. To do that, estimate the value of the irrational number. Then use that estimate to find the value of the entire expression.

When estimating the value of an expression that includes \( \pi \), it is helpful to remember that \( \pi \) can be approximated as 3.14 or \( \frac{22}{7} \).

Example 1

Estimate the value of \( \pi^2 \).

Strategy

Substitute 3.14 for \( \pi \).

Step 1

Write an expression that could be used.

\[ \pi^2 = \pi \cdot \pi \approx 3.14 \cdot 3.14 \]

Step 2

Multiply.

Since each factor has 2 decimal places, the product will have 2 + 2, or 4, decimal places.

\[ 3.14 \cdot 3.14 = 9.8596 \approx 9.86 \]

Solution

The value of \( \pi^2 \) can be estimated as 9.86.

When a number appears to the left of a radical, it means to multiply the number outside the radical by the square root. For example \( 3\sqrt{5} \) means to multiply 3 \( \times \sqrt{5} \).
Example 2
Estimate the value of $2\sqrt{7}$.

**Strategy**  Estimate the value of $\sqrt{7}$ to the nearest tenth.

**Step 1**  Approximate $\sqrt{7}$ to the nearest whole number.

$\sqrt{4} < \sqrt{7} < \sqrt{9}$, so $2 < \sqrt{7} < 3$.

$\sqrt{7}$ is slightly closer to 9 than to 4, so $\sqrt{7}$ is slightly closer to 3.

**Step 2**  Use guess and check to approximate $\sqrt{7}$ to the nearest tenth.

$2.6^2 = 2.6 \cdot 2.6 = 6.76$ ➞ close

$2.7^2 = 2.7 \cdot 2.7 = 7.29$ ➞ not as close as $2.6^2$

$\sqrt{7} \approx 2.6$

**Step 3**  Use that decimal approximation to estimate the value of the expression.

$2\sqrt{7} = 2 \cdot \sqrt{7} \approx 2 \cdot 2.6 \approx 5.2$

**Solution**  A good estimate of the value of $2\sqrt{7}$ is $5.2$.

Example 3
Estimate the value of this expression.

$\sqrt{43} - \frac{7}{11}$

**Strategy**  Approximate the value of each number. Then subtract.

**Step 1**  Approximate $\sqrt{43}$ to the nearest whole number.

$\sqrt{36} < \sqrt{43} < \sqrt{49}$, so $6 < \sqrt{43} < 7$.

$\sqrt{43}$ is slightly closer to 49 than to 36, so $\sqrt{43}$ is slightly closer to 7.

**Step 2**  Use guess and check to approximate $\sqrt{43}$ to the nearest tenth.

$6.5^2 = 6.5 \cdot 6.5 = 42.25$ ➞ close

$6.6^2 = 6.6 \cdot 6.6 = 43.56$ ➞ closer than $6.5^2$

$\sqrt{43} \approx 6.6$

**Step 3**  Approximate $\frac{7}{11}$ to the nearest tenth.

$\frac{7}{11} = 7 ÷ 11 = 0.63 \approx 0.6$

**Step 4**  Subtract the estimated values of the numbers.

$\sqrt{43} - \frac{7}{11} \approx 6.6 - 0.6 \approx 6$

**Solution**  The value of $\sqrt{43} - \frac{7}{11}$ is approximately 6.
Approximate the value of $\frac{\pi}{2}$. Plot a point to represent that value on a number line.

$\pi \approx \underline{\phantom{000}}$

Use the space below to divide that approximate value for $\pi$ by 2.

Now, plot and label a point representing the value of $\frac{\pi}{2}$ on the number line above.

The value of $\frac{\pi}{2}$ is approximately $\underline{\phantom{000}}$ and is represented on the number line above.
Exponents

Getting the Idea

A number in **exponential form** has a **base** and an **exponent**. The exponent indicates how many times the base is used as a factor. In \(a^s\), the base is \(a\) and the exponent is \(s\).

In \(5^4\), 5 is used as a factor 4 times: \(5^4 = 5 \times 5 \times 5 \times 5 = 625\).

The expression \(5^4\) can also be called a power of 5 and is read as “five to the fourth power.”

The value of a nonzero expression in which the exponent is 0 is 1, so \(5^0 = 1\).

Example 1

What is \(4^3\) written in standard form?

**Strategy**   Multiply the base by itself the number of times shown by the exponent.

\[4 \times 4 \times 4 = 64\]

**Solution**   \(4^3 = 64\) in standard form.

A base raised to a negative exponent is equal to the **reciprocal** of the expression with a positive exponent. Look at the examples below.

\[5^{-3} = \left(\frac{1}{5}\right)^3 = \frac{1}{5^3}\]
\[\frac{1}{6^{-2}} = \left(\frac{6}{1}\right)^2 = 6^2\]

To change the sign of an exponent, move the expression to the denominator of a fraction.

\[a^{-n} = \frac{1}{a^n}, \text{ if } a \neq 0\]

To change the sign of an exponent in a denominator, move the expression to the numerator.

\[\frac{1}{a^{-n}} = a^n = a^n, \text{ if } a \neq 0\]
Example 2
What is $8^{-3}$ written in standard form?

**Strategy** Write the reciprocal of the exponential expression with a positive exponent, then simplify.

**Step 1** Write the reciprocal of the exponential expression to eliminate the negative exponent.

$$8^{-3} = \left(\frac{1}{8}\right)^3 = \frac{1}{8^3}$$

**Step 2** Simplify.

$$\frac{1}{8^3} = \frac{1}{8 \times 8 \times 8} = \frac{1}{512}$$

**Solution** $8^{-3} = \frac{1}{512}$ in standard form.

Numbers in exponential form are sometimes called **powers**. There are some properties you can apply to simplify powers. In the table below, $a$ and $b$ are real numbers, and $m$ and $n$ are integers.

<table>
<thead>
<tr>
<th>Properties of Powers</th>
<th>Examples</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Product of Powers</strong></td>
<td>$a^m \times a^n = a^{m+n}$</td>
</tr>
<tr>
<td>To multiply two numbers with the same base, add the exponents.</td>
<td>$8^2 \times 8^7 = 8^{2+7} = 8^9$</td>
</tr>
<tr>
<td><strong>Quotient of Powers</strong></td>
<td>$a^m \div a^n = a^{m-n}$</td>
</tr>
<tr>
<td>To divide two numbers with the same base, subtract the exponents.</td>
<td>$3^{10} \div 3^2 = 3^{10-2} = 3^8$</td>
</tr>
<tr>
<td><strong>Power of a Power</strong></td>
<td>$(a^m)^n = a^{m \times n}$</td>
</tr>
<tr>
<td>To raise a power to a power, multiply the exponents.</td>
<td>$(6^4)^5 = 6^{4 \times 5} = 6^{20}$</td>
</tr>
<tr>
<td><strong>Power of Zero</strong></td>
<td>$a^0 = 1$, if $a \neq 0$</td>
</tr>
<tr>
<td>Any nonzero number raised to the power of zero is 1.</td>
<td>$4^0 = 1$</td>
</tr>
<tr>
<td><strong>Power of a Product</strong></td>
<td>$(ab)^m = a^m b^m$</td>
</tr>
<tr>
<td>To find a power of a product, find the power of each factor and multiply.</td>
<td>$(3 \times 2)^3 = 3^3 \times 2^3$</td>
</tr>
<tr>
<td><strong>Power of a Quotient</strong></td>
<td>$\left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$, if $b \neq 0$</td>
</tr>
<tr>
<td>To raise a quotient to a power, raise both the numerator and denominator to that power.</td>
<td></td>
</tr>
</tbody>
</table>
Example 3
What is $25^0$ written in standard form?

Strategy  Use the power of zero property.

$25 \neq 0$, so $25^0 = 1$.

Solution  $25^0 = 1$ in standard form.

Example 4
What is $3^2 \times 3^4$? Write the product in standard form.

Strategy  Use the product of powers property.

Step 1  The exponential terms have the same base, 3. Add the exponents.

$3^2 \times 3^4 = 3^{2+4} = 3^6$

Step 2  Evaluate. Write the number in standard form.

$3^6 = 3 \times 3 \times 3 \times 3 \times 3 \times 3 = 729$

Solution  $3^2 \times 3^4 = 3^6 = 729$

You could have solved the problem by evaluating each exponent and then multiplying. Since $3^2 = 9$ and $3^4 = 81$, $9 \times 81 = 729$.

Example 5
What is $5^5 \div 5^3$? Write the quotient in standard form.

Strategy  Use the quotient of powers property.

Step 1  The exponential terms have the same base, 5. Subtract the exponents.

$5^5 \div 5^3 = 5^{5-3} = 5^2$

Step 2  Evaluate. Write the number in standard form.

$5^2 = 5 \times 5 = 25$

Solution  $5^5 \div 5^3 = 5^2 = 25$
Example 6
What is $7^3 \times 7^{-5}$? Write the product in standard form.

Strategy  Use the product of powers property.

Step 1  The exponential terms have the same base. Add the exponents.
$$7^3 \times 7^{-5} = 7^{3+(-5)} = 7^{-2}$$

Step 2  Write the exponential expression without the negative exponent.
$$7^{-2} = \frac{1}{7^2}$$

Step 3  Evaluate. Write the number in standard form.
$$\frac{1}{7^2} = \frac{1}{7 \times 7} = \frac{1}{49}$$

Solution  $7^3 \times 7^{-5} = 7^{-2} = \frac{1}{49}$

Coached Example

What is $(10^2)^3$ in standard form?

To raise a power to a power, you must _______________ the exponents.

$(10^2)^3 = 10^{2 \cdot 3} = 10^6$

Use 10 as a factor ____ times.

Multiply to find the product in standard form.

(10$^6$) = _______________
Getting the Idea

Squaring a number means raising it to the power of 2. For example, $7^2$ is equivalent to $7 \times 7$, or 49. So, 49 is a perfect square.

The opposite, or inverse, of squaring a number is taking its square root. The radical symbol ($\sqrt{\phantom{x}}$) is used to represent square roots. To find the square root of a perfect square, think about what number, when multiplied by itself, will result in that perfect square.

Example 1
Solve for $y$.

$y^2 = 196$

**Strategy**  
Determine what number, multiplied by itself, results in 196.

**Step 1**  
Take the square root of both sides of the equation.

\[
\sqrt{y^2} = \sqrt{196}
\]

\[
y = \sqrt{196}
\]

**Step 2**  
Try squaring numbers until you find one that results in 196.

\[
12^2 = 12 \times 12 = 144 \quad \text{Too low}
\]

\[
13^2 = 13 \times 13 = 169 \quad \text{Too low}
\]

\[
14^2 = 14 \times 14 = 196 \quad \checkmark
\]

**Step 3**  
Solve for $y$.

\[
14^2 = 196, \text{ so } \sqrt{196} = 14.
\]

\[
y = \sqrt{196} = 14
\]

**Solution**  
$y = 14$. 


Cubing a number means raising it to the power of 3. For example, \(2^3\) is equivalent to \(2 \times 2 \times 2\), or 8. So, 8 is a **perfect cube**.

The opposite, or inverse, of cubing a number is taking its **cube root**. The symbol \(\sqrt[3]{\phantom{0}}\) is used to represent cube roots. To find the cube root of a perfect cube, think about what number, when multiplied by itself twice, will result in that perfect cube.

### Example 2

Solve for \(r\).

\[ r^3 = 125 \]

**Strategy**  
Determine what number, when cubed, results in 125.

**Step 1**  
Take the cube root of both sides of the equation.  
\[
\sqrt[3]{r^3} = \sqrt[3]{125} \\
\implies r = \sqrt[3]{125}
\]

**Step 2**  
Try cubing numbers until you find one that gives a result of 125.  
\[ 5^3 = 5 \times 5 \times 5 = 125 \quad \checkmark \]

**Step 3**  
Solve for \(r\).  
\[ 5^3 = 125, \text{ so } \sqrt[3]{125} = 5. \]

\[ r = \sqrt[3]{125} \]

\[ r = 5 \]

**Solution**  
\(r = 5\).

The number under a radical sign is called the **radicand**. If you do not have a calculator handy, you may need to estimate the value of a square root or a cube root.

To estimate a square root, find the two perfect squares between which the radicand lies. Take the square root of each to find the range of your estimate.

To estimate a cube root, find the two perfect cubes between which the radicand lies. Then take the cube root of each to find the range of your estimate.
Example 3

Between which two consecutive integers is \(3\sqrt[3]{500}\)?

**Strategy**  
Find the two perfect cubes between which 500 lies. Then take the cube root of each to make your estimate.

Try cubing consecutive positive integers.

\[
\begin{align*}
6^3 &= 6 \times 6 \times 6 = 216 \\
7^3 &= 7 \times 7 \times 7 = 343 \\
8^3 &= 8 \times 8 \times 8 = 512
\end{align*}
\]

The radicand, 500, is between the perfect cubes 343 and 512.

\[
7 < 3\sqrt[3]{500} < 8
\]

**Solution**  
\(3\sqrt[3]{500}\) has a value between 7 and 8.

---

Coached Example

The area of the square garden on the right is 121 square yards.  
What is the length, \(s\), of each side of the garden?

The formula for finding the area, \(A\), of a square is \(A = s^2\), where \(s\) is the length of a side.

The area of the garden above is ________ square yards.

To find the length of one side, take the ________ root of that area.

On the lines below, try squaring numbers until you find one that results in _________. That is the value of \(s\).

___________________________________________________________________________

___________________________________________________________________________

The length of each side, \(s\), of the garden is _________ yards.
Scientific Notation

Getting the Idea

Scientific notation is a way to abbreviate very large or very small numbers using powers of 10. A number written in scientific notation consists of two factors. The first factor is a number greater than or equal to 1, but less than 10. The second factor is a power of 10.

Here are some guidelines and examples of numbers written in scientific notation.

<table>
<thead>
<tr>
<th>Standard Form</th>
<th>Scientific Notation</th>
</tr>
</thead>
<tbody>
<tr>
<td>8,000,000</td>
<td>$8 \times 10^6$</td>
</tr>
<tr>
<td>3</td>
<td>$3 \times 10^0$</td>
</tr>
<tr>
<td>0.0007</td>
<td>$7 \times 10^{-4}$</td>
</tr>
</tbody>
</table>

A number raised to the power of 0 is equal to 1, so multiplying by $10^0$ is the same as multiplying by 1.

Example 1

Jupiter’s minimum distance from the Sun is about 460,100,000 miles. What is that number written in scientific notation?

Strategy  
Use the definition of scientific notation to find the two factors.

Step 1
Write the first factor, which must be greater than or equal to 1 and less than 10.

- Put the decimal point after the first nonzero digit, starting at the left.
- Drop all zeros after the last nonzero digit.

4.60100000

The first factor is 4.601.

Step 2
Find the exponent for the power of 10.

- Count the number of places that the decimal point was moved.

4.60100000

The decimal point was moved 8 places to the left.

Since the original number is greater than 10, the exponent will be positive.

8 is the exponent for the power of 10.
Step 3  Write the second factor.
   The exponent is positive 8. The second factor is $10^8$.

Step 4  Write the number in scientific notation.
   $4.601 \times 10^8$

Solution  
   Jupiter’s minimum distance from the Sun is about $4.601 \times 10^8$ miles.

Example 2
Mr. Kendall measured a specimen that was 0.00000045 millimeter long. What is the specimen’s length, in millimeters, written in scientific notation?

Strategy  
   Use the definition of scientific notation to find the two factors.

Step 1  Write the first factor, which must be greater than or equal to 1 and less than 10.
   Put the decimal point after the first nonzero digit, starting from the left.
   Drop the zeros that precede that digit.
   
   $0.00000045$
   The first factor is 4.5.

Step 2  Find the exponent for the power of 10.
   Count the number of places that the decimal point was moved.
   
   $0.00000045$
   The decimal point was moved 7 places to the right.
   Since the original number is less than 1, the exponent will be negative.
   $-7$ is the exponent for the power of 10.

Step 3  Write the second factor.
   The exponent is $-7$. The second factor is $10^{-7}$.

Step 4  Write the number in scientific notation.
   $4.5 \times 10^{-7}$

Solution  
   The specimen was $4.5 \times 10^{-7}$ millimeter long.
When converting from scientific notation to standard form, move the decimal point to the right for a positive power of 10 and to the left for a negative power of 10.

Example 3
What is \(3.5 \times 10^{-6}\) written in standard form?

**Strategy**

**Look at the exponent of the second factor to move the decimal point.**

**Step 1**
Look at the exponent of the second factor.
- The exponent is negative, so the decimal point will move to the left.
- The exponent is \(-6\), so move the decimal point 6 places to the left.

**Step 2**
Move the decimal point in 3.5 six places to the left.
- Add zeros as needed.
  \[0.0000035\]

**Step 3**
Use a scientific calculator to check your solution.
- To find \(3.5 \times 10^{-6}\):
  - Type 3.5.
  - Press the multiplication sign key.
  - Type 10.
  - Press the exponent key.
  - Type 6.
  - Press the positive/negative key to change the sign on the 6.
  - Press the equal sign key.
- The screen should show 0.0000035.
- The solution is correct.

**Solution**

\[3.5 \times 10^{-6} = 0.0000035\]

To multiply numbers in scientific notation, first multiply the decimal factors and then multiply the power-of-10 factors. Use the properties of powers when you multiply the power-of-10 factors. In the example below, \(a\) and \(b\) are the decimal factors.

\[(a \times 10^m)(b \times 10^n) = ab \times 10^{m+n}\]
To divide numbers in scientific notation, first divide the decimal factors. Then divide the power-of-10 factors, using the properties of powers. In the example below, \(a\) and \(b\) are the decimal factors and \(b \neq 0\).

\[
\frac{(a \times 10^m)}{(b \times 10^n)} = \frac{a}{b} \times 10^{m-n}
\]

When you multiply or divide numbers in scientific notation, your product or quotient may not be in scientific notation because the decimal factor is not greater than or equal to 1 and less than 10. To fix this, write the decimal factor in scientific notation and use the properties of powers to simplify the expression.

**Example 4**

Find the product in scientific notation.

\[(1.5 \times 10^3)(7.8 \times 10^{-7})\]

**Strategy**

Multiply the decimal-number factors. Then multiply the power-of-10 factors.

**Step 1**

Use the commutative and associative properties to regroup the factors.

\[(1.5 \times 10^3)(7.8 \times 10^{-7}) = (1.5 \times 7.8)(10^3 \times 10^{-7})\]

**Step 2**

Multiply the decimal factors.

\[1.5 \times 7.8 = 11.7\]

**Step 3**

Multiply the power-of-10 factors.

\[10^3 \times 10^{-7} = 10^{3+(-7)} = 10^{-4}\]

**Step 4**

Write the product using the products from Steps 2 and 3.

\[(1.5 \times 10^3)(7.8 \times 10^{-7}) = 11.7 \times 10^{-4}\]

**Step 5**

Write \(11.7 \times 10^{-4}\) in scientific notation.

Move the decimal point in 11.7 one place to the left.

Since you moved the decimal point one place to the left, the exponent increases by 1.

\[11.7 \times 10^{-4} = 1.17 \times 10^{-3}\]

**Solution**

\[(1.5 \times 10^3)(7.8 \times 10^{-7}) = 1.17 \times 10^{-3}\]
Example 5
What is \(4.2 \times (2.5 \times 10^{-6})\) written in standard form?

Strategy Use the associative property to regroup the factors. Then write the product in standard form.

Step 1 Use the associative property to regroup the factors.
\[
4.2 \times (2.5 \times 10^{-6}) = (4.2 \times 2.5) \times 10^{-6}
\]

Step 2 Multiply the decimal factors.
\[
4.2 \times 2.5 = 10.5
\]

Step 3 Rewrite the expression using the result from Step 2 and the power-of-10 factor.
\[
(4.2 \times 2.5) \times 10^{-6} = 10.5 \times 10^{-6}
\]

Step 4 Write the product in standard form.
Look at the power-of-10 factor.
The negative exponent means you move the decimal point to the left.
So, \(-6\) means you move the decimal point in 10.5 six places to the left.
\[
0.0000105
\]

Solution \(4.2 \times (2.5 \times 10^{-6}) = 0.0000105\)

Example 6
Find the quotient in scientific notation.
\[
\frac{8.82 \times 10^5}{3.6 \times 10^3}
\]

Strategy Divide the decimal-number factors and divide the power-of-10 factors.

Step 1 Rewrite the expression.
\[
\frac{8.82 \times 10^5}{3.6 \times 10^3} = \frac{8.82}{3.6} \times \frac{10^5}{10^3} = 2.45 \times 10^2
\]

Step 2 Divide the decimal-number factors.
\[
\frac{8.82}{3.6} = 2.45
\]

Step 3 Divide the power-of-10 factors.
\[
\frac{10^5}{10^3} = 10^{5-3} = 10^2
\]

Step 4 Write the result using the quotients from Steps 2 and 3.
\[
2.45 \times 10^2
\]

Solution \(\frac{8.82 \times 10^5}{3.6 \times 10^3} = 2.45 \times 10^2\)
Coached Example

In 2013, the Hartsfield-Jackson Atlanta International Airport ranked as the world’s busiest airport. In that year, approximately $9.4 \times 10^7$ passengers passed through this airport. What is that number written in standard form?

Since the exponent is positive, this is a number greater than _______.

The exponent of the second factor is _______.

The exponent tells you to move the decimal point in 9.4 _______ places to the ________.

The number $9.4 \times 10^7$ in standard form is ____________________.

About ___________________ passengers passed through the Hartsfield-Jackson Atlanta International Airport in 2013.
### Getting the Idea

Sometimes, you may need to multiply or divide numbers written in scientific notation in order to solve real-world problems.

### Example 1

A rectangular section of wilderness will be set aside as a new wildlife refuge. Its dimensions are $5 \times 10^5$ meters by $4 \times 10^4$ meters. Find the area of the land in square meters. Then convert the area into square kilometers using the conversion below.

$$1 \text{ square kilometer (km}^2\text{)} = 1 \times 10^6 \text{ square meters (m}^2\text{)}$$

Which unit is a better choice for measuring the area of the wildlife refuge, and why?

**Strategy** Multiply the dimensions. Convert the area into square kilometers. Compare the two units.

**Step 1** Multiply the dimensions to find the area, in square meters $A$.

$$A = (5 \times 10^5)(4 \times 10^4)$$

**Step 2** Multiply the first factors and then multiply the power-of-10 factors.

$$5 \times 4 = 20$$

$$10^5 \times 10^4 = 10^{5+4} = 10^9$$

So, $A = (5 \times 10^5)(4 \times 10^4) = 20 \times 10^9$.

**Step 3** Rewrite the number in scientific notation.

$$20 \times 10^9 = 2 \times 10^{10}$$

**Step 4** Convert the area into square kilometers.

To convert a smaller unit (square meters) to a larger unit (square kilometers), divide:

$$A \text{ (in km}^2\text{)} = \frac{2 \times 10^{10}}{1 \times 10^6}$$

Divide the first factors and then divide the power-of-10 factors.

$$\frac{2}{1} = 2$$

$$\frac{10^{10}}{10^6} = 10^{10-6} = 10^4$$

So, $A \text{ (in km}^2\text{)} = 2 \times 10^4$. 


Step 5
Which is the better unit to use?

\[ 2 \times 10^{10} \text{ square meters} = 20,000,000,000 \text{ m}^2 \]
\[ 2 \times 10^4 \text{ square kilometers} = 20,000 \text{ km}^2 \]

20,000 is a more reasonable number to work with in standard form. Also, square kilometers are larger units than square meters. Since the area is large, it is better to use the larger unit.

Solution
The area of the refuge is \( 2 \times 10^{10} \) square meters or \( 2 \times 10^4 \) square kilometers. Square kilometers is a better unit to use because the area is large.

Sometimes, you may use technology, such as a calculator, to generate a number. If the result is a number that is very large or very small, many calculators will automatically give the number in scientific notation.

Example 2
One cubic millimeter of Ms. Murphy’s blood contains about 5,000,000 red blood cells. There are about 4,900,000 cubic millimeters of blood in her entire body. Use a calculator to determine approximately how many red blood cells Ms. Murphy has in total. Interpret the number your calculator gives as the final answer.

Strategy
Use a calculator to determine the answer. Interpret the result.

Step 1
How can you find the total number of red blood cells?

Multiply the number of red blood cells in one cubic millimeter of blood (5,000,000) by the total number of cubic millimeters of blood in the body (4,900,000).

Step 2
Use a calculator to determine the answer.

Type 5000000.
Press \( \times \)

Type 4900000.
Press \( = \)

Step 3
Interpret the answer shown on the calculator display.

The screen shows this:

\[ 2.45 \times 10^{13} \]

Solution
Ms. Murphy has a total of about \( 2.45 \times 10^{13} \) red blood cells in her body.
Lesson 8: Solve Problems Using Scientific Notation

Example 3
California, the most populous state, has approximately $4 \times 10^7$ people living in it.
The population of the entire United States is approximately $3 \times 10^8$ people. About how many times greater is the population of the United States than the population of California?

Strategy Decide if you should multiply or divide. Then solve the problem.

Step 1 Decide on which operation to use.
To find how many times greater, divide $3 \times 10^8$ by $4 \times 10^7$.

Step 2 Divide the first factors and then divide the power-of-10 factors.
\[
\frac{3}{4} = 0.75 \quad \quad \frac{10^8}{10^7} = 10^1 = 10
\]
So, \[
\frac{3 \times 10^8}{4 \times 10^7} = 0.75 \times 10 = 7.5
\]

Solution The population of the United States is 7.5 times the population of California.

Coached Example
A computer was used to draw a rectangle with an area of 0.000007 square meter. Would it be better to measure the area in square meters or square millimeters? Use the conversion below to help determine your answer.

1 square meter (m$^2$) = $1 \times 10^6$ square millimeters (mm$^2$)

Rewrite 0.000007 in scientific notation. 0.000007
The decimal point was moved ________ places to the right.
The original number is less than _______, so the exponent will be negative.

\[
0.000007 = 7 \times 10^{-7}
\]
Multiply to convert that number of square meters to square millimeters:
\[
(7 \times 10^{-6})(1 \times 10^6)
\]
Multiply the first factors: $7 \times 1 = \underline{7}
Multiply the power-of-10 factors: \underline{10}
The area is ______ square millimeters.
It is better to measure the area in square ________________ because it is better to measure a small area using a ________________ unit.
Domain 2 • Lesson 9

Linear Equations in One Variable

Getting the Idea

An *equation* is a mathematical sentence that uses an equal (=) sign to show that two quantities are equal in value. A *variable* is a symbol or letter that is used to represent one or more numbers. A *constant* is a value that does not change.

A linear equation has one or more variables raised to the first power. You can use *inverse operations* and the properties of equality to solve a linear equation that has one variable.

<table>
<thead>
<tr>
<th>Properties of Equality</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Addition Property of Equality</strong></td>
</tr>
<tr>
<td>If you add the same number to both sides of an equation, the equation continues to be true.</td>
</tr>
<tr>
<td>If $a = c$, then $a + b = c + b$.</td>
</tr>
<tr>
<td><strong>Multiplication Property of Equality</strong></td>
</tr>
<tr>
<td>If you multiply both sides of an equation by the same number, the equation continues to be true.</td>
</tr>
<tr>
<td>If $a = c$, then $ab = cb$.</td>
</tr>
<tr>
<td><strong>Subtraction Property of Equality</strong></td>
</tr>
<tr>
<td>If you subtract the same number from both sides of an equation, the equation continues to be true.</td>
</tr>
<tr>
<td>If $a = c$, then $a - b = c - b$.</td>
</tr>
<tr>
<td><strong>Division Property of Equality</strong></td>
</tr>
<tr>
<td>If you divide both sides of an equation by the same nonzero number, the equation continues to be true.</td>
</tr>
<tr>
<td>If $a = c$ and $b \neq 0$, then $\frac{a}{b} = \frac{c}{b}$.</td>
</tr>
</tbody>
</table>

Whatever you do to one side of the equation, you must also do to the other side. That way, you can isolate the variable while still keeping the equation true.

**Example 1**

Find the value of $x$ in this equation.

\[-\frac{x}{6} + 9 = -1\]

**Strategy** Use inverse operations to isolate the variable.

**Step 1** Remove the constant.

Subtract 9 from both sides.

\[-\frac{x}{6} + 9 = -1\]

\[-\frac{x}{6} + 9 - 9 = -1 - 9\]

\[-\frac{x}{6} = -10\]
Step 2  
Isolate the variable.

Multiply both sides by $-6$.

$$\frac{-x}{6} \cdot -6 = -10 \cdot -6$$

$$x = 60$$

Solution  
The value of $x$ is 60.

To undo multiplication by a fraction, multiply by the reciprocal of that fraction.
Flip the fraction to find its reciprocal. For example, the reciprocal of $\frac{4}{5}$ is $\frac{5}{4}$.

Example 2
What is the value of $c$ in this equation?

$$\frac{2}{3}c - \frac{3}{5} = \frac{7}{10}$$

Strategy  
Use inverse operations to isolate the variable.

Step 1  
Remove the constant.

Add $\frac{3}{5}$ to both sides.

$$\frac{2}{3}c - \frac{3}{5} + \frac{3}{5} = \frac{7}{10} + \frac{3}{5}$$

$$\frac{2}{3}c = \frac{7}{10} + \frac{3}{5}$$

Give $\frac{7}{10}$ and $\frac{3}{5}$ the same denominator.

$$\frac{2}{3}c = \frac{7}{10} + \frac{6}{10}$$

$$\frac{2}{3}c = \frac{13}{10}$$

Step 2  
Isolate the variable.

Divide both sides by $\frac{2}{3}$ or multiply by the reciprocal, $\frac{3}{2}$.

$$\frac{2}{3}c \cdot \frac{3}{2} = \frac{13}{10} \cdot \frac{3}{2}$$

$$1c = \frac{39}{20}$$

$$c = \frac{19}{20}$$

Solution  
The value of $c$ is $1\frac{19}{20}$. 
You may also need to combine like terms if the same variable is on both sides of the equation. Like terms are terms that contain the same variables raised to the same power.

Example 3
What is the value of \( z \) in this equation?

\[
0.8z + 3.74 = z + 1.5
\]

**Strategy** Combine like terms. Then solve.

**Step 1** Combine like terms so there is only one variable term.

Subtract \( 0.8z \) from both sides.

\[
\begin{align*}
0.8z + 3.74 &= z + 1.5 \\
0.8z - 0.8z + 3.74 &= z - 0.8z + 1.5 \\
3.74 &= 0.2z + 1.5
\end{align*}
\]

**Step 2** Remove the constant.

Subtract 1.5 from both sides.

\[
\begin{align*}
3.74 - 1.5 &= 0.2z + 1.5 - 1.5 \\
3.74 - 1.50 &= 0.2z \\
2.24 &= 0.2z
\end{align*}
\]

**Step 3** Isolate the variable.

Divide both sides by 0.2.

\[
\begin{align*}
2.24 &= 0.2z \\
\frac{2.24}{0.2} &= \frac{0.2z}{0.2} \\
11.2 &= z
\end{align*}
\]

**Solution** The value of \( z \) is 11.2.
To solve some equations, you may need to use the distributive property.

**Distributive Property of Multiplication over Addition**
When a factor is multiplied by the sum of two numbers, multiply each of the two numbers by the factor and then add the products.

\[a(b + c) = ab + ac\]

**Distributive Property of Multiplication over Subtraction**
When a factor is multiplied by the difference of two numbers, multiply each of the two numbers by the factor and then subtract the products.

\[a(b - c) = ab - ac\]

**Example 4**
What is the value of \(y\) in this equation?

\[4y - 1 = 2(y - 2)\]

**Strategy**  
Use the distributive property. Then combine like terms.

**Step 1**  
Apply the distributive property to evaluate the right side of the equation.  
Distribute the 2 over \((y - 2)\).

\[
4y - 1 = 2(y - 2) \\
4y - 1 = (2 \cdot y) - (2 \cdot 2) \\
4y - 1 = 2y - 4
\]

**Step 2**  
Combine like terms so there is only one variable term in the equation.  
Subtract \(2y\) from both sides.

\[
4y - 1 - 2y = 2y - 4 - 2y \\
2y - 1 = -4
\]

**Step 3**  
Remove the constant.  
Add 1 to both sides.

\[
2y - 1 + 1 = -4 + 1 \\
2y = -3
\]
Step 4  Isolate the variable.

Divide both sides by 2.

\[
\frac{2y}{2} = \frac{\frac{3}{2}}{2}
\]

\[
y = \frac{3}{2}
\]

\[
y = -1\frac{1}{2}
\]

Solution  The value of \(y\) is \(-1\frac{1}{2}\).

In Examples 1–4, all the equations had one solution. However, linear equations may also have no solution or infinitely many solutions.

<table>
<thead>
<tr>
<th>One Solution</th>
<th>No Solution</th>
<th>Infinitely Many Solutions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Only one number, 1, makes the equation below true.</td>
<td>No number makes the equation below true.</td>
<td>Any number makes the equation below true.</td>
</tr>
<tr>
<td>Example: (2x = x + 1)</td>
<td>Example: (x + 1 = x + 2) (1 \neq 2 \Rightarrow) never true</td>
<td>Example: (x + 0 = x) (x = x \Rightarrow) always true</td>
</tr>
</tbody>
</table>

Example 5

Does this equation have one solution, no solutions, or infinitely many solutions?

\[10q - 15 = 5(2q + 4)\]

Strategy  Use the distributive property. Then combine like terms.

Decide how many solutions the equation has.

Step 1  Apply the distributive property to evaluate the right side of the equation.

Distribute 5 over \((2q + 4)\).

\[10q - 15 = 5(2q + 4)\]

\[10q - 15 = (5 \cdot 2q) + (5 \cdot 4)\]

\[10q - 15 = 10q + 20\]

Step 2  Combine like terms.

Subtract 10q from both sides.

\[10q - 15 = 10q + 20\]

\[10q - 10q - 15 = 10q - 10q + 20\]

\[-15 = 20 \Rightarrow\) never true\]
Step 3  Determine the solution of the equation.

Since $-15 \neq 20$, no value of $q$ makes the equation true.
The equation has no solutions.

Solution  The equation has no solutions.

Coached Example

Does the equation below have one solution, no solutions, or infinitely many solutions?

\[ n + 2 = \frac{1}{3}(3n + 6) \]

Apply the distributive property.

Distribute $\frac{1}{3}$ over $(3n + 6)$.

\[ n + 2 = \frac{1}{3}(3n + 6) \]
\[ n + 2 = \left( \frac{1}{3} \cdot \frac{3n}{1} \right) + \left( \frac{1}{3} \cdot \_ \right) \]
\[ n + 2 = \_ + \_ \]

Subtract 2 from both sides.

\[ n + 2 - 2 = \_ - 2 \]
\[ n = \_ \]

Is the equation above always true, never true, or sometimes true?

The equation is ______________ true, so any value of $n$ makes the equation true.

Does the equation have one solution, no solutions, or infinitely many solutions?

Since any value of $n$ makes the equation true, the equation has ______________ solution(s).
## Use One-Variable Linear Equations to Solve Problems

### Getting the Idea

Sometimes, you can solve a real-world problem by writing a linear equation to represent it and then solving the equation. Identifying which quantities are equal can help you as a first step. Writing an expression for each of the equal quantities can also help.

<table>
<thead>
<tr>
<th>Operation</th>
<th>Problem</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition</td>
<td>4 more than $x$</td>
<td>$4 + x$</td>
</tr>
<tr>
<td></td>
<td>the sum of $x$ and 4</td>
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<td>Subtraction</td>
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<td>Multiplication</td>
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<td>the product of 4 and $x$</td>
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<tr>
<td></td>
<td>$x$ groups of 4</td>
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<tr>
<td>Division</td>
<td>$x$ partitioned into 4 equal groups</td>
<td>$\frac{x}{4}$</td>
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<td>$x$ shared by 4 equally</td>
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Example 1
Diego has 60 CDs. This is 12 more than Heidi has. How many CDs does Heidi have?

Strategy  
Write and solve an equation.

Step 1  
Write an equation.
Let $h$ represent the number of CDs Heidi has.

$$60 = h + 12$$

Step 2  
Solve the equation.

$$60 = h + 12$$
$$60 - 12 = h + 12 - 12$$  Subtract 12 from both sides.
$$48 = h$$

Solution  
Heidi has 48 CDs.

Example 2
Simone has six less than $\frac{2}{3}$ of the number of baseball cards that Manuel has. Simone has 14 baseball cards. How many cards does Manuel have?

Strategy  
Write and solve an equation.

Step 1  
Identify the quantities that are equal.

$$14 = 6 \text{ less than } \frac{2}{3} \text{ the number of baseball cards Manuel has}.$$ 

Step 2  
Write an equation.

Let $m$ represent the number of cards Manuel has.

$$\frac{2}{3}m - 6 = 14$$

Step 3  
Solve the equation.

$$\frac{2}{3}m - 6 = 14$$
$$\frac{2}{3}m - 6 + 6 = 14 + 6$$  Add 6 to both sides.
$$\frac{2}{3}m = 20$$
$$\frac{2}{3}m \cdot \frac{3}{2} = \frac{20}{1} \cdot \frac{3}{2} = \frac{30}{1}$$  Multiply both sides by $\frac{3}{2}$, the reciprocal of $\frac{2}{3}$.

Solution  
Manuel has 30 baseball cards.
Example 3
A taxi charges $2.50 for each ride plus $1.25 per mile traveled. If the total charge for one ride was $8.75, how many miles were traveled?

Strategy  Write and solve an equation.

Step 1  Write an equation.
Let $m$ represent the number of miles traveled.

\[ 2.5 + 1.25m = 8.75 \]

Step 2  Solve the equation.

\[ 2.5 + 1.25m = 8.75 \] Subtract 2.5 from each side.
\[ 1.25m = 6.25 \] Divide both sides by 1.25.
\[ m = 5 \]

Solution  The taxi traveled 5 miles.

Coached Example
At a video store, for every DVD bought at the regular price, a customer can buy a second DVD for half the regular price. Nadia buys two DVDs, each of which regularly costs $d$ dollars, and pays $33 in all. What is the regular price of each DVD?

Translate the problem into an equation.

\[ \text{One DVD at the regular price of } d \text{ dollars} \quad + \quad \text{Second DVD at half the regular price of } d \text{ dollars} = 33 \text{ in all} \]

Rewrite the equation and solve for $d$.

The regular price of a DVD at the store is $\underline{_______}$.
The graph of a linear equation is a straight line. The steepness of the line is called its **slope**. The slope shows the rate at which two quantities are changing. Specifically, it is the **ratio** of the vertical change to the horizontal change, or \( \frac{\text{rise}}{\text{run}} \).

Look at the lines graphed above.

- A line that slants up from left to right has a positive slope.
  For this graph, as the \( x \)-values increase, the \( y \)-values also increase.
- A line that slants down from left to right has a negative slope.
  For this graph, as the \( x \)-values increase, the \( y \)-values decrease.
- A horizontal line has a slope of 0 because there is no vertical change.
- A vertical line has an undefined slope, because there is no horizontal change.
  A fraction with a denominator of 0 is undefined, and \( \frac{\text{rise}}{\text{run}} \) would have a denominator of 0.
Example 1
What is the slope of this line?

**Strategy** Find the vertical change and the horizontal change.

**Step 1** Choose two points on the line.
(1, 1) and (6, –3)

**Step 2** Identify the vertical change and horizontal change.
From (1, 1) to (6, –3), the line moves 4 units down and 5 units to the right.

**Step 3** Write the slope.
The line slants down from left to right, so the slope is negative.

\[
slope = \frac{\text{rise}}{\text{run}} = -\frac{4}{5}
\]

**Solution** The slope is \(-\frac{4}{5}\).
**Similar** figures have the same shape, but not necessarily the same size. The ratios of the lengths of their corresponding sides are equal.

**Example 2**

Does the slope of a non-vertical line change depending on which two points you use to determine it? Use the two similar triangles and the line graphed below to help you answer the question.

![Graph with points (2,2), (4,5), (8,11) and line graphed through them.](image)

**Strategy**

For each triangle, write the ratio of the vertical side length to the horizontal side length. Compare those ratios, and compare them to the slope of the line.

**Step 1**

Find the ratio of the vertical and horizontal side lengths for each triangle.

- Smaller triangle: \( \frac{\text{vertical side length}}{\text{horizontal side length}} = \frac{3}{2} \)
- Larger triangle: \( \frac{\text{vertical side length}}{\text{horizontal side length}} = \frac{6}{4} = \frac{3}{2} \)

**Step 2**

Compare the ratios.

The ratio of the lengths of the vertical and horizontal sides for both triangles is \( \frac{3}{2} \).
Step 3  Compare the ratios to the slope.

The slope is the ratio \( \frac{\text{rise}}{\text{run}} \) or \( \frac{\text{vertical change}}{\text{horizontal change}} \).

If you use the points (2, 2) and (4, 5), slope = \( \frac{\text{rise}}{\text{run}} = \frac{3}{2} \).

If you use the points (4, 5) and (8, 11), slope = \( \frac{\text{rise}}{\text{run}} = \frac{6}{4} = \frac{3}{2} \).

Step 4  Analyze the slope.

No matter which two points on the line you use, the slope is the same.
This is because the graph of a line changes at a constant rate.

Solution  The slope of a non-vertical line stays the same no matter which two points you use to determine it.

Example 2 demonstrates that the slope of a line represents a constant rate of change.
If you know any two points on a line, you can determine its slope using the formula below.

\[
\text{Slope} = \frac{\text{change in } y}{\text{change in } x} = \frac{y_2 - y_1}{x_2 - x_1}
\]

Example 3
What is the slope of a line that passes through \((-3, 6)\) and \((5, 1)\)?

Strategy  Use the slope formula.

Step 1  Identify the points.
Let \((x_1, y_1) = (-3, 6)\).
Let \((x_2, y_2) = (5, 1)\).

Step 2  Substitute the numbers into the slope formula.
\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{1 - 6}{5 - (-3)} = \frac{-5}{8}
\]

Solution  The slope is \(-\frac{5}{8}\).
The slope of a line represents a constant rate of change. For example, a slope of \( \frac{50}{3} \) could represent these rates:

- \( \frac{50}{3} \) miles per gallon
- \( \frac{50}{3} \) pages per minute

Example 4

A locksmith charges a flat fee for each house call plus an hourly rate, as shown by the graph below.

Find the slope of the graph. What does the slope represent in this problem?

**Strategy**
Find and interpret the slope of the line.

**Step 1**
Find the slope.
- Let \((x_1, y_1) = (1, 50)\).
- Let \((x_2, y_2) = (2, 70)\).
- \[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{70 - 50}{2 - 1} = \frac{20}{1} = 20
\]

**Step 2**
Interpret the slope.

The slope compares the total charge, in dollars, to the number of hours worked.

The slope shows a rate of change of \( \frac{20 \text{ dollars}}{1 \text{ hour}} \), or \$20 per hour.

So, the slope represents the hourly rate charged by the locksmith.

**Solution**

The slope is 20. It shows that the locksmith charges \$20 per hour for each job.
Coached Example

Joanie bought an airplane phone card that charges her a connection fee plus an additional rate for each minute a call lasts. The graph below represents this situation.

What is the slope of the graph, and what does it represent?

Choose any two points on the graph.

Let \((x_1, y_1) = (2, \_\_\_\_)\).

Let \((x_2, y_2) = (6, \_\_\_\_)\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\_\_\_\_\_ - \_\_\_\_\_}{6 - 2} = \_\_\_\_\_ = \_\_\_\_\_
\]

The y-axis shows the total cost in ______________.

The x-axis shows the time in ______________.

So, the slope shows a rate of change of ________ dollars to ______________, or ______ per minute.

Does the slope represent the connection fee or the rate per minute for the call? ______________

The slope is ________. It shows that Joanie must pay $________ per minute for the calls she makes.
Getting the Idea

Some linear equations have two variables. For example, the linear equation $y = 2x + 5$ includes the variables $x$ and $y$. All of the ordered pairs, in the form $(x, y)$, that make that equation true are solutions of the equation.

The graph of a linear equation is a straight line. The point at which the graph crosses the $y$-axis is called its $y$-intercept. Any point $(0, b)$ that is a solution of the equation is the $y$-intercept.

Example 1

Below is a graph of the linear equation $y = 2x + 5$. Identify its $y$-intercept. Then show that the $y$-intercept is a solution for the equation.

**Strategy** Identify the coordinates of the $y$-intercept. Then show that those $x$- and $y$-values make the equation true.

**Step 1** Identify the $y$-intercept.

The graph crosses the $y$-axis at $(0, 5)$. That is the $y$-intercept.

**Step 2** Show that $(0, 5)$ is a solution for the equation.

Substitute 0 for $x$ and 5 for $y$ into the equation.

$y = 2x + 5$

$5 = 2(0) + 5$

$5 = 0 + 5$

$5 = 5$ ✓

**Solution** The $y$-intercept is $(0, 5)$. Those coordinates are a solution for the equation, as shown in Step 2.
The equation \( y = 2x + 5 \) is written in **slope-intercept form**.

If a linear equation is in slope-intercept form, you can use the \( y \)-intercept and the slope to graph it.

The slope-intercept form of an equation is:

\[
y = mx + b,
\]

where \( m \) represents the slope and \( b \) represents the \( y \)-intercept.

**Example 2**  
Graph the equation \( y = \frac{2}{3}x + 1 \).

**Strategy** Identify the \( y \)-intercept and slope. Use the slope to find a second point on the line.

**Step 1** Identify the slope and the \( y \)-intercept.

The equation \( y = \frac{2}{3}x + 1 \) is in slope-intercept form, \( y = mx + b \).

\[
m = \frac{2}{3},\quad \text{so the slope is } \frac{2}{3}.
\]

\[
b = 1,\quad \text{so the } y\text{-intercept is } (0, 1).
\]

**Step 2** Use the slope to find a second point.

Plot a point at the \( y \)-intercept, \((0, 1)\).

Use the slope to find a second point.

\[
slope = \frac{\text{rise}}{\text{run}} = \frac{2}{3}
\]

Start at \((0, 1)\). Since the slope is positive, rise up 2 units and run 3 units to the right.

Plot a point at \((3, 3)\).

**Step 3** Draw a straight line through the points \((0, 1)\) and \((3, 3)\).

**Solution** The graph of \( y = \frac{2}{3}x + 1 \) is shown above.
A linear equation in the form $y = mx$ has $b = 0$. This means that its $y$-intercept is at $(0, 0)$, the origin.

To graph $y = mx + b$:
- First, graph $y = mx$.
- Shift each point on the graph up or down $b$ units.
  - If $b > 0$, shift the graph $b$ units up.
  - If $b < 0$, shift the graph $b$ units down.

**Example 3**
Graph $y = -\frac{1}{2}x$. On the same grid, graph $y = -\frac{1}{2}x - 3$. Compare the graphs.

**Strategy**
Graph $y = -\frac{1}{2}x$. Then shift the graph down or up to graph $y = -\frac{1}{2}x - 3$.

**Step 1**
Graph $y = -\frac{1}{2}x$.
Start at the $y$-intercept, $(0, 0)$.
The slope is negative, so count 1 unit down and 2 units to the right. Plot a point there at $(2, -1)$.
Draw a line through the points $(0, 0)$ and $(2, -1)$.

**Step 2**
Graph $y = -\frac{1}{2}x - 3$.
The slope is the same as for $y = -\frac{1}{2}x$.
Since $b < 0$, each point on the graph of $y = -\frac{1}{2}x$ is shifted 3 units down in the graph of $y = -\frac{1}{2}x - 3$.
So, $(0, 0)$ moves 3 units down to $(0, -3)$.
$(2, -1)$ moves 3 units down to $(2, -4)$, and so on.
Solution The graphs of \( y = -\frac{1}{2}x \) and \( y = -\frac{1}{2}x - 3 \) are shown on the right. Their slopes are the same. Every point in the graph of \( y = -\frac{1}{2}x - 3 \) is shifted three units down from the graph of \( y = -\frac{1}{2}x \).

When you know the coordinates of one point on a line and its slope, you can use the point-slope form to write the equation in slope-intercept form.

A line that passes through \((x_1, y_1)\) with a slope \(m\) can be written in point-slope form:

\[
y - y_1 = m(x - x_1)
\]

**Example 4**

A line has a slope of \(\frac{4}{3}\) and passes through \((3, 5)\). Write the equation of the line in slope-intercept form.

**Strategy** Use the point-slope form to write the equation.

The slope, \(m\), is \(\frac{4}{3}\). Let \((x_1, y_1) = (3, 5)\).

\[
y - y_1 = m(x - x_1)
\]

\[
y - 5 = \frac{4}{3}(x - 3)
\]

\[
y - 5 = \frac{4}{3}x - 4
\]

\[
\text{Distribute \(\frac{4}{3}\) over \((x - 3)\).}
\]

\[
y - 5 + 5 = \frac{4}{3}x - 4 + 5
\]

\[
\text{Add 5 to both sides.}
\]

\[
y = \frac{4}{3}x + 1
\]

**Solution** A line with a slope of \(\frac{4}{3}\) that passes through \((3, 5)\) has the equation \(y = \frac{4}{3}x + 1\).
What is the equation of the line graphed below?

Find the values of $m$ and $b$ to write an equation in slope-intercept form.

The $y$-intercept is the point at which the graph crosses the ___-axis.

The $y$-intercept of this graph is $(0, \underline{\hspace{2cm}})$. So, $b = \underline{\hspace{2cm}}$.

Choose two points on the graph to find the slope, $m$.

Use the $y$-intercept, $(0, \underline{\hspace{2cm}})$, and the point $(3, 0)$.

To move from the $y$-intercept to $(3, 0)$, count _____ units up and _____ units to the right.

$m = \frac{\text{rise}}{\text{run}} = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

Since the line slants ________ from left to right, the slope is positive.

Substitute those values of $m$ and $b$ into $y = mx + b$.

$y = \underline{\hspace{2cm}} x - \underline{\hspace{2cm}}$

The equation of the line is $y = \underline{\hspace{2cm}}$. 
Proportional Relationships

Getting the Idea

A ratio is a comparison of two numbers. For example, if there are 13 boys and 12 girls in a class, the ratio of boys to girls is 13 to 12. This can also be written with a colon, 13:12, or as a fraction, $\frac{13}{12}$. Since a ratio is a comparison of numbers, the ratio $\frac{13}{12}$ is not an improper fraction and cannot be rewritten as $1\frac{1}{12}$.

The ratio $\frac{13}{12}$ compares part of a class (the boys) to another part of the class (the girls). You can also use ratios to compare parts to totals.

Example 1

A bouquet contains only red and white roses. The ratio of red roses to white roses is 1:2. What is the ratio of red roses to the total number of roses in the bouquet?

Strategy  Use the part-to-part ratio to find the part-to-total ratio.

Step 1 Write the part-to-part ratio as a fraction.

\[
\frac{\text{red roses}}{\text{white roses}} = \frac{1}{2}
\]

Step 2 Use that ratio to write the part-to-total ratio.

The total includes all the red roses and all the white roses.

\[
\frac{\text{red roses}}{\text{red roses} + \text{white roses}} = \frac{1}{1 + 2} = \frac{1}{3}
\]

Solution The ratio of red roses to total roses is $\frac{1}{3}$.

A proportion shows that two ratios are equal in value. You can use proportional reasoning to solve for an unknown value in a proportion.

Example 2

The debate team won 3 out of 5 debates it participated in this semester. If the team participated in 20 debates, how many debates did it win? How many did it lose?

Strategy Set up a proportion and use proportional reasoning.

Step 1 What does the given ratio represent?

\[
\frac{3}{5}
\] is the ratio of debates won to total debates.
Step 2 Write a second ratio that includes the same terms.

Let $x$ represent the number of debates won. There were 20 debates total.

\[
\frac{\text{debates won}}{\text{total debates}} = \frac{x}{20}
\]

Step 3 Set the ratios equal to each other to form a proportion.

\[
\frac{3}{5} = \frac{x}{20}
\]

Step 4 Use proportional reasoning and think about equivalent fractions to find the value of $x$.

The denominators are 5 and 20, and $5 \times 4 = 20$.

So, multiply the numerator and denominator of the first ratio by 4.

\[
\frac{3 \times 4}{5 \times 4} = \frac{12}{20}
\]

\[
\frac{3}{5} \text{ is equivalent to } \frac{12}{20}, \text{ so the value of } x \text{ is 12.}
\]

Step 5 Find the number of debates that the team lost.

If the team won 12 out of 20 debates, then the number of debates lost was:

\[
20 - 12 = 8.
\]

Solution The team won 12 debates and lost 8 debates.

Another way to solve for an unknown value in a proportion is to use cross-multiplication. To cross-multiply, multiply the numerator of each ratio by the denominator of the other ratio and set them equal to each other. Then solve for the unknown value.

A rate is a ratio that compares quantities that use different units. Use the same strategies to work with rates as you use with other ratios.

Example 3

It costs $261 for 3 nights at Pavia Pavilion Hotels. At the same rate, how much will it cost to stay for 7 nights?

Strategy Set up a proportion and cross-multiply.

Step 1 Write a ratio comparing the cost to the number of nights.

\[
\frac{\text{cost}}{\text{number of nights}} = \frac{261}{3}
\]

Step 2 Write a second ratio that includes the unknown.

Let $x$ represent the unknown cost.

\[
\frac{\text{cost}}{\text{number of nights}} = \frac{x}{7}
\]

Step 3 Set the ratios equal to each other to form a proportion.

\[
\frac{261}{3} = \frac{x}{7}
\]
Step 4

Cross-multiply and solve for $x$.

$$\frac{261}{3} = \frac{x}{7}$$

$$261 \cdot 7 = 3 \cdot x$$

$$1,827 = 3x$$

$$\frac{1,827}{3} = \frac{3x}{3}$$

$$609 = x$$

Solution

A 7-night stay at the Pavia Pavilion Hotels will cost $609.

A **unit rate** is a rate that, when expressed as a fraction, has a 1 in the denominator. For example, 46 miles per gallon is a unit rate because it expresses the ratio $\frac{46 \text{ miles}}{1 \text{ gallon}}$. If a unit rate involves money, it is called a **unit price**.

**Example 4**

A 16-ounce box of Wheaty Puffs costs $3.52. A 64-ounce box of Wheaty Puffs is sold at the same unit price. What is the cost of the 64-ounce box?

**Strategy**

Find the unit price. Then use it to find the cost of the 64-ounce box.

**Step 1**

Find the unit price of the 16-ounce box.

The rate is $\frac{\$3.52}{16 \text{ ounces}}$. So, divide by 16 to find the unit price.

$$\frac{3.52}{16} = \frac{3.52 \div 16}{16 \div 16} = \frac{0.22}{1}$$

The unit price is $0.22 per ounce.

**Step 2**

Multiply to find the cost of the 64-ounce box.

Since 1 ounce costs $0.22, multiply by 64 to find the price for 64 ounces.

$$0.22 \times 64 = \$14.08$$

**Solution**

The cost of the 64-ounce box of Wheaty Puffs is $14.08.
Mr. Cipriati has driven 700 miles in 4 days. If he continues to drive at the same rate, how many miles will he drive in 22 days?

Write two ratios, each comparing the number of miles to the number of days.

Let \( x \) represent the unknown quantity.

\[
\frac{\text{miles}}{\text{days}} = \frac{700}{5} \quad \text{and} \quad \frac{\text{miles}}{\text{days}} = \frac{x}{5}
\]

Set the ratios equal to each other to form a proportion. Then cross-multiply to solve.

\[
700 \cdot 5 = x \cdot 5
\]

\[
\frac{700}{5} \cdot 5 = 4 \cdot x \quad \text{Cross-multiply.}
\]

\[
\frac{700}{5} = 4x
\]

\[
\frac{700}{4} = \frac{4x}{4} \quad \text{Divide by 4.}
\]

\[
\frac{700}{4} = x
\]

If he continues to drive at the same rate, Mr. Cipriati will drive ________ miles in 22 days.
A direct proportion is a special kind of linear equation. In a direct proportion, the ratio of two variables, such as $y$ and $x$, is a constant, $m$. That means that for every change in $x$, $y$ changes by a constant factor, $m$. We can say that $y$ is directly proportional to $x$.

A direct proportion may be written in one of the following forms:

$$y = mx \text{ or } \frac{y}{x} = m$$

where $m \neq 0$ and $m$ is the constant of proportionality, as well as the slope of the line that represents the equation.

**Example 1**

Which graph shows a direct proportion?

**Strategy**

Find the ratio of $y$ to $x$ for at least two points on each line. Determine if the ratio is constant.
Step 1
Find the ratio $\frac{y}{x}$ for two points on the line in Graph 1.
For $(-1, 6)$, the ratio is: $\frac{6}{-1} = -6$.
For $(4, -4)$, the ratio is: $\frac{-4}{4} = -1$.  \[ \text{different ratio} \]
The ratio $\frac{y}{x}$ is not constant. So, Graph 1 does not show a direct proportion.

Step 2
Find the ratio $\frac{y}{x}$ for two points on the line in Graph 2.
For $(-3, 1)$, the ratio is: $\frac{1}{-3} = -\frac{1}{3}$.
For $(6, -2)$, the ratio is: $\frac{-2}{6} = -\frac{1}{3} = -\frac{1}{3}$.  \[ \text{same ratio} \]
The ratio $\frac{y}{x}$ is constant for both points.
Since all points on a straight line change by the same constant rate, the slope, you only need to test those two points.
Graph 2 shows a direct proportion.

**Solution**
Graph 2 shows a direct proportion because the ratio of $y$ to $x$, $-\frac{1}{3}$, is a constant.

If the graph of an equation is a non-vertical, straight line that passes through the origin, the graph shows a direct proportion. So a direct proportion is linear. The constant of variation, $m$, is the slope of the line, and the $y$-intercept is $(0, 0)$.
A direct proportion in the form $y = mx$ can also be used to represent a real-world situation.
The constant $m$ represents a unit rate and tells how many units of $y$ per unit of $x$.

**Example 2**
During a trip, a car is driven at a constant rate of 60 miles per hour on the highway.
Write an equation and make a graph to display the total distance that the car will travel if it maintains that speed for at least 3 hours. What does the slope of the graph represent?

**Strategy**
Write an equation to represent the situation. Then graph the equation.

**Step 1**
Will the equation you write be a direct proportion?
As the hours increase, the distance traveled increases.
The car travels at a constant rate.
So, the total distance traveled is directly proportional to the number of hours that the car is driven.
Step 2  Write an equation in the form \( y = mx \).

Let \( x \) represent the number of hours.
Let \( y \) represent the total distance, in miles.

The car travels at a constant rate of speed: \( \frac{60 \text{ miles}}{1 \text{ hour}} \).
So, the constant, \( m \), is 60.

The equation is \( y = 60x \).

Step 3  Draw and label a coordinate grid.

The car cannot travel a negative number of miles or drive for a negative number of hours, so use only the 1st quadrant.

Title the graph and label its axes.

Step 4  Find two points and connect them with a line.

The graph is a direct proportion, so it must pass through \((0, 0)\).

Since you must show at least 3 hours, substitute 3 for \( x \) in the equation to find another point.
\[
y = 60(3) \quad y = 180
\]
Plot a second point at \((3, 180)\). Draw a line through the points.

Step 5  What does the slope of the graph represent?

In \( y = 60x \), \( m = 60 \). So, the slope of the graph, 60, shows the speed of the car.

Solution  The equation \( y = 60x \) and the graph in Step 4 represent this situation.

The slope of the graph represents the speed of the car, 60 miles per hour.

A direct proportion can be represented using an equation, a graph, or a table. Sometimes, you may need to compare two different representations.
Example 3
Cassie has to buy several pounds of tomatoes at a farmer’s market. The graph shows the cost of buying tomatoes at Farm Stand 1.

The equation \( y = 4x \) gives the cost of buying \( x \) pounds of tomatoes at Farm Stand 2. Which farm stand offers the better price?

**Strategy**  Find the value of \( m \) for each direct proportion. Then compare.

**Step 1**  Find the value of \( m \) for the graph.

The \( y \)-axis shows cost in dollars and the \( x \)-axis shows pounds.

So, the ratio \( \frac{y}{x} \) compares the number of dollars to the number of pounds.

The point (2, 9) lies on the graph. \( m = \frac{y}{x} = \frac{9}{2} = 4.5 \)

This shows a unit rate of $4.50 for 1 pound of tomatoes at Farm Stand 1.

**Step 2**  Find the value of \( m \) for the equation.

In \( y = 4x \), \( m = 4 \) or \( \frac{4}{1} \).

This shows a unit rate of $4.00 for 1 pound of tomatoes at Farm Stand 2.

**Step 3**  Which farm stand offers a better unit price?

\( $4.00 < $4.50 \), so Farm Stand 2 has the better unit price.

**Solution**  Farm Stand 2 offers a better price.

---

**Coached Example**

The table below shows the distance, \( y \), in meters, that Ariel can run during the time, \( x \), in minutes. Does the table show a direct proportion?

<table>
<thead>
<tr>
<th>Ariel's Running Record</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
</tr>
<tr>
<td>( y )</td>
</tr>
</tbody>
</table>

If the table shows a direct proportion, then the ratio \( \frac{y}{x} \) will be equal for all ordered pairs.

For (1, 350): \( \frac{y}{x} = \frac{350}{1} = \underline{350} \)  \( \frac{700}{2} = \underline{350} \)

For (3, 1050): \( \frac{y}{x} = \underline{350} \)  \( \frac{1400}{4} = \underline{350} \)

The ratio \( \frac{y}{x} \) \underline{is constant}, so the table \underline{is} a direct proportion.
### Pairs of Linear Equations

**Getting the Idea**

You can use what you know about the slope of a line to help you classify a pair of lines.

<table>
<thead>
<tr>
<th>Type of Lines</th>
<th>Example</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Intersecting lines</strong></td>
<td><img src="image" alt="Graph of intersecting lines" /></td>
</tr>
<tr>
<td>Cross one another at a point. They have different slopes.</td>
<td></td>
</tr>
<tr>
<td>$y = -x$ has a slope of $-1$.</td>
<td></td>
</tr>
<tr>
<td>$y = 2x - 3$ has a slope of $2$.</td>
<td></td>
</tr>
</tbody>
</table>

| **Parallel lines**     | ![Graph of parallel lines](image) |
| Lie in the same plane and never intersect. They have the same slope but different $y$-intercepts. |
| $y = \frac{1}{2}x + 2$ has a slope of $\frac{1}{2}$. |
| $y = \frac{1}{2}x$ has a slope of $\frac{1}{2}$. |

| **Coincident lines**   | ![Graph of coincident lines](image) |
| Lie on top of one another. They have the same slope and the same $y$-intercept. In fact, they have all points in common. |
| The lines for the graph of $x + 3y = -3$ and $y = -\frac{1}{3}x - 1$ coincide. |
Example 1
Do these equations represent parallel lines?

\[ y = 4x - 2 \]
\[ y = 4x + 3 \]

**Strategy** Find the slope and \( y \)-intercept for each line. Then use the definition of parallel lines.

**Step 1** Identify \( m \) and \( b \) for the equation \( y = 4x - 2 \).

- \( m = 4 \), so the slope is 4.
- \( b = -2 \), so the \( y \)-intercept is \((0, -2)\).

**Step 2** Identify \( m \) and \( b \) for the equation \( y = 4x + 3 \).

- \( m = 4 \), so the slope is 4.
- \( b = 3 \), so the \( y \)-intercept is \((0, 3)\).

**Step 3** Are the lines parallel?

- Both lines have the same slope, 4, but different \( y \)-intercepts.
- That describes parallel lines.

**Solution** The equations \( y = 4x - 2 \) and \( y = 4x + 3 \) represent parallel lines.

Example 2
Determine if the lines below are parallel, intersecting, or coincident.

\[ y = \frac{3}{4}x + 2 \]
\[ -6x + 8y = 16 \]

**Strategy** Make sure both equations are in slope-intercept form. Then identify the slopes and intercepts, and compare them.

**Step 1** Rewrite the second equation in slope-intercept form.

Get \( y \) by itself on one side of the equation.

\[-6x + 8y = 16\]
\[-6x + 8y + 6x = 16 + 6x\] Add 6x to both sides.
\[8y = 6x + 16\]
\[\frac{8y}{8} = \frac{6x + 16}{8}\] Divide both sides by 8.
\[y = \frac{6}{8}x + 2\]
\[y = \frac{3}{4}x + 2\] This is identical to the first equation.
Step 2

Compare the slopes and \( y \)-intercepts.

The slope-intercept form of both equations is \( y = \frac{3}{4}x + 2 \).

So, both equations have the same slope, \( \frac{3}{4} \), the same \( y \)-intercept, (0, 2), and share all points in common. They are coincident lines.

Solution

The lines represented by \( y = \frac{3}{4}x + 2 \) and \(-3x + 4y = 8\) are coincident lines.

Example 3

Does the line that passes through (0, \(-3\)) and (2, 7) intersect the line that passes through (5, 1) and (10, 2)?

Strategy

Find and compare the slopes.

Step 1

Find the slope of the line that passes through (0, \(-3\)) and (2, 7).

\[
\begin{align*}
m &= \frac{y_2 - y_1}{x_2 - x_1} \\
m &= \frac{7 - (-3)}{2 - 0} \\
m &= \frac{10}{2} \\
m &= 5
\end{align*}
\]

Step 2

Find the slope of the line that passes through (5, 1) and (10, 2).

\[
\begin{align*}
m &= \frac{y_2 - y_1}{x_2 - x_1} \\
m &= \frac{2 - 1}{10 - 5} \\
m &= \frac{1}{5} \quad \text{Different than a slope of 5.}
\end{align*}
\]

Step 3

Do the lines intersect?

The lines have different slopes, so they must intersect.

Solution

The line that passes through (0, \(-3\)) and (2, 7) intersects the line that passes through (5, 1) and (10, 2).
Lesson 15: Pairs of Linear Equations

**Coached Example**

Does the line that passes through (0, 2) and (5, 5) intersect the line that passes through (−10, −4) and (−5, −1)? If not, are the two lines parallel or coincident?

Find the slope of the line that passes through (0, 2) and (5, 5).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 2}{5 - 0} = \underline{\text{______}}
\]

Find the slope of the line that passes through (−10, −4) and (−5, −1).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \underline{\text{______}} = \underline{\text{______}}
\]

Are the slopes the same or different?
The slopes are \underline{__________}. So, the lines are not intersecting lines.

To decide if the lines are parallel or coincident, compare their y-intercepts.

You know the first line passes through (0, 2). That is its y-intercept.

Use the point-slope form to find the y-intercept of the other line.

The slope, \(m\), is \underline{______}. Let \((x_1, y_1) = (−5, −1)\).

\[
y - y_1 = m(x - x_1)
\]

\[
y - (-1) = \left(-\frac{3}{5}\right)(x - \underline{\text{____}})
\]

\[
y + 1 = \underline{-}x + \underline{\text{____}}
\]

\[
y + 1 - 1 = \underline{\text{______}} - 1
\]

Subtract 1 from both sides.

\[
y = \underline{\text{___________}}
\]

The equation above is in slope-intercept form.

Since \(b = \underline{______}\), the y-intercept of that line is (0, \underline{______}).

Is that different or the same as the y-intercept of the first line? \underline{__________}

The lines have the same \underline{________} and \underline{________}, so they \underline{_______} intersect.

They are \underline{___________} lines.
Solve Systems of Equations Graphically

Getting the Idea

A system of linear equations is two or more linear equations with the same variables. You can use what you’ve learned about pairs of linear equations to solve systems of linear equations.

One way to solve a system of linear equations is to graph both equations. If the lines intersect, the ordered pair that names the point of intersection is the solution for the system of equations. Since both lines pass through that point of intersection, that ordered pair of values satisfies both equations at the same time.

A system of equations may have one unique solution, no solution, or infinitely many solutions, as shown below.
Example 1
Solve the system of equations graphically.

\[3x + y = -2\]
\[y = \frac{1}{2}x + 5\]

**Strategy**  
Graph each line. If the lines intersect, identify the coordinates of their point of intersection.

**Step 1**  
Graph \(3x + y = -2\) on the coordinate plane.

Rewrite the equation in slope-intercept form.

\[3x + y = -2\]
\[3x + y - 3x = -3x - 2\]
\[y = -3x - 2\]

Graph the line with a slope of \(-3\) and a \(y\)-intercept of \((0, -2)\).

**Step 2**  
Graph \(y = \frac{1}{2}x + 5\) on the same coordinate plane.

Graph the line with a slope of \(\frac{1}{2}\) and a \(y\)-intercept of \((0, 5)\).

**Step 3**  
Identify the solution for the system of linear equations.

The two lines intersect at \((-2, 4)\).

So, only when \(x = -2\) and \(y = 4\) are both equations true.

**Step 4**  
Check the solution.

Substitute \((-2, 4)\) into each equation.

\[3x + y = -2\]
\[y = \frac{1}{2}x + 5\]
\[3(-2) + (4) = -2\]
\[4 = \frac{1}{2}(-2) + 5\]
\[-6 + 4 = -2\]
\[4 = -1 + 5\]
\[-2 = -2\]  \(\checkmark\)
\[4 = 4\]  \(\checkmark\)

**Solution**  
The solution for the system of equations is \((-2, 4)\).
Example 2
Graph the system of equations below.

\[4x + 2y = 8\]
\[y = -2x - 1\]

How many solutions does this system have?

**Strategy**  Graph each line. Then determine how many solutions the system has.

**Step 1**  Graph \(4x + 2y = 8\) on the coordinate plane.

Rewrite the equation in slope-intercept form.

\[4x + 2y = 8\]

\[2y = -4x + 8\] Subtract 4x from both sides.

\[y = -2x + 4\] Divide both sides by 2.

Graph the line with a slope of \(-2\) and a \(y\)-intercept of \((0, 4)\).
Lesson 16: Solve Systems of Equations Graphically

**Step 2**  
Graph \( y = -2x - 1 \) on the coordinate plane.  
- Graph the line with a slope of \(-2\) and a \(y\)-intercept of \((0, -1)\).

**Step 3**  
Determine the number of solutions.  
- Both lines have the same slope, \(-2\).
- The lines are parallel, so there is no ordered pair that names a point on both lines.
- The system has no solution.

**Solution**  
The system of linear equations graphed above has no solution.

**Example 3**  
Graph and estimate the solution for the system of equations below.  
\[
\begin{align*}
  y &= \frac{1}{2}x + 2 \\
  y &= \frac{3}{2}x + 1
\end{align*}
\]

**Strategy**  
Graph each line. If the lines intersect, find their point of intersection and visually estimate its coordinates.

**Step 1**  
Graph a line for each equation on the coordinate plane.  
- For \( y = \frac{1}{2}x + 2 \), graph a line with a slope of \(\frac{1}{2}\) and a \(y\)-intercept of \((0, 2)\).
- For \( y = \frac{3}{2}x + 1 \), graph a line with a slope of \(\frac{3}{2}\) and a \(y\)-intercept of \((0, 1)\).

**Step 2**  
Visually estimate the solution to the graph.  
- The lines appear to intersect when \( x = 1 \) and when the value of \( y \) is halfway between 2 and 3.
- A good estimate of the solution is \((1, 2.5)\) or \((1, \frac{5}{2})\).

**Solution**  
A good estimate of the solution of the system of equations is \((1, 2.5)\).  

\[
\begin{align*}
  y &= \frac{1}{2}x + 2 \\
  y &= \frac{3}{2}x + 1
\end{align*}
\]
Graph $-\frac{1}{4}x + y = -2$ and $x - 4y = 8$ on the coordinate plane.

Use the graph to determine how many solutions this system of linear equations has.

Rewrite the equation $-\frac{1}{4}x + y = -2$ in slope-intercept form.

\[-\frac{1}{4}x + y = -2\]
\[-\frac{1}{4}x + ____ + y = -2 + ____ \quad \text{Add } \frac{1}{4}x \text{ to both sides.}\]

\[y = \underline{\quad\quad}\]

This line has a slope of _____ and a $y$-intercept of $(0, ____ )$.

Graph and label $-\frac{1}{4}x + y = -2$ on the grid above.

Rewrite the equation $x - 4y = 8$ in slope-intercept form.

\[x - ____ - 4y = 8 - ____ \quad \text{Subtract } x \text{ from both sides.}\]
\[-4y = ____ + 8\]
\[\frac{-4y}{-4} = ____ + \frac{8}{-4}\quad \text{Divide both sides by } -4.\]

\[y = \underline{\quad\quad}\]

This line has a slope of _____ and a $y$-intercept of $(0, ____ )$.

Graph and label $x - 4y = 8$ on the grid above.

What do you notice about the graphs?

The second line I graphed lies ___________ the first line.

So, are the two lines intersecting, parallel, or coincident? ______________

Does the system have one solution, no solution, or infinitely many solutions? __________

The lines share all points in common, so the system of equations has ______________ solution(s).
Solve Systems of Equations Algebraically

Getting the Idea

A system of linear equations can be solved using algebra.

One method you can use is called the substitution method. In this method, you replace one variable with an expression that is equivalent to the other variable. This creates a one-variable equation that you can solve to find the value of the other variable.

Example 1

Solve this system of linear equations using substitution.

\[ \begin{align*}
  y &= 2x - 1 \\
  2x + 2y &= 10
\end{align*} \]

Strategy

Replace the variable \( y \) using substitution.

Step 1
Write an equation that contains only one variable.

The first equation is solved for \( y \). It shows that the expression \( 2x - 1 \) is equivalent to \( y \).

So, substitute \( 2x - 1 \) for \( y \) in the second equation.

\[ \begin{align*}
  2x + 2y &= 10 \\
  2x + 2(2x - 1) &= 10
\end{align*} \]

Step 2
Solve that equation for \( x \).

\[ \begin{align*}
  2x + 4x - 2 &= 10 \\
  6x - 2 &= 10 \\
  6x &= 12 \\
  x &= 2
\end{align*} \]

Step 3
Substitute that value for \( x \) into one of the original equations. Solve for \( y \).

\[ \begin{align*}
  y &= 2x - 1 \\
  y &= 2(2) - 1 \\
  y &= 3
\end{align*} \]

So, the solution is \( (2, 3) \).

Solution

The solution for this system of linear equations is \( (2, 3) \).
A system of linear equations may have one unique solution, no solutions, or infinitely many solutions. You can use algebra to determine how many solutions a system has.

**Example 2**

Does this system of equations have one solution, no solution, or infinitely many solutions?

\[
3x + 4y = -4 \\
\frac{3}{4}x + y = -1
\]

**Strategy**  Solve the system using substitution.

**Step 1**  Find an expression that is equivalent to one of the variables.

Solve the second equation for \( y \).

\[
\frac{3}{4}x + y = -1 \\
y = -\frac{3}{4}x - 1
\]

**Step 2**  Substitute that expression for \( y \) in the first equation.

\[
3x + 4y = -4 \\
3x + 4\left(-\frac{3}{4}x - 1\right) = -4 \quad \text{Substitute } -\frac{3}{4}x - 1 \text{ for } y. \\
3x + (-3x) + (-4) = -4 \quad \text{Apply the distributive property.} \\
0x - 4 = -4 \quad \text{Combine like terms.} \\
-4 = -4 \quad \rightarrow \quad \text{always true}
\]

This means that this system has infinitely many solutions.

**Solution**  This system of linear equations has infinitely many solutions.

If you write both equations in slope-intercept form, you will see that they each have the same slope, \(-\frac{3}{4}\), and the same \( y \)-intercept, \((0, -1)\). So, they are coincident lines.
Another algebraic method you could use is called the elimination method. In this method, you add or subtract the equations to eliminate one variable and solve for the remaining variable. Then use that value to find the value of the other variable.

A coefficient is a number that is multiplied by a variable.

**Example 3**
Use elimination to determine if this system has one solution, no solution, or infinitely many solutions.

\[
\begin{align*}
2x - 9y &= 18 \\
x + 3y &= -21
\end{align*}
\]

**Strategy** Add the equations to eliminate one variable. Solve for the remaining variable and use its value to find the value of the other variable.

**Step 1** Eliminate one variable.
- If you multiply the second equation by 3, the coefficients of \( y \) will be opposites (\(-9\) and \(9\)).
- Their sum is zero, so that will eliminate \( y \).
- If you multiply the second equation by \(-2\), the coefficients of \( x \) will be opposites (\(2\) and \(-2\)).
- Their sum is zero, so that will eliminate \( x \).
- It does not matter which variable you eliminate.

**Step 2** Multiply the second equation by 3.
\[
\begin{align*}
x + 3y &= -21 \\
3(x + 3y) &= 3(-21) \\
3x + 9y &= -63
\end{align*}
\]

Apply the distributive property.

Because you multiplied both sides of the equation by 3, the equation is still true.

**Step 3** Add the result of Step 2 to the original first equation.
\[
\begin{align*}
2x - 9y &= 18 \\
+ 3x + 9y &= -63 \\
\hline
5x &= -45
\end{align*}
\]

**Step 4** Solve for \( x \).
\[
\begin{align*}
\frac{5x}{5} &= \frac{-45}{5} \\
x &= -9
\end{align*}
\]
Step 5 Substitute $-9$ for $x$ into one of the original equations and solve for $y$.

\[
\begin{align*}
x + 3y &= -21 \\
-9 + 3y &= -21 \\
3y &= -12 \\
y &= -4
\end{align*}
\]

So, the solution is $(-9, -4)$.

Solution The solution for this system of linear equations is $(-9, -4)$.

Sometimes, you can solve a system of linear equations using your reasoning skills.

Example 4
Does this system of equations have one solution, no solution, or infinitely many solutions?

\[
\begin{align*}
5x - 4y &= 2 \\
5x - 4y &= 3
\end{align*}
\]

Strategy Since the equations look very similar, try to reason the answer.

Step 1 Compare the equations.

Both equations have the same quantity, $5x - 4y$, on the left side.

Each equation has a different value on the right side.

Step 2 How many solutions do the equations have?

Since the quantity $5x - 4y$ cannot be equivalent to 2 and equivalent to 3, this system of equations has no solution.

Solution This system of equations has no solution.

If you graphed the equations, you would see that they are parallel lines and therefore have no point of intersection. If you subtract the second equation from the first equation, you get the equation $0 = -1$, which is never true.
Example 5
A line passes through the points (1, -3) and (-4, 2). A second line passes through the points (1, 3) and (-2, -3). At what point do the two lines intersect?

Strategy  Write a system of equations to represent the two pairs of points. Then solve the system using substitution.

Step 1  Write the equation of the line that passes through the first pair of points.

Use the slope formula to find the slope of the line that passes through (1, -3) and (-4, 2).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{2 - (-3)}{-4 - 1} = \frac{5}{-5} = -1
\]

Use the point (1, -3) and the value of \( m \) to write an equation in point-slope form.

\[
y - y_1 = m(x - x_1)
\]

\[
y - (-3) = -1(x - 1)
\]

\[
y + 3 = -x + 1 \quad \text{Apply the distributive property.}
\]

\[
y = -x - 2 \quad \text{Subtract 3 from both sides to solve for } y.
\]

Step 2  Write the equation of the line that passes through the second pair of points.

Use the slope formula to find the slope of the line that passes through (1, 3) and (-2, -3).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-3 - 3}{-2 - 1} = \frac{-6}{-3} = 2
\]

Use the point (1, 3) and the value of \( m \) to write an equation in point-slope form.

\[
y - y_1 = m(x - x_1)
\]

\[
y - 3 = 2(x - 1)
\]

\[
y - 3 = 2x - 2 \quad \text{Apply the distributive property.}
\]

\[
y = 2x + 1 \quad \text{Add 3 to both sides to solve for } y.
\]

Step 3  Solve the system of equations using substitution.

Both equations are solved for \( y \). The second equation shows that the expression \( 2x + 1 \) is equivalent to \( y \). Substitute \( 2x + 1 \) for \( y \) in the first equation.

\[
y = -x - 2
\]

\[
2x + 1 = -x - 2 \quad \text{Substitute } 2x + 1 \text{ for } y.
\]

\[
3x + 1 = -2 \quad \text{Add } x \text{ to both sides.}
\]

\[
3x = -3 \quad \text{Subtract 1 from both sides.}
\]

\[
x = -1 \quad \text{Divide both sides by 3.}
\]
Step 4 Substitute the value of \( x \) into either equation to find the value of \( y \).

Use the first equation.

\[
y = -x - 2 = -(1) - 2 = 1 - 2 = -1
\]

Step 5 Write the solution to the system of equations as an ordered pair.

\[ x = -1, \ y = -1 \]

So, the solution written as an ordered pair is \((-1, -1)\).

The solution to a system of two linear equations is the point of intersection.

Solution The lines intersect at \((-1, -1)\).

Coached Example

Solve by substitution.

\[
\begin{align*}
3x - 2y &= -16 \\
y &= 8 - x
\end{align*}
\]

The second equation shows that \( y = 8 - x \).

So, substitute \( \quad _____ \quad \) for \( y \) in the first equation and solve for \( x \).

\[
\begin{align*}
3x - 2y &= -16 \\
3x - 2(\quad _____ \quad) &= -16 \\
3x - \quad _____ \quad &= -16 \quad \text{Apply the distributive property.} \\
\quad _____ \quad &= \quad _____ \quad \text{Combine like terms.} \\
\quad _____ \quad &= \quad _____ \quad \text{Add _____ to both sides.} \\
\quad _____ \quad &= \quad _____ \quad \text{Divide both sides by _____}. \\
\end{align*}
\]

Substitute that value for \( x \) into the second equation.

\[
y = 8 - x = 8 - \quad _____ \quad = \quad _____
\]

Check your solution using each of the original equations.

\[
\begin{align*}
3x - 2y &= -16 \\
y &= 8 - x \\
3(\quad _____ \quad) - 2(\quad _____ \quad) &= -16 \\
\quad _____ \quad 8 - \quad _____ \\
\end{align*}
\]

The solution for the system of linear equations is \((\quad _____ \quad, \quad _____ \quad)\).
Use Systems of Equations to Solve Problems

Getting the Idea

You can use a system of linear equations to model and solve problem situations in which you are given two different relationships between two unknown variables.

Example 1

Chelsea and Zack are both dog sitters. Chelsea charges $2 per day plus a sign-up fee of $3. Zack charges a flat rate of $3 per day. The system of linear equations below represents $y$, the total amount earned in dollars for $x$ days of dog sitting.

\[
y = 2x + 3 \\
y = 3x
\]

After how many days do Chelsea and Zack earn the same amount for dog sitting? What is that amount?

Strategy

Graph a line for each equation.

**Step 1**

Graph a line for each equation.

For $y = 2x + 3$, graph a line with a slope of 2 and a $y$-intercept of (0, 3).

Since Chelsea charges $2 per day plus a $3 sign-up fee, this line represents Chelsea’s earnings.

For $y = 3x$, graph a line with a slope of 3 and a $y$-intercept of (0, 0).

Since Zack charges $3 per day, this line represents Zack’s earnings.

**Step 2**

Identify and interpret the solution.

The $x$-axis shows numbers of days.

The $y$-axis shows amounts earned.

The lines intersect at (3, 9).

So, if Chelsea and Zack dog sit for 3 days, they each earn $9.

Solution

Chelsea and Zack charge the same amount, $9, for 3 days of dog sitting.
Example 2
Maria works for a gardener after school. Each week, she is paid an hourly rate plus a fixed amount to cover travel expenses. During the first week, Maria worked 10 hours and was paid $115. During the second week, Maria worked 5 hours and was paid $65. What is Maria’s hourly rate of pay? What is the fixed amount she gets each week for travel expenses?

Strategy Write a system of linear equations. Use the elimination method to solve it.

Step 1 Write a system of linear equations.
Let \( x \) represent Maria’s hourly rate in dollars.
Let \( y \) represent the fixed amount for travel expenses.
During Week 1, she worked 10 hours and was paid $115. \( 10x + y = 115 \)
During Week 2, she worked 5 hours and was paid $65. \( 5x + y = 65 \)

Step 2 Multiply the second equation by \(-1\).
This works because \( y \) and \(-y\) are opposites.
\( 5x + y = 65 \)
\( -1(5x + y) = -1(65) \)
\( -5x - y = -65 \)

Step 3 Add the result of Step 2 to the original first equation.
\( 10x + y = 115 \)
\( + -5x - y = -65 \)
\( 5x = 50 \)

Step 4 Solve for \( x \).
\( 5x = 50 \)
\( x = 10 \)
\( \rightarrow \) This represents Maria’s hourly rate of pay.

Step 5 Substitute 10 for \( x \) into one of the original equations and solve for \( y \).
\( 5x + y = 65 \)
\( 5(10) + y = 65 \)
\( 50 + y = 65 \)
\( y = 15 \)
\( \rightarrow \) This represents the fixed amount she is given for travel expenses.

Solution Maria earns $10 per hour and is paid a fixed amount of $15 each week for travel expenses.
Inside the stables, there are only horses and people. The number of horses and people combined is 9. A boy inside the stables added the number of legs of all the horses and the number of legs of all the people combined. He determined that there are 30 legs in total. How many horses and how many people are in the stables?

Write a system of linear equations to represent this problem.

Let $x$ represent the number of horses. Let $y$ represent the number of people. Remember that a horse has 4 legs and a person has 2 legs.

The number of horses and people combined is 9.  

There are 30 legs in total.

Solve the first equation you wrote for $y$.

$y = \phantom{0}$

Substitute the equation you just rewrote for $y$ into the second equation and solve for $x$.

Substitute that value for $x$ into the first equation, $x + y = 9$.

The solution for the system of linear equations is (_____, ____).

It shows that there are _____ horses and _____ people in the stables.
A relation is a set of ordered pairs. A function is a relation in which each input value, or x-value, corresponds to exactly one output value, or y-value. A function or other relation can be represented as a set of ordered pairs in a table, as an equation, or by a graph.

Example 1
Which table below represents a function?

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Table 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>x</td>
<td>y</td>
</tr>
<tr>
<td>−2</td>
<td>−4</td>
</tr>
<tr>
<td>−1</td>
<td>−1</td>
</tr>
<tr>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
</tr>
</tbody>
</table>

Strategy Compare the x- and y-values.

Step 1 Compare the x- and y-values in Table 1.

The x-value −2 corresponds to only one y-value, −4.
The x-value −1 corresponds to only one y-value, −1.
The x-value 0 corresponds to only one y-value, 2.
The x-value 1 corresponds to only one y-value, 5.
Since each x-value has exactly one y-value, Table 1 shows a function.

Step 2 Compare the x- and y-values in Table 2.

The x-value 4 corresponds to only one y-value, −2.
The x-value 2 corresponds to two y-values, −1 and 1.
Since there is an x-value that corresponds to more than one y-value, this relation is not a function.

Solution Table 1 represents a function. Table 2 represents a relation that is not a function.
In a function, the set of all the input values, or x-values, is called the **domain**. The set of all the output values, or y-values, is called the **range**. Braces, \{ \}, are often used when listing the domain and range.

**Example 2**
Identify the domain and range for the function shown below.

<table>
<thead>
<tr>
<th>Sale Prices</th>
<th>Regular Price, x</th>
<th>$5</th>
<th>$10</th>
<th>$15</th>
<th>$20</th>
<th>$25</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sale Price, y</td>
<td>$1</td>
<td>$2</td>
<td>$3</td>
<td>$4</td>
<td>$5</td>
<td></td>
</tr>
</tbody>
</table>

**Strategy**
Identify the domain and range of the function.

**Step 1**
Identify the domain.
List the x-values.
5, 10, 15, 20, 25

**Step 2**
Identify the range.
List the y-values.
1, 2, 3, 4, 5

**Solution**
The domain of the function is 5, 10, 15, 20, 25. The range is 1, 2, 3, 4, 5.

Every function follows a rule that maps each element in its domain to exactly one element in its range. So, another way to determine if a relation is a function is to draw a mapping diagram. List the domain elements and the range elements in order. Then draw an arrow from each domain value to its range value. A mapping diagram for the function $y = x^2$ is shown below.

```
<table>
<thead>
<tr>
<th>Domain</th>
<th>Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>-2</td>
<td>1</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>2</td>
<td>9</td>
</tr>
<tr>
<td>3</td>
<td></td>
</tr>
</tbody>
</table>
```

The x-values of -2 and 2 both map to 4, but there is still exactly one y-value for each x-value in the set. So, this relation is a function.
Example 3
Create a mapping diagram for the relation below.

(2, 6), (3, 9), (3, 12), (4, 15), (5, 10)

Is the relation a function?

Strategy  Create a mapping diagram.

Step 1  List the domain elements and the range elements in order.
        List the domain values, 2, 3, 4, 5, in a box on the left.
        List the range values, 6, 9, 10, 12, 15, in a box on the right.

Step 2  Draw an arrow from each domain value to its range value.
        (2, 6) is part of the relation. So, draw an arrow from 2 to 6.
        (3, 9) and (3, 12) are part of the relation. So, draw arrows from 3 to 9 and to 12.
        Represent (4, 15) and (5, 10) with arrows, too.

Step 3  Is the relation a function?
        The domain element 3 maps to two different range elements, 9 and 12.
        So, the relation is not a function.

Solution  The mapping diagram in Step 2 shows that one of the domain elements maps to two range elements. So, the relation is not a function.
The graph of a function is the set of ordered pairs consisting of input values and their corresponding output values. To determine whether a graph represents a function, you can use the **vertical line test**.

Imagine drawing vertical lines through the graph. If no vertical line intersects the graph in more than one point, the graph shows a function. For example, in the left-hand graph below, no vertical dashed line crosses the graph in more than one point, so the graph shows a function.

If you can draw a vertical line that intersects the graph in two or more points, the graph does not show a function. In the right-hand graph below, the vertical dashed line crosses the graph in two points, so the graph does not show a function.

**Example 4**

Which graph represents a function?
Strategy  Use the vertical line test on each graph.

Step 1  Use the vertical line test on Graph 1.

No matter where you draw a vertical line, it only crosses the graph once. This graph represents a function.

Step 2  Use the vertical line test on Graph 2.

This graph includes a vertical segment at $x = 1$. So, there is more than one $y$-value paired with the $x$-value, 1. This graph does not represent a function.

Solution  Graph 1 represents a function.
The points shown in the table below represent a relation. Plot the points and determine if the relation is a function.

<table>
<thead>
<tr>
<th>x</th>
<th>1</th>
<th>2</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td>−1</td>
<td>−2</td>
<td>−4</td>
<td>−4</td>
<td>−5</td>
<td>−6</td>
</tr>
</tbody>
</table>

The data in the table correspond to the points (1, −1), (2, −2), (2, ____), (____, ____), (____, ____), and (____, ____).

Plot those points on the coordinate grid below.

Draw a vertical line through (1, −1).
Does it pass through more than one point on the graph? _____

Draw a vertical line through (2, −1).
Does it pass through more than one point on the graph? _____

When \( x = 2 \), the relation corresponds to ____ \( y \)-value(s).
So, the relation ________ a function.
Work with Linear Functions

Getting the Idea

A **linear function** has a graph that is a straight line. It can be represented by a linear equation in the form $y = mx + b$. A **nonlinear function** is any function that is not linear.

In a linear equation, no variable is raised to a power greater than 1.

Example 1

Is the function $y = x^3$ linear or nonlinear?

**Strategy**  
Look at the exponent for each variable in the equation.

In a linear equation, no variable is raised to a power greater than 1.  
In $y = x^3$, the variable $x$ is raised to the power of 3.  
So, the equation is not a linear equation, and the function is nonlinear.

**Solution**  
The function represented by $y = x^3$ is a nonlinear function.

You can also determine if a function is linear or nonlinear by graphing it. For example, the graph of the function from Example 1, $y = x^3$, is shown below. The points on the graph, $(-2, -8)$, $(-1, -1)$, $(0, 0)$, $(1, 1)$, and $(2, 8)$, do not lie on a straight line, so the function is nonlinear.
A function can describe a situation in which one quantity determines another. In a function, the output value, or \( y \)-value, depends on the input value, or \( x \)-value. That is why the \( y \)-value is often called the dependent variable, and the \( x \)-value is often called the independent variable.

**Example 2**

Given the linear function \( y = -\frac{1}{2}x + 7 \), find the missing output values in the table below. Then identify the independent and dependent variables.

<table>
<thead>
<tr>
<th>Input (( x ))</th>
<th>Output (( y ))</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>9</td>
</tr>
<tr>
<td>0</td>
<td>7</td>
</tr>
<tr>
<td>2</td>
<td>?</td>
</tr>
<tr>
<td>12</td>
<td>?</td>
</tr>
</tbody>
</table>

**Strategy** Substitute each \( x \)-value into the equation and solve for \( y \). Then identify the independent and dependent variables.

**Step 1** Find the output when the input is 2.

\[
y = -\frac{1}{2}x + 7
\]

\[
y = -\frac{1}{2}(2) + 7
\]

\[
y = -1 + 7
\]

\[
y = 6
\]

**Step 2** Find the output when the input is 12.

\[
y = -\frac{1}{2}x + 7
\]

\[
y = -\frac{1}{2}(12) + 7
\]

\[
y = -6 + 7
\]

\[
y = 1
\]

**Step 3** Identify the independent and dependent variables.

The output, or \( y \)-value, depends on the input, or \( x \)-value.

So, the dependent variable is \( y \), and the independent variable is \( x \).

**Solution** When the input is 2, the output is 6. When the input is 12, the output is 1. The dependent variable is \( y \), and the independent variable is \( x \).
A linear function can be represented in many ways. If you are given a table of values or a graph, you can use it to write an equation for the function. The equation for a function gives the rule that shows how each input value relates to each output value.

You can determine the rate of change and initial value for a linear function from a table of values, a graph, or an equation. The initial value is the value of $y$ when $x$ equals 0. In a graph, the rate of change is the same as the slope. Since the graph of a linear function is a straight line, the rate of change is constant, and you can determine it from any two pairs of $(x, y)$ values for the function. You can find the initial value of a linear function by identifying the $y$-intercept.

**Example 3**
The input-output table below represents a linear function.

<table>
<thead>
<tr>
<th>Input ($x$)</th>
<th>Output ($y$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>-1</td>
</tr>
<tr>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
</tr>
</tbody>
</table>

Write an equation for the function and identify the rate of change and initial value.

**Strategy**  
Find the rule that relates each $x$-value to its corresponding $y$-value. Then write the equation.

**Step 1**  
Find a rule that relates the first pair of values in the table.

In the first column, the $x$-values are increasing by 1s.
In the second column, each $y$-value is 3 more than the previous $y$-value.
Look for a rule that involves multiplying by 3.
Consider $(0, -1)$. $0 \cdot 3 = 0$, not $-1$. But, if you subtract 1, you get $0 - 1 = -1$.
So, the rule may be: multiply each $x$-value by 3 and then subtract 1.

**Step 2**  
See if the rule works for the other pairs of values in the table.

$1 \cdot 3 - 1 = 3 - 1 = 2$, and $(1, 2)$ is in the table.
$2 \cdot 3 - 1 = 6 - 1 = 5$, and $(2, 5)$ is in the table.
$3 \cdot 3 - 1 = 9 - 1 = 8$, and $(3, 8)$ is in the table.

**Step 3**  
Use the rule to write an equation.

To find each $y$-value, multiply each $x$-value by 3 and then subtract 1.
So, $y = 3x - 1$.  

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Step 4  
Determine the rate of change for the function.
Let \((x_1, y_1) = (1, 2)\).
Let \((x_2, y_2) = (2, 5)\).
rate of change \(= \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 2}{2 - 1} = \frac{3}{1} = 3\)

Step 5  
Determine the initial value.
The initial value is the value of \(y\) when \(x\) equals 0. The \(y\)-value = -1 when \(x = 0\).

Solution  
The equation \(y = 3x - 1\) describes the linear function.
Its rate of change is 3, and the initial value is -1.

You could also have determined the rate of change by looking at the equation. In \(y = 3x - 1\), \(m = 3\). So, the slope of the graph and its rate of change must be 3.

Coached Example

The graph represents a linear function. Find the rate of change for the function. Then write an equation for the function.

The rate of change for the function is equal to its \(\underline{\text{_______,}}\), \(m\).

Choose two points on the graph to find the rate of change.
Let \((x_1, y_1) = (0, \underline{\text{____}})\).
Let \((x_2, y_2) = (2, \underline{\text{____}})\).
\(m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\underline{\text{____}}}{2 - 0} = \underline{\text{____}}\)

The equation for the line graphed above shows the equation of the linear function.
The \(y\)-intercept of the graph is \((0, \underline{\text{____}})\). That is the initial value. So, \(b = \underline{\text{____}}\).

You already know that \(m = \underline{\text{____}}\).

Substitute those values into the slope-intercept form, \(y = mx + b\).
\(y = \underline{\text{____}}x + \underline{\text{____}}\)

The rate of change for the linear function is \(\underline{\text{____}}\), and its equation is \(y = \underline{\text{____}}\).
Sometimes, linear functions are used to model and solve real-world problems.

Example 1
The relationship between a side length of a square, s, and its perimeter, P, can be modeled by the function \( P = 4s \). Find the missing perimeters in the table below. Then identify the dependent and independent variables.

<table>
<thead>
<tr>
<th>Side Length (s)</th>
<th>Perimeter (P)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>4</td>
</tr>
<tr>
<td>4</td>
<td>16</td>
</tr>
<tr>
<td>8</td>
<td>?</td>
</tr>
<tr>
<td>9.5</td>
<td>?</td>
</tr>
</tbody>
</table>

Strategy Substitute each s-value into the equation and solve for P. Then identify the independent and dependent variables.

Step 1 Find the perimeter when the side length is 8 units.
\[
P = 4s \\
P = 4(8) \\
P = 32
\]

Step 2 Find the perimeter when the side length is 9.5 units.
\[
P = 4s \\
P = 4(9.5) \\
P = 38
\]

Step 3 Identify the independent and dependent variables.
The perimeter, P, depends on the length of a side of the square, s.
So, the dependent variable is P, and the independent variable is s.

Solution When a square has sides 8 units long, its perimeter is 32 units.
When a square has sides 9.5 units long, its perimeter is 38 units.
In this function, the dependent variable is P, and the independent variable is s.
A graph may also be used to represent a real-world situation that is modeled by a linear function. If so, the slope of the graph shows the rate at which the two quantities in the problem are changing.

**Example 2**
The graph below shows how the cost of buying gas at Steve’s Service Station changes, based on the number of gallons that are purchased.

Find the price per gallon of gas.

**Strategy** Find and interpret the rate of change.

**Step 1** Find the rate of change.

This is the same as the slope of the graph.

Let \((x_1, y_1) = (1, 3)\).

Let \((x_2, y_2) = (2, 6)\).

rate of change \[ \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 3}{2 - 1} = \frac{3}{1} = 3 \]

**Step 2** Interpret the rate of change.

Since the \(x\)-axis shows number of gallons and the \(y\)-axis shows total cost in dollars, the rate of change is \(\frac{3 \text{ dollars}}{1 \text{ gallon}}\), or $3 per gallon.

**Solution** The gas costs $3 per gallon.
Example 3
To bowl at Cavanaugh Lanes, it costs $2 per game plus a $3 shoe rental. The total cost, \( y \), in dollars, depends on \( x \), the number of games played. Write an equation to represent this situation. Then make a table of values to represent the situation.

**Strategy**
Write an equation for the situation. Then make a table of values.

**Step 1**
Translate the words into an equation.

$2\text{ per game} \quad \text{plus} \quad $3\text{ shoe rental} \quad \text{total cost}$

\[2x + 3 = y\]

**Step 2**
Decide which \( x \)-values you should include in the table.

You cannot bowl fewer than 0 games. Also, if you go to the trouble of renting bowling shoes, you will probably bowl at least 1 game.

So, the initial \( x \)-value should be 1. The other \( x \)-values should be whole numbers because you can only pay for a whole number of games. In other words, the domain is limited to 1, 2, 3, 4, ….

**Step 3**
Make a table of values.

<table>
<thead>
<tr>
<th>Games Bowled (x)</th>
<th>( y = 2x + 3 )</th>
<th>Total Cost in Dollars (y)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( y = 2(1) + 3 = 5 )</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>( y = 2(2) + 3 = 7 )</td>
<td>7</td>
</tr>
<tr>
<td>3</td>
<td>( y = 2(3) + 3 = 9 )</td>
<td>9</td>
</tr>
<tr>
<td>4</td>
<td>( y = 2(4) + 3 = 11 )</td>
<td>11</td>
</tr>
</tbody>
</table>

**Solution**
The equation \( y = 2x + 3 \) and the table of values in Step 3 represent this problem situation.
Example 4

A marching band needs to raise $1,200 in order to attend the regional marching band festival. The band members are selling tickets to a fundraising breakfast. Tickets are $10 for adults and $6 for children. The equation $10x + 6y = 1,200$, where $x$ represents the number of adult tickets sold and $y$ represents the number of children’s tickets sold, can be used to model the situation. The graph of this equation is shown below.

Explain the meaning of the $y$-intercept in terms of the number of adult and children’s tickets sold.

**Strategy**  Identify and interpret the $y$-intercept.

**Step 1** Identify the $y$-intercept.

It is located at $(0, 200)$.

**Step 2** Interpret the $y$-intercept.

Since $y$ represents the number of children’s tickets sold, the coordinates of the $y$-intercept, $(0, 200)$, represent a situation where no adult tickets are sold (0 tickets) and only children’s tickets are sold (200 tickets).

**Solution** The $y$-intercept represents a situation in which the marching band sells 200 children’s tickets and 0 adult tickets to raise $1,200.
Example 5
To rent a limousine from Deluxe Limousines, a customer must pay a set fee plus an additional amount per hour, as shown by the graph below.

![Graph showing limousine rental costs]

a. Identify and interpret the initial value and the rate of change.
b. Determine the cost of renting a limousine for 8 hours.

**Strategy** Use the graph and the slope formula to identify and interpret the y-intercept and the rate of change.

**Step 1** Find and interpret the initial value.

The initial value is the value of \( y \) when \( x \) is equal to 0. It is the y-intercept. The y-intercept is at (0, 50).

If the limousine is rented for 0 hours indicates that $50 is the set fee for a limousine rental before any hourly charges are added, or the initial value.

**Step 2** Find and interpret the rate of change.

Let \((x_1, y_1) = (0, 50)\).
Let \((x_2, y_2) = (5, 350)\).

rate of change \( = \frac{y_2 - y_1}{x_2 - x_1} = \frac{350 - 50}{5 - 0} = \frac{300}{5} = 60 \frac{\text{money}}{\text{hour}} \)

Since the x-axis shows number of hours and the y-axis shows total cost in dollars, the rate of change is $60 per hour.

This is the hourly rate charged for a limousine rental.
**Step 3** Determine the cost of renting a limousine for 8 hours.

The cost, $y$, is a $50 fee plus $60 per hour for $x$ hours, so:

$$50 + 60x = y$$

Substitute 8 for $x$ and find the value of $y$.

$$50 + 60(8) = y$$

$$50 + 480 = y$$

$$530 = y$$

**Solution**

The initial value, or set fee, is $50. The rate of change is $60 per hour. Renting a limousine for 8 hours costs $530.

---

**Coached Example**

Students have dining cards at a boarding school. Each time a student gets a meal at the dining hall, 5 points are deducted from his or her dining card. Tyeisha’s dining card had a value of 630 points at the beginning of the semester. If her card now has 570 points left on it, how many meals has she eaten at the dining hall?

Translate the words into an equation.

Let $x$ represent the number of meals she has eaten.

Let $y$ represent the total amount left on the card.

```
value of 630 points each meal, 5 points are deducted total amount left on card

_________ – _________ = y
```

Substitute 570 for $y$ and solve for $x$.

$$_________ – _________ = 570$$

$$_________ = 570 – _______$$

Subtract _________ from both sides.

$$______ = _____$$

$$x = _____$$

Divide both sides by _________.

If Tyeisha has 570 points left on her card, she has eaten _____ meals at the dining hall.
Getting the Idea

To represent some real-world situations, you may need to break a graph into pieces to show a sequence of events. It may not be as simple as drawing or interpreting a straight line.

Example 1

On Thursday, Maksim went for a long nature walk, stopping for lunch at one point. The graph below represents his walk.

Describe what Maksim did during each interval shown.

Strategy  
Look at the graph piece by piece.

Step 1  
Look at the first piece of the graph.

The first piece is a line segment slanting up from (0, 0) to (2, 6).

Since the x-axis shows time, in hours, and the y-axis shows total distance, in miles, this segment shows that Maksim walked 6 miles during the first 2 hours.

Find and interpret the rate of change for this segment.

rate of change = \frac{\text{miles}}{\text{hours}} = \frac{6 - 0}{2 - 0} = \frac{6}{2} = 3

So, Maksim walked at a speed of 3 miles per hour for the first 2 hours.
Step 2
Look at the second piece of the graph.
The second piece is a horizontal line segment from (2, 6) to (3, 6).
The rate of change for a horizontal segment is 0.
So, Maksim walked no additional distance during that hour.
That is probably when he stopped for lunch.

Step 3
Look at the third piece of the graph.
The third piece is a line segment slanting up from (3, 6) to (5, 11).
11 \(-\) 6 \(=\) 5, so Maksim walked 5 more miles during the last 2 hours of his walk.
rate of change = \(\frac{\text{miles}}{\text{hours}}\) = \(\frac{11 - 6}{5 - 3} = \frac{5}{2} = 2.5\)
So, Maksim walked at a speed of 2.5 miles per hour for the last 2 hours.

Solution
The graph shows that Maksim walked at a speed of 3 miles per hour during the first 2 hours, stopped for lunch between hours 2 and 3, and walked at a slightly slower speed of 2.5 miles per hour between hours 3 and 5.

Be careful when you interpret the meaning of the term constant.
• If a graph is increasing at a constant rate, it is represented by a line segment that slants up.
• If a graph is decreasing at a constant rate, it is represented by a line segment that slants down.
• If a piece of a graph is constant, it is represented by a horizontal line segment.
  Here, constant means that it is neither increasing nor decreasing.
Example 2
A function decreases at a constant rate from \((-6, 5)\) to \((-1, -3)\). It then increases at a constant rate from \((-1, -3)\) to \((2, 1)\). Finally, it is constant from \(x = 2\) to \(x = 4\). Sketch the graph.

**Strategy**  
Graph the function piece by piece.

**Step 1**  
Graph the first piece.
Plot points at \((-6, 5)\) and \((-1, -3)\) on a coordinate grid.
Since the graph decreases at a constant rate, draw a line segment to connect the points.

**Step 2**  
Graph the second piece.
Plot a point at \((2, 1)\).
Since the graph increases at a constant rate, draw a line segment to connect \((-1, -3)\) to \((2, 1)\).

**Step 3**  
Graph the final piece.
Since the graph is constant from \(x = 2\) to \(x = 4\), it will neither increase nor decrease during that interval.
\((2, 1)\) is on the graph, so plot a point with an \(x\)-value of 4 and the same \(y\)-coordinate as \((2, 1)\). That point is \((4, 1)\). Connect the two points with a horizontal line segment.

**Solution**  
The graph in Step 3 fits the verbal description of the function.
Use that information to describe each part of the graph.

Decide which line segment is steeper.

The line segment from (0, 13) to (30, 1) is __________________ than the line segment from (30, 1) to (50, 0).

The steeper line segment shows Mr. Kowalski moving toward home at a faster rate.

Since a person travels faster on a bus than on foot, the line segment from (0, 13) to (30, 1) shows that Mr. Kowalski ______________________________________.

30 − 0 = 30, so that segment represents _____ minute(s) of his commute.

13 − 1 = ____, so it represents a distance of _____ mile(s) traveled.

Look at the line segment from (30, 1) to (50, 0).

Does this line segment show Mr. Kowalski taking the bus or walking? __________

50 − 30 = ____, so that segment represents _____ minute(s) of his commute.

1 − 0 = 1, so it represents a distance of _____ mile(s) traveled.

The graph shows that during his commute, Mr. Kowalski ______________ for the first _____ minute(s) and traveled a distance of _____ mile(s). He then _______ for the next _____ minute(s) and traveled a distance of _____ mile(s).
Getting the Idea

Functions can be represented in different ways—as a set of ordered pairs, in a table, with a verbal or algebraic rule, as an equation, or by a graph. Sometimes, you may need to compare two different functions represented in different ways.

Example 1

A brother and sister are racing 30 meters to a tree. Since Justin is younger, his sister Cami lets him have a 6-meter head start. The graph below shows the distance that Justin runs during the race.

![Graph of Justin's Race]

The equation $y = 3x$ can be used to represent $y$, the total distance in meters that Cami has run after $x$ seconds have passed. Who is running at a faster speed? How much faster?

Strategy

Compare the rate of change shown in the graph to the rate of change shown by the equation.

Step 1

Find Justin’s speed.

The $x$-axis shows time, in seconds, and the $y$-axis shows distance, in meters.

So, the rate of change for the graph compares seconds to meters.

Use the points $(0, 6)$ and $(12, 30)$ to find the rate of change.

$$\text{rate of change} = \frac{\text{meters}}{\text{seconds}} = \frac{30 - 6}{12 - 0} = \frac{24}{12} = 2$$

So, Justin’s speed is 2 meters per second.
Step 2 Find Cami’s speed.

The equation $y = 3x$ is in the form $y = mx$.

Since $m = 3$, the rate of change is 3 meters per second.

Step 3 Find the person running at a faster speed. How much faster?

3 meters per second $> 2$ meters per second, and $3 - 2 = 1$.

So, Cami is running 1 meter per second faster than Justin.

Solution Cami’s speed is 1 meter per second faster than Justin’s speed.

Example 2

Compare the rates of change for the two linear functions described below.

Function 1: Any $y$-value can be found using the expression $\frac{1}{3}x + 2$.

Function 2:

<table>
<thead>
<tr>
<th>$x$</th>
<th>-4</th>
<th>-2</th>
<th>0</th>
<th>2</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$y$</td>
<td>1</td>
<td>1.5</td>
<td>2</td>
<td>2.5</td>
<td>3</td>
</tr>
</tbody>
</table>

Which function has a greater rate of change, or are they the same?

Strategy Identify the rate of change for each function. Then compare them.

Step 1 Find the rate of change for Function 1.

Any $y$-value can be found using the expression $\frac{1}{3}x + 2$, so find two ordered pairs for the function.

If $x = 0$, then the $y$-value is $\frac{1}{3}(0) + 2$, or 2.

If $x = 3$, then the $y$-value is $\frac{1}{3}(3) + 2$, or 3.

Use the points (0, 2) and (3, 3) to find the rate of change.

rate of change $= \frac{3 - 2}{3 - 0} = \frac{1}{3}$

Step 2 Find the rate of change for Function 2.

Use the points (0, 2) and (4, 3) to find the rate of change.

rate of change $= \frac{3 - 2}{4 - 0} = \frac{1}{4}$

Step 3 Compare the rates of change.

$\frac{1}{3} > \frac{1}{4}$, so Function 1 has the greater rate of change.

Solution The rate of change for Function 1 is greater than for Function 2.
Example 3
The price of buying \( x \) baseball caps at Sporty’s is shown by the graph below. The advertisement shows the cost of buying the same baseball caps at Hats ‘R Us.

When does it make sense to buy caps at Sporty’s? at Hats ‘R Us?

Strategy
List ordered pairs for each store and compare them.

Step 1
List ordered pairs for each store.

To find the ordered pairs for Hats ‘R Us, use the words in the advertisement.

The cost is $10 for the first cap and $6 for each cap after that.

<table>
<thead>
<tr>
<th>Number of Caps ((x))</th>
<th>Price at Sporty’s ((y))</th>
<th>Price at Hats ‘R Us ((y))</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>$7.50</td>
<td>$10.00</td>
</tr>
<tr>
<td>2</td>
<td>$15.00</td>
<td>$10 + $6 = $16.00</td>
</tr>
<tr>
<td>3</td>
<td>$22.50</td>
<td>$10 + $6(2) = $22.00</td>
</tr>
<tr>
<td>4</td>
<td>$30.00</td>
<td>$10 + $6(3) = $28.00</td>
</tr>
</tbody>
</table>
Step 2  Compare the $y$-values, or prices, in the table.

$7.50 < $10.00, so buying 1 cap is cheaper at Sporty’s.
$15.00 < $16.00, so buying 2 caps is also cheaper at Sporty’s.
$22.50 > $22.00, so buying 3 caps is cheaper at Hats ‘R Us.
The price of buying caps continues to be cheaper at Hats ‘R Us for any number of caps over 3.

Solution  If a customer wants to buy 1 or 2 caps, it is cheaper to go to Sporty’s.
If a customer wants 3 or more caps, Hats ‘R Us is the better choice.

Coached Example

Compare the rates of change for the two linear functions represented below.

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-4</td>
<td>-12</td>
</tr>
<tr>
<td>-2</td>
<td>-9</td>
</tr>
<tr>
<td>0</td>
<td>-6</td>
</tr>
<tr>
<td>2</td>
<td>-3</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>$x$</th>
<th>$y$</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1</td>
<td>-2</td>
</tr>
<tr>
<td>-2</td>
<td>-3</td>
</tr>
<tr>
<td>-3</td>
<td>-4</td>
</tr>
<tr>
<td>-4</td>
<td>-5</td>
</tr>
<tr>
<td>-5</td>
<td>-6</td>
</tr>
</tbody>
</table>

Which function has a greater rate of change, or are they the same?

Find the rate of change for Function 1.
Use the points (0, ____ ) and (____, 0).

$\frac{0 - (-12)}{0 - 0} = \frac{12}{0}$

Find the rate of change for Function 2.
Use the points (____, 2) and (____, 5).

$\frac{5 - 2}{5 - \text{____}} = \frac{3}{5}$

Which function has a greater rate of change, or are they the same? ___________________

Function 1 has a rate of change of _____ and Function 2 has a rate of change of _____, so ________________________________.
Congruence Transformations

Getting the Idea

Congruent figures have the same shape and the same size. A rigid transformation is a change in the position of a figure. It does not change the size or the shape of a figure.

Three types of rigid, or congruent, transformations are:

- A translation is a slide of a figure to a new position.
- A rotation is a turn of a figure about a point.
- A reflection is a flip of a figure over a line.

If a figure is translated, rotated, or reflected across a line, the figure and its image are congruent. This means that line segments in the figures are the same length, angles have the same measure, and parallel lines remain parallel.

The symbol for congruent is \( \equiv \). Congruent figures have the following properties:

- The corresponding sides of congruent figures are congruent.
- The corresponding angles of congruent figures are congruent.

Example 1

Triangle \( FGH \) is congruent to triangle \( STU \).

What type of transformation of \( \triangle FGH \) was used to create \( \triangle STU \)?

Strategy

Identify the transformation.

- \( FG \) was flipped over the line to create \( ST \).
- \( GH \) was flipped over the line to create \( TU \).
- \( FH \) was flipped over the line to create \( SU \).

A flip is a reflection.

Solution

Triangle \( FGH \) was reflected over a line to create triangle \( STU \).
When a figure is transformed, it creates an image. On a coordinate grid, the vertices of the original figure are labeled with letters, such as $ABCD$. The vertices of the image are labeled with a prime symbol, ('), such as $A'B'C'D'$.

Use these rules to rotate a figure about the origin.

<table>
<thead>
<tr>
<th>Clockwise Rotations About the Origin</th>
</tr>
</thead>
<tbody>
<tr>
<td>90°: $(x, y)$ → $(y, -x)$</td>
</tr>
<tr>
<td>180°: $(x, y)$ → $(-x, -y)$</td>
</tr>
<tr>
<td>270°: $(x, y)$ → $(-y, x)$</td>
</tr>
</tbody>
</table>

Use these rules to reflection a figure across an axis.

<table>
<thead>
<tr>
<th>Reflections Across an Axis</th>
</tr>
</thead>
<tbody>
<tr>
<td>x-axis: $(x, y)$ → $(x, -y)$</td>
</tr>
<tr>
<td>y-axis: $(x, y)$ → $(-x, y)$</td>
</tr>
</tbody>
</table>

To translate a figure, add if moving right or up and subtract if moving left or down.
Example 2
Rectangle $EFGH$ is translated 5 units right and 6 units down to create rectangle $E'F'G'H'$.

Which pairs of sides are congruent?
Which pairs of sides are parallel?
What are the lengths of the sides?

Strategy  Use the transformation to identify congruent parts.

Step 1
Identify the corresponding sides.
- $EF$ and $E'F'$ are corresponding sides.
- $FG$ and $F'G'$ are corresponding sides.
- $EH$ and $E'H'$ are corresponding sides.
- $HG$ and $H'G'$ are corresponding sides.

Step 2
Identify the congruent sides.
The corresponding sides are congruent.
The opposite sides of a rectangle are congruent.

$EF \cong E'F' \cong HG \cong H'G'$.
$EH \cong E'H' \cong FG \cong F'G'$.

Step 3
Identify the parallel sides.
The opposite sides of a rectangle are parallel.

$EF$ is parallel to $HG$, so $E'F'$ is parallel to $H'G'$.
$EH$ is parallel to $FG$, so $E'H'$ is parallel to $F'G'$.

Step 4
Find the lengths of the sides.
$EF = E'F' = HG = H'G' = 3$ units
$EH = E'H' = FG = F'G' = 4$ units

Solution
$EF \cong E'F' \cong HG \cong H'G'$ and $EH \cong E'H' \cong FG \cong F'G'$.
$EF$ is parallel to $HG$, $E'F'$ is parallel to $H'G'$,
$EH$ is parallel to $FG$, and $E'H'$ is parallel to $F'G'$.
$EF = E'F' = HG = H'G' = 3$ units
$EH = E'H' = FG = F'G' = 4$ units
Example 3
Figure $ABCDE$ is rotated $90^\circ$ counterclockwise about the origin.

Draw the image of the transformation after it has been rotated.

**Strategy**  
Think about the definition of rotation.

**Step 1**  
Describe the rotation.

A $90^\circ$ counterclockwise turn is the same as a $270^\circ$ clockwise turn.

**Step 2**  
Use the rule for a $270^\circ$ rotation: $(x, y) \mapsto (-y, x)$.

Each $x$-value becomes $y$ and the opposite of each $y$-value becomes $x$.

- The vertex at $A(3, 7)$ moves to $A'(−7, 3)$.
- The vertex at $B(4, 5)$ moves to $B'(−5, 4)$.
- The vertex at $C(4, 2)$ moves to $C'(−2, 4)$.
- The vertex at $D(2, 2)$ moves to $D'(−2, 2)$.
- The vertex at $E(2, 5)$ moves to $E'(−5, 2)$.

**Step 3**  
Plot the vertices and draw the image.

**Solution**  
Figure $A'B'C'D'E'$, the image of Figure $ABCDE$ after a $90^\circ$ counterclockwise rotation, is shown in Step 3.
Trapezoid \(QRST\) is rotated \(180^\circ\) counterclockwise about the origin to create trapezoid \(Q'R'S'T'\).

Which sides of the trapezoids are congruent? Which sides are parallel?

Trapezoid \(Q'R'S'T'\) is a rotation of trapezoid \(QRST\).

So, the figures are the same shape and the same size, or ________________.

How many units long is \(QR\)? ______

How many units long is \(Q'R'\)? ______

How many units long is \(ST\)? ______

How many units long is \(S'T'\)? ______

How many units long is \(TQ\)? ______

How many units long is \(T'Q'\)? ______

The corresponding sides of congruent figures are ________________.

\(QR\) corresponds to ________, so \(QR \cong ________\).

\(RS\) corresponds to ________, so \(RS \cong ________\).

\(ST\) corresponds to ________, so \(ST \cong ________\).

\(TQ\) corresponds to ________, so \(TQ \cong ________\).

Which sides of trapezoid \(QRST\) are parallel? ________________

So, which sides of trapezoid \(Q'R'S'T'\) are parallel? ________________

\(QR \cong ________, RS \cong ________, ST \cong ________, and TQ \cong ________.\)

In trapezoid \(QRST\), ________ is parallel to ________.

In trapezoid \(Q'R'S'T'\), ________ is parallel to ________.\)
A dilation is a transformation that changes the size of a figure, but not its shape. A dilation stretches or shrinks a figure. A stretch is called an enlargement. A shrink is called a reduction.

The dilated image is similar to the original figure. This means that the image has sides that are proportional in length to the corresponding sides of the original figure.

The size of a figure after a dilation depends on the scale factor used to create it.

- If the scale factor is greater than 1, the dilation will enlarge the original figure.
- If the scale factor is between 0 and 1, the dilation will shrink the original figure.

When working with a dilation, you need to know the scale factor and where the center of dilation is. If the center of dilation is at the origin, you can multiply the coordinates of each vertex by the scale factor to find the vertices of the dilated figure.

**Example 1**

Rectangle $JKLM$ was dilated to form rectangle $J'K'L'M'$.

The center of dilation was at the origin. What scale factor was used?

**Strategy** Look at the side lengths of the figure and those of its image.
Step 1

Decide if the dilation is an enlargement or a reduction.

Rectangle $J'K'L'M'$ is larger than rectangle $JKLM$.
So, it is an enlargement.

Step 2

Look at the side lengths of one set of corresponding sides.

$JK$ and $J'K'$ are horizontal, so you can determine their lengths by counting units.

$J'K' = 6$ units and $JK = 3$ units.
$J'K'$ is 2 times as long as $JK$. The scale factor is 2.

Step 3

Use the vertices to check your answer.

If you multiply the coordinates of the vertices of the original figure by 2, you should get the coordinates of the vertices of the dilated image.

$J(2, 6) \rightarrow (2 \times 2, 6 \times 2) \rightarrow J'(4, 12)$
$K(5, 6) \rightarrow (5 \times 2, 6 \times 2) \rightarrow K'(10, 12)$
$L(5, 2) \rightarrow (5 \times 2, 2 \times 2) \rightarrow L'(10, 4)$
$M(2, 2) \rightarrow (2 \times 2, 2 \times 2) \rightarrow M'(4, 4)$

Solution

The scale factor for the dilation was 2.

Example 2

Dilate $\triangle FGH$ with the center of dilation at the origin and a scale factor of $\frac{1}{3}$.

Draw the dilated image and identify the coordinates of its vertices.

Strategy

Use the scale factor to identify the coordinates of the vertices of the image.
Lesson 25: Dilations

**Step 1** Multiply the coordinates of each vertex of \( \triangle FGH \) by the scale factor, \( \frac{1}{3} \).

- \( F(3, 9) \rightarrow \left\{ 3 \cdot \frac{1}{3}, 9 \cdot \frac{1}{3} \right\} \rightarrow F'(1, 3) \)
- \( G(9, 12) \rightarrow \left\{ 9 \cdot \frac{1}{3}, 12 \cdot \frac{1}{3} \right\} \rightarrow G'(3, 4) \)
- \( H(6, 6) \rightarrow \left\{ 6 \cdot \frac{1}{3}, 6 \cdot \frac{1}{3} \right\} \rightarrow H'(2, 2) \)

**Step 2** Plot and connect the vertices of the image, \( \triangle F'G'H' \).

Solution The vertices of the dilated image are \( F'(1, 3) \), \( G'(3, 4) \), and \( H'(2, 2) \). Its graph is shown in Step 2.

The center of dilation is not always at the origin.

**Example 3**

Dilate \( \triangle ABC \) with the center of dilation at point \( B \) and a scale factor of 3.

Draw the dilated image and identify the coordinates of its vertices.

**Strategy** Use the scale factor to find the lengths of the vertical and horizontal sides of the dilated image.

**Step 1** Think about how the side lengths of the figure change with a scale factor of 3.

Each side of the dilated image will be 3 times as long as its corresponding side on \( \triangle ABC \).
Step 2  Find the length of side \( \overline{A'B} \).

The length of \( \overline{AB} \) is 2 units.

\[ \overline{A'B} = 2 \times 3 = 6 \text{ units} \]

The center of dilation is at point \( B \), so count up 6 units from point \( B \).

Plot point \( A' \) there, at \((-6, 2)\).

Do not write a prime symbol after the letter \( B \) because the location of point \( B \) is the same for the original triangle and its image.

Step 3  Find the length of side \( \overline{BC'} \).

The length of \( \overline{BC} \) is 4 units.

\[ \overline{BC'} = 4 \times 3 = 12 \text{ units} \]

Count 12 units to the right of point \( B \).

Plot point \( C' \) there, at \((6, -4)\).

Connect the vertices.

Solution  The vertices of the dilated image are \( A'(-6, 2), B(-6, -4), \) and \( C'(6, -4) \).

Its graph is shown in Step 2.

Coached Example

Dilate \( \triangle XYZ \) with the origin as the center of dilation and a scale factor of \( \frac{1}{2} \).

Identify the coordinates of the vertices of the dilated image.

Multiply the coordinates of each vertex of \( \triangle XYZ \) by the scale factor, \( \frac{1}{2} \), to find the vertices of the dilated image, \( \triangle X'Y'Z' \).

\[
\begin{align*}
X(2, 4) & \rightarrow \left[ \frac{1}{2} \times 2, \frac{1}{2} \times 4 \right] \rightarrow X'(1, \underline{\phantom{0}}) \\
Y(\underline{\phantom{0}}, \underline{\phantom{0}}) & \rightarrow \left[ \frac{1}{2} \times \underline{\phantom{0}}, \frac{1}{2} \times \underline{\phantom{0}} \right] \rightarrow Y'(\underline{\phantom{0}}, \underline{\phantom{0}}) \\
Z(\underline{\phantom{0}}, \underline{\phantom{0}}) & \rightarrow \left[ \frac{1}{2} \times \underline{\phantom{0}}, \frac{1}{2} \times \underline{\phantom{0}} \right] \rightarrow Z'(\underline{\phantom{0}}, \underline{\phantom{0}})
\end{align*}
\]

Plot the vertices of \( \triangle X'Y'Z' \) on the grid above. Then connect them.

The dilated image, with vertices \( X'(1, \underline{\phantom{0}}), Y'(\underline{\phantom{0}}, \underline{\phantom{0}}), \) and \( Z'(\underline{\phantom{0}}, \underline{\phantom{0}}) \), is graphed above.
Similar triangles have the same shape but not necessarily the same size. They also have the following properties:

- Their corresponding angles are congruent.
- Their corresponding sides are proportional in length. This means that corresponding sides in similar triangles have the same ratio.

You already know that a dilation results in an image that is similar to the original figure. You can use that fact, and some other knowledge such as the Pythagorean theorem, to help you understand why two triangles are similar.

**Example 1**

Right triangle $ABC$ has vertices $A(1, 4)$, $B(1, 1)$ and $C(5, 1)$. Draw that triangle and dilate it with the origin as the center of dilation and a scale factor of 2. Name the image $\triangle DEF$. Compare the side lengths and angle measures of the triangles. What do they tell you about the triangles?

**Strategy**  
Graph the result of the dilation. Then compare the corresponding parts of the triangles.

**Step 1**
Multiply the coordinates of the vertices of $\triangle ABC$ by the scale factor, 2.

- $A(1, 4) \rightarrow (2, 8)$
- $B(1, 1) \rightarrow (2, 2)$
- $C(5, 1) \rightarrow (10, 2)$

**Step 2**
Draw $\triangle ABC$ and its dilated image.
Plot and connect the coordinates above. Name the image $\triangle DEF$.

$\triangle DEF$ is similar to $\triangle ABC$ because it is the result of a dilation.
Step 3: Compare the vertical and horizontal side lengths of the two triangles.

You can count units to determine the horizontal and vertical side lengths of $\triangle ABC$ and $\triangle DEF$.

$AB = 3$ units, and its corresponding side, $DE$, measures 6 units.

$BC = 4$ units, and its corresponding side, $EF$, measures 8 units.

Step 4: Compare the other side lengths.

Since you know the lengths of two legs of both triangles, you can use the Pythagorean theorem to find the lengths of the hypotenuses.

Hypotenuse of $\triangle ABC$:

$$a^2 + b^2 = c^2$$

$3^2 + 4^2 = c^2$

$$25 = c^2$$

$$5 = c$$

Hypotenuse of $\triangle DEF$:

$$a^2 + b^2 = c^2$$

$6^2 + 8^2 = c^2$

$$100 = c^2$$

$$10 = c$$

So, $AC = 5$ units, and its corresponding side, $DF$, measures 10 units.

Step 5: Compare the corresponding sides.

$$\frac{AB}{DE} = \frac{BC}{EF} = \frac{AC}{DF}$$

$$\frac{3}{6} = \frac{4}{8} = \frac{5}{10}$$

The ratio of each pair of corresponding sides is equivalent to $\frac{1}{2}$.

So, corresponding sides have proportional lengths.

Step 6: Compare the known angle measures.

Angle $E$ measures $90^\circ$ because it is a right angle, just like its corresponding angle, $\angle B$.

Step 7: Use a protractor to compare the other angle measures.

$\angle A$ is congruent to $\angle D$, and $\angle C$ is congruent to $\angle F$.

This means that all corresponding angles of the two triangles are congruent.

Solution: The two triangles are similar because $\triangle DEF$ is the dilated image of $\triangle ABC$. Their corresponding sides have proportional lengths, and their corresponding angles are congruent.
To prove that two triangles are similar, you do not need to show that all three pairs of angles are congruent and all three pairs of sides have proportional lengths. That would take a long time. Example 2 shows a shorter way.

**Example 2**

Right triangle \(NOP\) has vertices \(N(2, 10)\), \(O(14, 10)\) and \(P(14, 15)\). Draw that triangle and determine the coordinates of its image after a dilation with the center of dilation at the origin and a scale factor of \(\frac{1}{2}\). Plot those coordinates and draw two sides of the smaller triangle, named \(\triangle QRS\). Can you prove that the two triangles are similar?

**Strategy** Graph the original triangle and the vertices of its dilation. Then compare corresponding parts of the triangles.

**Step 1** Multiply the coordinates of the vertices of \(\triangle NOP\) by the scale factor, \(\frac{1}{2}\).

\[
N(2, 10) \rightarrow (1, 5) \\
O(14, 10) \rightarrow (7, 5) \\
P(14, 15) \rightarrow (7, 7.5)
\]

**Step 2** Draw \(\triangle NOP\) and plot the coordinates of its dilated image.

Plot the coordinates and connect the points for \(N\), \(O\), and \(P\).

Then, name and connect points \(Q\) and \(R\) and points \(S\) and \(R\) to form two sides of the dilated image.

**Step 3** Compare the sides and angles.

Count units to determine if the side lengths are proportional.

\[
\frac{NO}{QR} = \frac{PO}{SR} \\
\frac{12}{6} = \frac{5}{2.5}
\]

Both ratios above are equivalent to 2.

So, two pairs of corresponding sides have proportional lengths.

The angles between them are congruent because when right triangle \(NOP\) is dilated, its image is a right triangle.

So, \(\angle O = \angle R = 90^\circ\).
Step 4  Decide if it is possible for $QS$ to have a length that is not proportional to $NP$.

There is only one possible length for the third side of $\triangle QRS$.

That length will be proportional to $NP$.

You can prove this using the Pythagorean theorem.

Hypotenuse of $\triangle NOP$:

\[
a^2 + b^2 = c^2
\]

\[
12^2 + 5^2 = c^2
\]

\[
169 = c^2
\]

\[
13 = c
\]

Hypotenuse of $\triangle QRS$:

\[
a^2 + b^2 = c^2
\]

\[
6^2 + 2.5^2 = c^2
\]

\[
42.25 = c^2
\]

\[
6.5 = c
\]

$NP = 13$ units, and its corresponding side, $QS$, measures 6.5 units.

\[
\frac{NP}{QS} = \frac{13}{6.5} = 2
\]

This is the same ratio as the other pairs of corresponding sides.

Solution  When two pairs of corresponding sides have proportional lengths and the corresponding angles between them are congruent, triangles are similar.

Note: Example 2 shows the **SAS (Side-Angle-Side) similarity theorem**.

Another way to prove that two triangles are similar is to prove that two pairs of corresponding angles have the same measure. Remember that the sum of the measures of the angles of a triangle is $180^\circ$. 
Example 3

Triangle $KLM$ is the result of a dilation of $\triangle GHI$.

Compare corresponding angles to prove that these two triangles are similar. How many pairs of corresponding angles do you need to compare in order to prove that the two triangles are similar?

**Strategy**  Use a protractor to compare corresponding angle measures.

**Step 1** Measure and compare angles $G$ and $K$.
Using a protractor, angles $G$ and $K$ each measure $110^\circ$.

**Step 2** Measure and compare angles $I$ and $M$.
Using a protractor, angles $I$ and $M$ each measure $45^\circ$.

**Step 3** Decide if it is necessary to measure the third pair of angles.
It is not necessary because the sum of the measure of the angles in a triangle is $180^\circ$.
So, the measures of $\angle H$ and $\angle L$ must each be equal to:

$$180^\circ - (110^\circ + 45^\circ) = 25^\circ.$$  

$$m\angle H = m\angle L$$

**Step 4** Summarize what you know.
If you determine that two pairs of corresponding angles of two triangles are congruent, you know that all three pairs of angles must be congruent.

**Solution** Yes, the two triangles must be similar. Since two pairs of corresponding angles are congruent, all three pairs of angles must be congruent.

Note: Example 3 shows the **AA (Angle-Angle) similarity theorem**.
Now that you know how to use dilations and coordinate grids to prove that two triangles are similar, you can apply that knowledge to find missing side lengths or missing angle measures in two similar triangles.

**Example 4**

What is the value of \( x \) in the diagram below?

Strategy

Determine if the two triangles are similar. If so, set up and solve a proportion to find the value of \( x \).

**Step 1**

Decide if the two triangles are similar.

If two pairs of corresponding sides have proportional lengths and the angles between them are congruent, then the triangles are similar.

The diagram gives lengths for corresponding sides \( XY \) and \( RS \) and for corresponding sides \( YZ \) and \( ST \).

It also shows that the corresponding angles \( Y \) and \( S \) between these sides are congruent.

\[
\frac{XY}{RS} = \frac{YZ}{ST}
\]

\[
\frac{12}{18} = \frac{16}{24}
\]

Both ratios are equivalent to \( \frac{2}{3} \). So, those sides have proportional lengths.

Triangle \( \triangle XYZ \) is similar to Triangle \( \triangle RST \).

**Step 2**

Set up a proportion that could be used to find the value of \( x \).

\[
\frac{XZ}{RT} = \frac{14}{x}, \text{ so:}
\]

\[
\frac{14}{x} = \frac{2}{3}
\]

**Step 3**

Cross-multiply to solve for \( x \).

\[
14 \cdot 3 = 2x
\]

\[
42 = 2x
\]

Divide both sides by 2.

\[
21 = x
\]

**Solution**

The value of \( x \) is 21 units.
Coached Example

Are the two triangles shown below similar? Explain how you know.

The sum of the measures of the angles in a triangle is ____°.
Use that fact to find the measure of angle $T$ in $\triangle PET$.

$$180 - (50 + ____) = 180 - ____ = _____$$

So, $m\angle T = ____°$.

Angle $E$ corresponds to $\angle ____$. Are they congruent? ____

Angle $T$ corresponds to $\angle ____$. Are they congruent? ____

Since the triangles have two pairs of corresponding angles that are congruent, they must be ___________ triangles.
Interior and Exterior Angles of Triangles

Getting the Idea

The sum of the measures of the angles of a triangle is 180°. The diagram below shows this.

Look at the triangle on the left. If you make three copies of the triangle and rotate them so that the 3 angles are aligned, you can see that the angles form a straight angle, which has a measure of 180°.

Example 1

What is the missing angle measure in the triangle below?

Strategy

Use the sum of the angle measures of a triangle.

Step 1 Find the sum of the two known angle measures.

\[ 110° + 25° = 135° \]

Step 2 Subtract the sum from the total angle measures of a triangle.

\[ 180° - 135° = 45° \]

Solution The missing angle measure is 45°.
Example 2
The angles of a triangle measure $5x + 8$ degrees, $9x + 2$ degrees, and $15x - 4$ degrees.

What is the value of $x$? What are the measures of the angles of the triangle?

**Strategy**  Use the sum of the angle measures of a triangle.

**Step 1** Write an equation.

Write an equation that shows the sum of the angle measures of a triangle equals $180^\circ$.

$$(5x + 8) + (9x + 2) + (15x - 4) = 180$$

**Step 2** Solve the equation.

$$(5x + 8) + (9x + 2) + (15x - 4) = 180$$

$$29x + 6 = 180$$  Combine like terms.

$$29x + 6 - 6 = 180 - 6$$  Subtract 6 from both sides.

$$29x = 174$$

$$\frac{29x}{29} = \frac{174}{29}$$  Divide both sides by 29.

$$x = 6$$

**Step 3** Substitute 6 for $x$ in each expression for the angle measures.

$$5x + 8 = (5 \times 6) + 8 = 30 + 8 = 38^\circ$$

$$9x + 2 = (9 \times 6) + 2 = 54 + 2 = 56^\circ$$

$$15x - 4 = (15 \times 6) - 4 = 90 - 4 = 86^\circ$$

**Step 4** Check that the angle measures total $180^\circ$.

$$38^\circ + 56^\circ + 86^\circ = 94^\circ + 86^\circ = 180^\circ$$  ✓

**Solution**  The value of $x$ is 6. The angle measures of the triangle are $38^\circ$, $56^\circ$, and $86^\circ$. 

121
An interior angle of a polygon is an angle that is on the inside of the polygon and has its vertex formed by two sides of the polygon. An exterior angle of a polygon is an angle formed by a side of the polygon and an extension of an adjacent side.

In the diagram below, \( \angle 1 \) is an interior angle of the triangle and \( \angle 2 \) is an exterior angle of the triangle.

An exterior angle and its corresponding interior angle form a straight angle. So, an exterior angle and the corresponding interior angle are supplementary, or have a sum of 180°.

**Example 3**
An exterior angle of a triangle measures 128°, as shown in the diagram.

What is the measure of the corresponding interior angle of the triangle?

**Strategy** Use the fact that an exterior angle and its corresponding interior angle are supplementary.

**Step 1** Understand the meaning of supplementary angles.
Supplementary angles have a sum of 180°.

**Step 2** Subtract the exterior angle measure from 180°.
\[ 180° - 128° = 52° \]

**Solution** The measure of the corresponding interior angle is 52°.
An angle of a triangle has a measure of $3x + 1$ degrees. The corresponding exterior angle has a measure of $5x - 13$ degrees. What is the value of $x$? What are the measures of the interior angle and the exterior angle?

An interior angle and the corresponding exterior angle of a triangle are ______________.

Supplementary angles add up to ________.

Write an equation that shows the sum of the measures of the given interior and exterior angles equals ________.

____________________________________________________________________________________

Solve the equation for $x$.

The value of $x$ is ________.

To find the measure of the interior angle, substitute the value of $x$ into __________.

Solve.

____________________________________________________________________________________

To find the measure of the exterior angle, substitute the value of $x$ into __________.

Solve.

____________________________________________________________________________________

Check that the sum of the angle measures is ________.

____________________________________________________________________________________

The value of $x$ is ________. The interior angle measures ________°.

The corresponding exterior angle measures ________°.
Parallel Lines and Transversals

Getting the Idea

Parallel lines lie in the same plane and never intersect. They are always the same distance apart. A transversal is a line that intersects two or more lines.

In the diagram below, line \( j \) is parallel to line \( k \) and line \( t \) is a transversal.

Corresponding angles lie on the same side of a transversal and on the same side of each parallel line. One pair of corresponding angles is \( \angle 1 \) and \( \angle 5 \).

Corresponding angles are congruent.

\[
\begin{align*}
\angle 1 &\cong \angle 5 \\
\angle 3 &\cong \angle 7 \\
\angle 2 &\cong \angle 6 \\
\angle 4 &\cong \angle 8
\end{align*}
\]

Interior angles lie between lines \( j \) and \( k \).

The interior angles are \( \angle 3, \angle 4, \angle 5, \) and \( \angle 6 \).

Alternate interior angles lie inside parallel lines and on opposite sides of the transversal. Alternate interior angles are congruent.

\[
\begin{align*}
\angle 4 &\cong \angle 6 \\
\angle 3 &\cong \angle 5
\end{align*}
\]

Exterior angles lie outside lines \( j \) and \( k \).

The exterior angles are \( \angle 1, \angle 2, \angle 7, \) and \( \angle 8 \).

Alternate exterior angles lie outside parallel lines and on opposite sides of the transversal. Alternate exterior angles are congruent.

\[
\begin{align*}
\angle 1 &\cong \angle 7 \\
\angle 2 &\cong \angle 8
\end{align*}
\]
In the diagram below, line $j$ is parallel to line $k$ and line $t$ is a transversal.

The three triangles shown are congruent. The angle measures of the triangles are $45^\circ$, $45^\circ$, and $90^\circ$.

You can use the diagram to find the measures of angles 1–4.

- $m \angle 1 = 90^\circ + 45^\circ = 135^\circ$
- $m \angle 3 = 180^\circ - 135^\circ = 45^\circ$
- $m \angle 4 = 45^\circ$
- $m \angle 2 = 180^\circ - 45^\circ = 135^\circ$

$\angle 1 \cong \angle 2$, and they are alternate interior angles.

$\angle 3 \cong \angle 4$, and they are alternate interior angles.

You can use similar reasoning and supplementary angle relationships to show that the alternate exterior angles are congruent and that the corresponding angles are congruent.

**Example 1**

Lines $c$ and $d$ are parallel, and line $t$ is a transversal.

If $m \angle 1 = 75^\circ$, what is $m \angle 8$?

**Strategy** Use the properties of angle pairs formed by parallel lines and a transversal.
Step 1  Find the corresponding angle to $\angle 1$.

$\angle 1$ and $\angle 5$ are corresponding angles.

Step 2  Find the measure of $\angle 5$.

$\angle 1 \cong \angle 5$, so $m\angle 5 = m\angle 1 = 75^\circ$.

Step 3  Find the relationship between $\angle 5$ and $\angle 8$.

$\angle 5$ and $\angle 8$ are supplementary angles.

Step 4  Find the measure of $\angle 8$.

The sum of the measures of supplementary angles is $180^\circ$.

$m\angle 5 + m\angle 8 = 180$

$75 + m\angle 8 = 180$

$75 - 75 + m\angle 8 = 180 - 75$

$m\angle 8 = 105^\circ$

Solution  The measure of $\angle 8$ is $105^\circ$.

There are other ways to solve Example 1. For example, you can use alternate exterior angles $1$ and $7$ and then the fact that angles $7$ and $8$ are supplementary to find the measure of $\angle 8$.

Example 2

Lines $r$ and $s$ are parallel, and line $a$ is a transversal. The measure of $\angle 4 = 140^\circ$.

What is the measure of $\angle 5$?

Strategy  Use the properties of angle pairs formed by parallel lines and a transversal.

Step 1  Find an angle that is congruent to $\angle 4$.

$\angle 4$ and $\angle 6$ are alternate interior angles.

$\angle 4 \cong \angle 6$

Step 2  Find the measure of $\angle 6$.

$m\angle 6 = m\angle 4 = 140^\circ$
Step 3  Find the relationship between $\angle 5$ and $\angle 6$.

$\angle 5$ and $\angle 6$ are supplementary angles.

Step 4  Use the properties of supplementary angles to find the measure of $\angle 5$.

The sum of the measures of supplementary angles is $180^\circ$.

$\begin{align*}
m\angle 5 + m\angle 6 &= 180 \\
m\angle 5 + 140 &= 180 \\
m\angle 5 + 140 - 140 &= 180 - 140 \\
m\angle 5 &= 40^\circ
\end{align*}$

Solution  The measure of $\angle 5$ is $40^\circ$.

Example 3

Lines $a$ and $b$ are parallel, and line $m$ is a transversal. If $m\angle 1 = 67^\circ$ and $m\angle 7 = (2x + 5)^\circ$, what is the value of $x$?

Strategy  Use the properties of angle pairs formed by parallel lines and a transversal.

Step 1  Look for a relationship between $\angle 1$ and $\angle 7$.

$\angle 1$ and $\angle 7$ are alternate exterior angles.

$\angle 1 \cong \angle 7$

Step 2  Use the properties of alternate exterior angles to set up an equation.

$m\angle 1 = m\angle 7$, so $67 = 2x + 5$.

Step 3  Solve the equation.

$\begin{align*}
67 &= 2x + 5 \\
62 &= 2x & \text{Subtract 5 from both sides.} \\
\frac{62}{2} &= \frac{2x}{2} & \text{Divide both sides by 2.} \\
31 &= x
\end{align*}$

Solution  The value of $x$ is $31$. 
Lines $m$ and $n$ are parallel, and line $t$ is a transversal.

If $m\angle 6 = 130^\circ$, what is $m\angle 3$?

Is $\angle 6$ an interior angle or an exterior angle? ________________

$\angle 6$ and $\angle 2$ are ________________ ________________ angles.

So, $\angle 6$ is ________________ to $\angle 2$.

The measure of $\angle 6$ is $130^\circ$, so the measure of $\angle 2$ is ____________.$^\circ$.

$\angle 2$ and $\angle 3$ are ________________ angles.

The sum of the measures of $\angle 2$ and $\angle 3$ is ________________.$^\circ$.

Find the measure of $\angle 3$.

$\underline{____} - \underline{____} = \underline{____}$

The measure of $\angle 3$ is ______°.
The Pythagorean Theorem

Getting the Idea
The side lengths in a right triangle are related by the **Pythagorean theorem**.
The formula for this theorem allows you to find missing side lengths in right triangles.

The Pythagorean Theorem
In a right triangle, the sum of the squares of the lengths of the legs is equal to the square of the length of the **hypotenuse**.

\[ a^2 + b^2 = c^2 \]

Example 1
Verify the Pythagorean theorem using a model.

**Strategy**
Use graph paper to model the sides of a right triangle.

**Step 1**
Cut out three squares with sides 3, 4, and 5 units long. Make a right triangle with the squares.

Find the area of each square.
The formula for the area of a square is: \( A = s^2 \), where \( s \) is the length of a side.
The area of a square with sides 3 units long is \( 3^2 = 9 \text{ units}^2 \).
The area of a square with sides 4 units long is \( 4^2 = 16 \text{ units}^2 \).
The area of a square with sides 5 units long is \( 5^2 = 25 \text{ units}^2 \).
Step 3  Show how the areas of those squares are related.

\[(3 \text{ units})^2 + (4 \text{ units})^2 = (5 \text{ units})^2\]
\[9 \text{ units}^2 + 16 \text{ units}^2 = 25 \text{ units}^2\]

The sum of the areas of the two smaller squares is equal to the area of the largest square.

So, the sum of the squares of the two smaller side lengths is equal to the square of the largest side length. That is the Pythagorean theorem.

Solution  Steps 1–3 verify the Pythagorean theorem.

Below is another proof for this theorem.

A right triangle with legs \(a\) and \(b\) and hypotenuse \(c\) is shown on the right. Its area, in square units, is \(\frac{1}{2}ab\).

Now, imagine two congruent squares, each with sides \((a + b)\) units long. See Figures 1 and 2 below.

Figure 1 is divided into 4 congruent right triangles and two shaded squares. One shaded square has sides \(a\) units long, so its area is \(a^2\). The other shaded square has sides \(b\) units long, so its area is \(b^2\).

Figure 2 is divided into 4 congruent right triangles and one shaded square with sides \(c\) units long. So, the area of that shaded square is \(c^2\).

\[
\text{Figure 1} \\
(a + b)^2 = a^2 + b^2 + (4 \times \frac{1}{2} ab) - (4 \times \frac{1}{2} ab) \\
a^2 + b^2 = c^2
\]

\[
\text{Figure 2} \\
(a + b)^2 = a^2 + b^2 + (4 \times \frac{1}{2} ab) - (4 \times \frac{1}{2} ab) \\
c^2 = c^2
\]

Both figures have the same area. Subtract \((4 \times \frac{1}{2} ab)\) from both sides.

This is the formula for the Pythagorean Theorem

Each large square contains four congruent white triangles. Since the large squares are congruent and the subtracted area of the white triangles is also congruent, then the remaining, shaded areas are congruent as well. The sum of the shaded areas in Figure 1 is equal to the shaded area in Figure 2.
Example 2
What is the length of $b$ in this right triangle?

![Right triangle with sides 15, b, and 25]

**Strategy**
Use the formula for the Pythagorean theorem.

**Step 1**
Identify the legs and the hypotenuse.

The legs have lengths of 15 units and $b$ units. The hypotenuse is 25 units long.

**Step 2**
Use the formula.

Substitute 15 for $a$ and 25 for $c$ into the formula. Solve for $b$.

\[ a^2 + b^2 = c^2 \]
\[ 15^2 + b^2 = 25^2 \]
\[ 225 + b^2 = 625 \]
\[ b^2 = 400 \]
\[ b = \sqrt{400} \]
\[ b = 20 \]

**Solution**
The length of $b$ is 20 units.

You can use the Pythagorean theorem to determine if a triangle is a right triangle.

**Converse of the Pythagorean Theorem**
If a triangle has sides of length $a$, $b$, and $c$ such that $a^2 + b^2 = c^2$, then the triangle is a right triangle with a right angle opposite side $c$. 
Example 3
Triangle $ABC$ has side lengths of 5 centimeters, 8 centimeters, and 10 centimeters.

Is $\triangle ABC$ a right triangle?

**Strategy** Use the converse of the Pythagorean theorem.

**Step 1** Identify the shorter sides and the longest side.
The shorter sides are 5 cm and 8 cm long. Let $a = 5$ and $b = 8$.
The longest side is 10 cm long. Let $c = 10$.

**Step 2** Test the values in the equation.

\[
a^2 + b^2 = c^2\\
5^2 + 8^2 \neq 10^2\\
25 + 64 \neq 100\\
89 \neq 100
\]

So, $\triangle ABC$ is not a right triangle.

**Solution** $\triangle ABC$ is **not** a right triangle.

---

Coached Example

Triangle $XYZ$ has side lengths of 7 inches, 24 inches, and 25 inches.

Is $\triangle XYZ$ a right triangle?

Identify the shorter sides and the longest side.
The shorter sides are 7 in. and ____ in. long. Let $a = 7$ and $b = ____$.
The longest side is ____ in. long. Let $c = ____$.

Substitute those values into the formula.

\[
a^2 + b^2 = c^2\\
7^2 + (____)^2 \neq (____)^2\\
49 + ____ \neq ____\\
____ = ____
\]

Do those values make the equation true? _______

Triangle $XYZ$ ____ a right triangle.
Distance

Getting the Idea

Sometimes, you may need to find distances between two points \((x_1, y_1)\) and \((x_2, y_2)\) on a coordinate grid.

- A horizontal distance is equal to the \textbf{absolute value} of the difference of the \(x\)-coordinates of the two points, or \(|x_2 - x_1|\).
- A vertical distance is equal to the absolute value of the difference of the \(y\)-coordinates of the two points, or \(|y_2 - y_1|\).

Absolute value is the distance between a number and zero on the number line. Because of that, absolute values are always positive.

To find a distance that is neither horizontal nor vertical on the coordinate grid, you can apply the Pythagorean theorem.

On the coordinate grid on the right, the length of the hypotenuse, \(d\), can be found as follows:

\[
(\text{leg}^2) + (\text{leg}^2) = (\text{hypotenuse}^2)
\]

\[
(x_2 - x_1)^2 + (y_2 - y_1)^2 = d^2
\]

To find \(d\), you can take the square root of both sides of the equation.

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]

The Distance Formula

The distance \(d\) between two points \((x_1, y_1)\) and \((x_2, y_2)\) is:

\[
d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}
\]
Example 1
What is the length of $\overline{AB}$ on the coordinate grid?

![Coordinate Grid]

**Strategy**
Use the distance formula.

**Step 1**
Identify the coordinates of the endpoints.
The line segment has endpoints $A(3, 2)$ and $B(9, 10)$.
Let $(x_1, y_1) = (3, 2)$ and $(x_2, y_2) = (9, 10)$.

**Step 2**
Substitute the values into the distance formula and solve for $d$.
\[ d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \]
\[ d = \sqrt{(9 - 3)^2 + (10 - 2)^2} \]
\[ d = \sqrt{6^2 + 8^2} \]
\[ d = \sqrt{36 + 64} \]
\[ d = \sqrt{100} \]
\[ d = 10 \]

**Solution**
The length of $\overline{AB}$ is 10 units.

The distance between two points is not always a whole number of units. Sometimes, the distance formula yields an irrational number. If this happens, you can approximate the number of units to the nearest tenth.
Example 2
What is the length of $\overline{CD}$?

**Strategy**
Use the distance formula.

**Step 1**
Identify the coordinates of the endpoints.
- The line segment has endpoints $C(-3, -5)$ and $D(9, 1)$.
- Let $(x_1, y_1) = (-3, -5)$ and $(x_2, y_2) = (9, 1)$.

**Step 2**
Substitute the values into the distance formula and solve for $d$.
\[
\begin{align*}
    d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
    &= \sqrt{(9 - (-3))^2 + (1 - (-5))^2} \\
    &= \sqrt{12^2 + 6^2} \\
    &= \sqrt{144 + 36} \\
    &= \sqrt{180} \\
    &\approx 13.4
\end{align*}
\]

**Solution**
The length of $\overline{CD}$ is $\sqrt{180}$ units, or approximately 13.4 units.

Example 3
What is the length of $\overline{RS}$ with endpoints $R(-6, 2)$ and $S(-2, -3)$?

**Strategy**
Use the distance formula.

Substitute the values into the distance formula and solve for $d$.
- Let $(x_1, y_1) = (-6, 2)$.
- Let $(x_2, y_2) = (-2, -3)$.
\[
\begin{align*}
    d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
    &= \sqrt{(-2 - (-6))^2 + (-3 - 2)^2} \\
    &= \sqrt{4^2 + (-5)^2} \\
    &= \sqrt{16 + 25} \\
    &= \sqrt{41} \\
    &\approx 6.4
\end{align*}
\]

**Solution**
The length of $\overline{RS}$ is $\sqrt{41}$ units, or approximately 6.4 units.
Which is closer to point $J$: point $K$, point $L$, or is each the same distance from point $J$?

Find the horizontal distance between points $J$ and $K$ by subtracting their ___-coordinates and then taking the absolute value.

Let point $J(3, 4)$ be $(x_1, y_1)$.
Let point $K(\text{__}, 4)$ be $(x_2, y_2)$.

Horizontal distance $= |x_2 - x_1| = |\text{__} - 3| = \text{__________}

The distance between points $J$ and $K$ is _____ units.

Use the distance formula to find the distance between points $J$ and $L$.

Let point $J(3, 4)$ be $(x_1, y_1)$.
Let point $L(\text{__}, \text{__})$ be $(x_2, y_2)$.

Substitute the values into the distance formula and solve for $d$.

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
$d = \sqrt{\text{__}^2 + \text{__}^2}$
$d = \text{__________}$
$d = \text{__________}$
$d = \text{__________}$

The distance between points $J$ and $L$ is _____ units.

Which is closer to point $J$: point $K$, point $L$, or is each the same distance from point $J$?

Points $K$ and $L$ are __________________________________________ from point $J$. 

Apply the Pythagorean Theorem

Getting the Idea

Sometimes, you may need to apply the Pythagorean theorem to solve problems involving geometric figures, real-world distances, or other situations. Drawing a diagram may help you realize that the Pythagorean theorem could be the key to solving a particular problem.

Example 1

A rectangular prism is 4 feet long, 3 feet wide, and 12 feet high. What is the length of a diagonal of the prism?

Strategy

Find the diagonal $EG$ of the bottom face. Then find the hypotenuse of right triangle $ECG$.

Step 1

The diagonal of the bottom face is $EG$.

Apply the Pythagorean Theorem to find the length of $EG$.

$$EG^2 = EH^2 + HG^2$$

$$= 3^2 + 4^2$$

$$= 9 + 16$$

$$= 25$$

$$EG = \sqrt{25} = 5$$

Step 2

Look for a right triangle that includes the diagonal.

$EG$, $CG$, and $EC$ form a right triangle. $EC$ is the hypotenuse and the diagonal of the prism.

Step 3

Apply the Pythagorean Theorem to find the length of $EC$.

$$EC^2 = EG^2 + CG^2$$

$$= 5^2 + 12^2$$

$$= 25 + 144$$

$$= 169$$

$$EC = \sqrt{169} = 13$$

Solution

The length of the diagonal of the prism is 13 feet.
Example 2
A team of archaeologists fenced off an ancient ruin they are exploring. They created a grid to represent the area, so they could label the locations of several artifacts.

For example, they found a pot at point \( P \) and a pitcher at point \( R \). If each unit on the grid represents 2 meters, approximately how many meters apart were the pitcher and the pot found?

**Strategy**

**Use the distance formula.**

**Step 1** Identify the coordinates of the points.

- The pot was found at \( P(4, 16) \).
- Let that be \((x_1, y_1)\).
- The pitcher was found at \( R(8, 4) \).
- Let that be \((x_2, y_2)\).

**Step 2** Substitute those values into the distance formula, and solve.

\[
\begin{align*}
  d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\
  d &= \sqrt{(8 - 4)^2 + (4 - 16)^2} \\
  d &= \sqrt{16 + 144} \\
  d &= \sqrt{160} \\
  d &\approx 12.6
\end{align*}
\]

**Solution** The pot and pitcher were found approximately 12.6 meters apart.
Sometimes, you may need to apply more than one piece of knowledge at a time to solve a problem. For example, you may need to use what you know about rectangles and finding distances on a coordinate grid in order to solve a problem.

Example 3
Determine whether or not parallelogram $ABCD$ is a rectangle.

**Strategy** Use what you know about rectangles and the distance formula.

**Step 1**
Think about rectangles.
The diagonals of a rectangle are congruent.

**Step 2**
Find the length of one of the diagonals, $\overline{AC}$.
Let $A(-6, 2)$ be $(x_1, y_1)$ and $C(4, -3)$ be $(x_2, y_2)$.
\[
d = \sqrt{(-3 - 2)^2 + [4 - (-6)]^2}
\]
\[
d = \sqrt{25 + 100}
\]
\[
d = \sqrt{125}
\]

**Step 3**
Find the length of the other diagonal, $\overline{BD}$.
Let $B(-2, 5)$ be $(x_1, y_1)$ and $D(0, -6)$ be $(x_2, y_2)$.
\[
d = \sqrt{(-6 - 5)^2 + [0 - (-2)]^2}
\]
\[
d = \sqrt{121 + 4}
\]
\[
d = \sqrt{125} \quad \text{That’s the same length as } \overline{AC}.
\]

**Solution** Since the diagonals are congruent, parallelogram $ABCD$ must be a rectangle.
Simon leans a 20-foot ladder against the side of his house so that the base of the ladder is 5 feet from the house.

About how high up the side of the house does the ladder reach? Round your answer to the nearest tenth of a foot.

The ladder forms a right triangle with the house, so use the Pythagorean theorem.

The ladder is the ______________ of the triangle, so let $c = \underline{\hspace{2cm}}$.

The distance from the base of the ladder to the house is a leg, so let $a = \underline{\hspace{2cm}}$.

Substitute those values into the formula and solve for $b$.

$$a^2 + b^2 = c^2$$

$$(\underline{\hspace{2cm}})^2 + b^2 = (\underline{\hspace{2cm}})^2$$

$\underline{\hspace{2cm}} + b^2 = \underline{\hspace{2cm}}$

$b^2 = \underline{\hspace{2cm}}$

$b = \underline{\hspace{2cm}}$

$b \approx \underline{\hspace{2cm}}$

The ladder reaches a height of about ______ feet.
The volume of a three-dimensional figure is the number of cubic units that fit inside it.

The table shows several three-dimensional figures and the formulas for calculating their volumes.

<table>
<thead>
<tr>
<th>Formula for Volume, $V$</th>
<th>Diagram</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Cylinder</strong></td>
<td><img src="image" alt="Cylinder Diagram" /></td>
</tr>
<tr>
<td>$V = Bh$, where $B$ represents the area of the base and $h$ represents the height.</td>
<td></td>
</tr>
<tr>
<td>Note: The base is a circle. The formula for finding $A$, the area of a circle, is: $A = \pi r^2$, where $r$ represents the <strong>radius</strong>.</td>
<td></td>
</tr>
<tr>
<td><strong>Cone</strong></td>
<td><img src="image" alt="Cone Diagram" /></td>
</tr>
<tr>
<td>$V = \frac{1}{3}Bh$, where $B$ represents the area of the base and $h$ represents the height.</td>
<td></td>
</tr>
<tr>
<td>Note: The base is a circle.</td>
<td></td>
</tr>
<tr>
<td><strong>Sphere</strong></td>
<td><img src="image" alt="Sphere Diagram" /></td>
</tr>
<tr>
<td>$V = \frac{4}{3}\pi r^3$, where $r$ represents the radius.</td>
<td></td>
</tr>
</tbody>
</table>
Example 1
To the nearest cubic centimeter, what is the volume of the cone on the right? Use 3.14 for $\pi$.

Strategy  Use the formula for the volume of a cone.

Step 1  Find the approximate area of the base.
Let $B$ represent the area of the base.
Substitute 4 for $r$. Use 3.14 for $\pi$.

\[
B = \pi r^2
\]
\[
B \approx 3.14 \times 4^2
\]
\[
B \approx 50.24 \text{ cm}^2
\]

Step 2  Find the volume.
Substitute 50.24 for $B$ and 6 for $h$.

\[
V = \frac{1}{3}Bh
\]
\[
V \approx \frac{1}{3} \times 50.24 \times 6
\]
\[
V \approx 100.48 \text{ cm}^3
\]
100.48 cm$^3$ rounds down to 100 cm$^3$.

Solution  The volume is approximately 100 cubic centimeters.

The area of the base of a cone, $B$, is equal to $\pi r^2$.
So, you can write the formula for finding the volume of a cone as:
\[
V = \frac{1}{3}Bh \quad \text{or} \quad V = \frac{1}{3}\pi r^2h.
\]
Notice that in the second formula, $\pi r^2$ is substituted for $B$.
For the same reason, you can use either of the formulas below to find the volume of a cylinder:
\[
V = Bh \quad \text{or} \quad V = \pi r^2h.
\]
Example 2
The cylindrical can shown on the right has a height of 24 inches and a volume of $864\pi$ cubic inches. What is its radius, $r$?

Strategy **Substitute the known values into the formula for the volume of a cylinder. Solve for $r$.**

**Step 1** Substitute the known values into the formula.

Substitute $864\pi$ for the volume, $V$. Substitute 24 for $h$.

$$V = \pi r^2 h$$

$$864\pi = \pi \times r^2 \times 24$$

$$864\pi = 24\pi r^2$$

**Step 2** Solve for $r$.

$$\frac{864\pi}{\pi} = \frac{24\pi r^2}{\pi}$$

$$864 = 24r^2$$

$$36 = r^2$$

$$6 = r$$

Solution **The radius of the cylinder is 6 inches.**

Sometimes, in a diagram, the **diameter** of a figure is given instead of the radius. The radius of a figure is equal to half its diameter.

Example 3
A museum wants to build a small planetarium. The dome for the planetarium will be a hemisphere with a diameter of 36 feet.

What will be the approximate volume of the dome to the nearest cubic foot? Use 3.14 for $\pi$.

Strategy **Use the formula for the volume of a sphere to find the volume of the hemisphere.**

**Step 1** Determine the radius.

The radius, $r$, is half the diameter, $d$, and $d = 36$ feet.

$$r = \frac{d}{2} = \frac{36}{2} = 18 \text{ ft}$$

**Step 2** Decide what a hemisphere is.

A hemisphere is a half-sphere.

So, multiply the formula for finding the volume of a sphere by $\frac{1}{2}$. 
Step 3  
Substitute the known values into the formula.  
Substitute 18 for \( r \). Use 3.14 for \( \pi \).  
\[ V = \frac{1}{2} \times \frac{4}{3} \pi r^3 \]  
\[ V \approx \frac{1}{2} \times \frac{4}{3} \times 3.14 \times 18^3 \]

Step 4  
Calculate the approximate volume.  
\[ V \approx \frac{1}{2} \times \frac{4}{3} \times 3.14 \times 18^3 \]  
\[ V \approx \frac{1}{2} \times \frac{4}{3} \times 3.14 \times 5,832 \]  
\[ V \approx 12,208.32 \]  
12,208.32 ft\(^3\) rounds down to 12,208 ft\(^3\).

Solution  
The dome will have a volume of approximately 12,208 cubic feet.

Coached Example  
To the nearest tenth of a cubic centimeter, what is the volume of the soup can below?

![Soup Can Diagram]

The diameter, \( d \), is _____ cm. Calculate the radius, \( r \).  
\[ r = \frac{d}{2} = \text{______________} \]  
Substitute _____ for the height, \( h \), and _____ for the radius, \( r \), into the formula.  
Use 3.14 for \( \pi \) and calculate the volume.  
\[ V = \pi r^2 h \]  
\[ V \approx 3.14 \times (\______)^2 \times \______ \]  
\[ V \approx 3.14 \times \______ \times \______ \]  
Evaluate the exponent.  
\[ V \approx \text{__________} \]  
Multiply.  
That number rounded to the nearest tenth is _______.  
The volume of the can is approximately _____ cubic centimeters.
Getting the Idea

A scatter plot is a graph in which ordered pairs of data are plotted. You can use a scatter plot to determine if a relationship, or an association, exists between two sets of data. There are different kinds of associations.

- **Positive association**
  - The points slant up from left to right, as if on a line. So, this is an example of a linear association. As the $x$-values increase, the $y$-values also tend to increase.

- **Negative association**
  - The points slant down from left to right, as if on a line. So, this is another example of a linear association. As the $x$-values increase, the $y$-values tend to decrease.

- **No association**
  - The ordered pairs look randomly scattered. The plot shows no relationship between the $x$- and $y$-values.

- **Nonlinear association**
  - The ordered pairs are related, but do not resemble a straight line. For example, this plot shows that as the $x$-values increase, the $y$-values increase at first and then decrease.

An association does not have to be true for every pair of values in a scatter plot. It should be true for most of the data points. Look at how the data cluster together to help you decide.
Example 1
The scatter plot shows the number of hours that students studied for a test and their scores on the test. Does the scatter plot show an association between the number of hours studied and test scores? If so, describe the association.

Strategy  Determine if the data points look related.

Step 1  Decide whether the data points look like a straight line, a curve, or look randomly scattered.

The data points resemble a straight line that slants up from left to right.
So, the data show a positive, linear association.

Step 2  Describe and interpret the association.

The $x$-axis shows hours of studying, and the $y$-axis shows test scores.
The plot shows that as $x$-values increase, $y$-values tend to increase.
In general, the more hours students studied, the higher their test scores.

Solution  The scatter plot shows a positive, linear association. The data indicate that, in general, the more hours students studied, the higher their test scores.

The scatter plot has a break from 0 to 65 on the $y$-axis. This indicates that there are no data from 0 to 65 on the $y$-axis.
Example 2

The data below show the average quarterly stock price for a company during its first three years in business.

<table>
<thead>
<tr>
<th>Quarter</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>9</th>
<th>10</th>
<th>11</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>Price per Share</td>
<td>$2</td>
<td>$4</td>
<td>$8</td>
<td>$11</td>
<td>$13</td>
<td>$14</td>
<td>$13</td>
<td>$12</td>
<td>$9</td>
<td>$8</td>
<td>$5</td>
<td>$1</td>
</tr>
</tbody>
</table>

Create a scatter plot for these data and identify the association, if any.

**Strategy** Choose a scale. Then title and label a grid. Plot each point on the grid. Identify the association, if any.

**Step 1** Choose a title and axis labels for the grid.
- Give the graph the same title as the table.
- There are four quarters in one year. So, a quarter is a unit of time.
- The other variable is price per share.
- The quarter cannot be affected by the price per share, but the price can change each quarter.
- So, the quarter is the independent variable. Label the $x$-axis “Quarter.”
- The price per share is the dependent variable. Label the $y$-axis “Price Per Share.”

**Step 2** Choose a range and scale for each axis.
- Since the greatest $x$-value in the table is 12, use 0 to 12 with intervals of 1.
- The greatest $y$-value in the table is 14, so a scale of 0 to 16, with intervals of 2, is a good scale.

**Step 3** Plot each data point on the grid.
Step 4  Analyze the data for an association.

The data points resemble a U-shaped curve.

Since the data points resemble a curve instead of a straight line, this shows a nonlinear association.

Solution  The scatter plot is shown in Step 3. It shows a nonlinear association.

When you look at a set of data on a scatter plot, you may notice that some data points do not give a good indication of the association shown by almost all the other data points. An outlier is a data point with values that are significantly different from the other data points in the set. It is often helpful to ignore an outlier when determining the association shown by a scatter plot.

Example 3

The manager of an outdoor theater space wanted to find out if ticket prices affect the number of people who attend a concert. The scatter plot below shows the data she collected.

What association is shown by the scatter plot?

Strategy  Determine how most of the points are related. Identify and exclude any outliers.

Step 1  Decide whether the data points look like a straight line, a curve, or look randomly scattered.

Most of the data points resemble a straight line that slants down from left to right.

Only the outlier at (20, 260) does not seem to fit near such a line. Exclude that point.

The scatter plot shows a negative association.
Step 2  Describe and interpret the association.

The $x$-axis shows ticket prices and the $y$-axis shows concert attendance. The plot shows that, in general, as ticket prices increase, concert attendance decreases. The outlier at (20, 260) is an exception. It shows that one concert that had $20 tickets still had high attendance. Perhaps that was just a very popular concert.

Solution  The scatter plot shows a negative association. The outlier at (20, 260) was excluded because it did not seem to fit with the other data.

Coached Example

The scatter plot shows the heights, in inches, of a class of students and their scores on a test. What type of association, if any, is shown by the scatter plot?

Do most of the data points resemble a straight line? ______
Do most of the data points resemble a curve? ______
Do the data points appear to be randomly scattered? ______

Since the data points appear ________________, this scatter plot shows that there is __________ association between students’ heights and their test scores.
Trend Lines

Getting the Idea

If there is a linear association between the data on a scatter plot, you can draw a line of best fit to show the general trend of the data. This line is also called a trend line. There is usually no line that will fit every data point exactly, but the line should be as close to as many of the points on the scatter plot as possible, with about as many points above the line as below it and including at least a few points on the line.

Example 1

The scatter plot shows the heights and weights of players on a basketball team. Draw a line of best fit for these data and discuss how well the line you drew models the trend of the data.

Strategy

Draw a line of best fit. Then describe the general trend.

Step 1

Draw a line of best fit to show the general trend of the data.

Try to draw a line that has about as many points above it as below it.

Step 2

Analyze your line of best fit.

The line shows a positive association. So, the taller a player is, the heavier his weight is.

The line includes two of the data points and has three points above it and four points below it. The points that do not lie on the line are not very close to the line.

So, it is a decent model, but not a great model, for these data.

Solution

The line of best fit drawn in Step 2 shows the general trend that, the taller the player, the greater the weight.
Drawing a line of best fit also helps you make predictions based on the scatter plot.

**Example 2**
The scatter plot below shows the ages of 16 cars listed for sale online and their selling prices.

<table>
<thead>
<tr>
<th>Age of Car and Selling Price</th>
<th>Age (in years)</th>
<th>Selling Price (in dollars)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0</td>
<td>36,000</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>32,000</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>28,000</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>24,000</td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>20,000</td>
</tr>
<tr>
<td></td>
<td>5</td>
<td>16,000</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>12,000</td>
</tr>
<tr>
<td></td>
<td>7</td>
<td>8,000</td>
</tr>
<tr>
<td></td>
<td>8</td>
<td>4,000</td>
</tr>
<tr>
<td></td>
<td>9</td>
<td>0</td>
</tr>
</tbody>
</table>

Suppose that someone listed a car for sale that is 4 years old. Make a prediction about the selling price.

**Strategy** Draw a line of best fit. Then use the line to estimate the selling price for a 4-year-old car.

**Step 1** Draw a line of best fit to show the general trend of the data.

**Step 2** Use the line to predict the selling price for a 4-year-old car.

Find 4 on the $x$-axis. Follow it up to the line.

The point $(4, 16,000)$ is on the line. So, a good prediction is $16,000.

**Solution** According to the line of best fit, a 4-year-old car would have a selling price of about $16,000.
Coached Example

The scatter plot below shows the relationship between the high temperature in degrees Fahrenheit (°F) and the elevation for 16 towns within a degree of latitude on the same day. Predict the high temperature for a town that has an elevation of 350 feet.

On the scatter plot above, draw a line of best fit to show the general trend of the data. Try to draw a line that has about as many points above it as below it.

Find 350 on the y-axis. Move your finger to the right until you reach the line of best fit.

What temperature corresponds to 350 feet? ________

I predict the temperature will be about ________ °F at an elevation of 350 feet.
Interpret Linear Models

Getting the Idea
A linear function can be represented in different ways—in a table, by a graph, or as an equation. You can also use different representations to show the trend in a set of data. You could draw a scatter plot and a line of best fit, or you could use an equation or other representation.

Example 1
Elena bought small potted plants and transferred them to her garden. She collected data last year and discovered that turning on the sprinkler for a short time each night was associated with increased plant growth. The equation \( y = 0.5x \) shows \( y \), the increase in plant growth in centimeters, if she turns on the sprinkler for \( x \) nights.

What is the increase in plant growth each night that she turns on the sprinkler? What is the increase if she turns on the sprinkler each night for a week? for 2 weeks?

**Strategy**
Interpret the equation. Then use it to calculate the increases in plant growth over time.

**Step 1**
What is the increase in plant growth each night?

Since \( x \) stands for the number of nights, substitute 1 for \( x \) in the equation.

\[ y = 0.5x = 0.5(1) = 0.5 \]

Turning on the sprinkler is associated with a 0.5-centimeter increase in plant growth each night.

**Step 2**
What is the increase in plant growth if Elena turns on the sprinkler for 1 or 2 weeks?

One week = 7 days, so \( y = 0.5(7) = 3.5 \) cm.

Two weeks = 14 days, so \( y = 0.5(14) = 7 \) cm.

**Solution**
Turning on the sprinkler is associated with an additional 0.5 centimeter of daily plant growth. If Elena turns on the sprinkler each night for 1 week, her plants will grow an additional 3.5 centimeters. They will grow 7 additional centimeters if she does so for 2 weeks.
The equation $y = 0.5x$ in Example 1 is a direct proportion in the form $y = mx$. The value of $m$, 0.5, represents the increase in plant growth per day.

When a linear model is used to represent what has been learned from collected pairs of data, you can look at the slope and intercepts to help you understand the data. You know already that the $y$-intercept is the point at which the graph crosses the $y$-axis. The $x$-intercept may also be important. It shows the point at which the graph crosses the $x$-axis.

**Example 2**

Last year, when Dayshawn bought a new computer, he collected pairs of data in a scatter plot to help him understand how the value of a computer changes over time. He used the scatter plot on the left to create the linear model on the right.

Identify the slope and intercepts of Dayshawn’s graph. What do they represent in this problem?

**Strategy**

Identify the slope and intercepts and what they represent in the problem.

**Step 1**

Identify the slope and what it represents.

Use the points (0, 800) and (5, 0).

$$m = \frac{0 - 800}{5 - 0}$$

$$m = -\frac{800}{5}$$

$$m = -160$$

The slope compares value in dollars to age in years. So, it shows that a computer’s value decreases by approximately $160 per year.
Step 2 Identify the intercepts and what they represent.

The y-intercept at (0, 800) shows an initial computer value of $800.

The x-intercept at (0, 5) shows that after 5 years a computer is not worth much, if anything. This type of information could help Dayshawn decide whether to upgrade a current computer or buy a new one.

Solution The slope shows that the age of a computer is associated with a decrease of about $160 per year in value. The intercepts show an initial computer value of $800 and that after 5 years, a computer will not have much, if any, value.

Coached Example

Raquel determines that the number of points she scores in a basketball game is related to the number of minutes she plays in a game. The table below represents several ordered pairs in her linear model.

<table>
<thead>
<tr>
<th>Raquel's Basketball Games</th>
</tr>
</thead>
<tbody>
<tr>
<td>Minutes Played (x)</td>
</tr>
<tr>
<td></td>
</tr>
<tr>
<td>Points Scored (y)</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

What is the slope, and what does it represent in the problem?

Find the slope. Use the points (____, 3) and (10, 6).

\[ m = \frac{6 - 3}{10 - ____} \]

\[ m = _____ \]

The slope compares points scored to minutes played.

So, her model shows that for each additional _____ minutes she plays, she tends to score _____ additional points.
Patterns in Data

Getting the Idea

The frequency of a piece of data is the number of times it appears in a data set. A frequency distribution is a way of grouping data so that meaningful patterns can be found. A frequency distribution table is used to show the total for each category or group.

Example 1

A sporting goods store recorded the number of tents sold each week in February and used the data to make a frequency table.

<table>
<thead>
<tr>
<th>Week</th>
<th>Tallies</th>
<th>Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>####</td>
<td>5</td>
</tr>
<tr>
<td>2</td>
<td>#######</td>
<td>12</td>
</tr>
<tr>
<td>3</td>
<td>####</td>
<td>7</td>
</tr>
<tr>
<td>4</td>
<td>#######</td>
<td>10</td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td>34</td>
</tr>
</tbody>
</table>

What percentage of all the tents sold in February were sold during the first two weeks of the month?

Strategy Use the frequency distribution table.

Step 1 Look at the frequency column for Weeks 1 and 2.

5 tents were sold during Week 1.
12 tents were sold during Week 2.

So, 5 + 12, or 17, tents were sold during that two-week period.

Step 2 Find the percentage.

The table shows that a total of 34 tents were sold in February.

$$\frac{17}{34} = 0.5 = 0.5 \times 100\% = 50\%$$

Solution Exactly 50% of the tents sold in February were sold during the first two weeks of the month.

In Example 1, you calculated the percentage of tents that were sold during a certain time period. Sometimes, it is helpful to show percentages in a frequency table by including a column for relative frequency. The relative frequency of a given category is found by dividing the frequency of that category by the sum of all the frequencies.
Example 2
The number of computers owned by the family of each student in Ms. Fontana’s class is shown below.

0, 2, 1, 3, 2, 4, 0, 2, 2, 3, 1, 2, 3, 2, 0, 1, 4, 2, 3, 5, 2, 1, 5, 0

Here is how that information looks in a partially-completed frequency distribution table.

<table>
<thead>
<tr>
<th>Computers Owned</th>
<th>Frequency</th>
<th>Relative Frequency</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>4</td>
<td>16%</td>
</tr>
<tr>
<td>1</td>
<td>4</td>
<td>16%</td>
</tr>
<tr>
<td>2</td>
<td>8</td>
<td>32%</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>2</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>25</strong></td>
<td><strong>100%</strong></td>
</tr>
</tbody>
</table>

Complete the last column of the table. Check that the relative frequencies total 100%.

**Strategy**  
Find the relative frequencies that are missing from the table.

**Step 1**  
Find the relative frequency for the “3 computers” row.

Five of 25 students’ families have 3 computers.

$$\frac{5}{25} = 0.2 = 0.2 \cdot 100\% = 20\%$$

**Step 2**  
Find the relative frequency for the other two rows.

Since both rows show frequencies of 2, their relative frequencies will be the same.

$$\frac{2}{25} = 0.08 = 0.08 \cdot 100\% = 8\%$$

**Step 3**  
Be sure that all the relative frequencies add up to 100%.

$$16\% + 16\% + 32\% + 20\% + 8\% + 8\% = 100\% \quad \checkmark$$

**Solution**  
The row for “3 computers” should show 20% as the relative frequency. The rows for “4 computers” and “5 computers” should each show 8% as the relative frequency.

Sometimes, you may want to look at two variables at the same time and determine if there is a relationship between them. One way to do that is to create a **two-way table**. You can enter either frequency counts or relative frequencies in the cells of the table.
Example 3
Ten students in a class were asked two questions. They were asked to tell if they do chores at home or not. They were then asked if they receive an allowance or not. The results are shown below.

### Student Survey

<table>
<thead>
<tr>
<th>Student</th>
<th>Abby</th>
<th>Bella</th>
<th>Chris</th>
<th>Deb</th>
<th>Erin</th>
<th>Frank</th>
<th>Gus</th>
<th>Hal</th>
<th>Isadore</th>
<th>John</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chores</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
</tr>
<tr>
<td>Allowance</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>No</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
<td>Yes</td>
<td>No</td>
</tr>
</tbody>
</table>

Create a two-way table to show the frequency counts for these data.

**Strategy**
Determine how the table will look. Then fill in the frequencies.

**Step 1**
Determine how the table will look.
- Include a column for “allowance” and a column for “no allowance” along the top.
- Include a row for “chores” and a row for “no chores” along the left side.

**Step 2**
Decide how to fill in the first row of cells.
- Four students (Abby, Bella, Frank, and Gus) do chores and get an allowance. The first cell shows “chores” and “allowance.” Record 4 in the first cell.
- Two students (Hal and John) do chores and get no allowance. Record 2 in the second cell.
- Find and record the total for that row: 4 + 2 = 6.

<table>
<thead>
<tr>
<th></th>
<th>Allowance</th>
<th>No Allowance</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chores</td>
<td>4</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>No Chores</td>
<td>1</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Total</td>
<td>5</td>
<td>5</td>
<td>10</td>
</tr>
</tbody>
</table>

**Step 3**
Complete the second row in the table. Then add the columns and record those totals.

<table>
<thead>
<tr>
<th></th>
<th>Allowance</th>
<th>No Allowance</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chores</td>
<td>4</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>No Chores</td>
<td>1</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Total</td>
<td>5</td>
<td>5</td>
<td>10</td>
</tr>
</tbody>
</table>

**Solution**
The two-way table in Step 3 organizes the data.
You can also use a two-way table to display relative frequencies instead of actual frequencies. You can show the relative frequencies for each row or for each column.

**Example 4**
Look back at the two-way table in Example 3. Can you conclude that students who get an allowance are more likely to do chores than students who do not? Find the relative frequencies for the columns in the table and see.

<table>
<thead>
<tr>
<th></th>
<th>Allowance</th>
<th>No Allowance</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chores</td>
<td>4</td>
<td>2</td>
<td>6</td>
</tr>
<tr>
<td>No Chores</td>
<td>1</td>
<td>3</td>
<td>4</td>
</tr>
<tr>
<td>Total</td>
<td>5</td>
<td>5</td>
<td>10</td>
</tr>
</tbody>
</table>

**Strategy**  
Recreate the two-way table, showing the relative frequencies for the columns.

**Step 1**  
Find the relative frequencies.

Of all students who get an allowance, \( \frac{4}{5} \), or 80%, also do chores, while \( \frac{1}{5} \), or 20%, do not.

Of all students who do not get an allowance, \( \frac{2}{5} \), or 40%, also do chores, while \( \frac{3}{5} \), or 60%, do not.

<table>
<thead>
<tr>
<th></th>
<th>Allowance</th>
<th>No Allowance</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chores</td>
<td>80%</td>
<td>40%</td>
<td>60%</td>
</tr>
<tr>
<td>No Chores</td>
<td>20%</td>
<td>60%</td>
<td>40%</td>
</tr>
<tr>
<td>Total</td>
<td>100%</td>
<td>100%</td>
<td>100%</td>
</tr>
</tbody>
</table>

**Step 2**  
What conclusions can you draw?

The data do show that students who do chores are more likely to get an allowance than students who do not do chores, because 80% of the students who get an allowance also do chores.

**Solution**  
The data indicate a positive association between getting an allowance and doing chores.
Coached Example

The P.E. teachers want to offer students a choice of several electives: yoga, flag football, or ultimate Frisbee. They wanted to see if the boys and girls had different first choices. So, they surveyed 200 students and made a two-way table that showed the relative frequencies for the rows, as shown below.

<table>
<thead>
<tr>
<th></th>
<th>Yoga</th>
<th>Flag Football</th>
<th>Frisbee</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>Boys</td>
<td>0.05</td>
<td>0.65</td>
<td>0.30</td>
<td>1.00</td>
</tr>
<tr>
<td>Girls</td>
<td>0.60</td>
<td>0.05</td>
<td>0.35</td>
<td>1.00</td>
</tr>
<tr>
<td>Total</td>
<td>0.325</td>
<td>0.35</td>
<td>0.325</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Is there any P.E. elective that seems to be about as popular among both boys and girls?

Since the table shows the relative frequencies for each row, you can look at the relationship between gender and P.E. elective preferences.

The probability that a girl will prefer yoga is ________, while the probability that a boy will prefer yoga is ________. So, yoga is more popular among the ________.

Similarly, ________ are more likely to prefer flag football.

The probability that a boy will prefer Frisbee is ________, and the probability that a girl will prefer Frisbee is ________.

What does that tell you about that elective choice?

Of all the electives, __________________ seems to be similarly popular among both boys and girls.
Lesson 1
Coached Example
Starting at the tick mark for 1, you count 5 tick marks from 1 to point P.
So, point P represents the mixed number $1 \frac{5}{6}$.
Instead of converting the mixed number to an improper fraction, just convert the fractional part, $\frac{5}{6}$, to a decimal.
Divide the numerator, 5, by the denominator, 6.

1. 5
---
6) 5.00
- 4
---
10
- 8
---
2

If the decimal repeats, write it with a bar over the repeating digit: $0.8\overline{3}$
Add 1 to the decimal: $1 + 0.8\overline{3} = 1.8\overline{3}$
The decimal represented by point P is $1.8\overline{3}$.

Lesson 2
Coached Example
So, the length of one side, s, can be found by taking the square root of $s^2$ or $A$.
The exact length of each side of the square is $\sqrt{67}$ meters.
67 lies between the perfect squares 64 and 81.
$\sqrt{64} = 8$, and the square root of the other perfect square is 9.
So, $\sqrt{67}$ lies between the whole numbers 8 and 9, but is closer to 8.

$8.1^2 = 8.1 \cdot 8.1 = 65.61 \rightarrow$ close, but less than 67.
$8.2^2 = 8.2 \cdot 8.2 = 67.24 \rightarrow$ close, and more than 67.
Which is closer to 67: $8.1^2$ or $8.2^2$? $8.2^2$
So, $\sqrt{67}$ is between 8.1 and 8.2, but is closer to 8.2.

$\sqrt{67}$

The exact length of one side of the square is $\sqrt{67}$ meters.

Lesson 3
Coached Example
The negative numbers are: $-\frac{1}{9}$ and $-0.8$.
$-\frac{1}{9} = -1 \div 9 = -0.11\ldots = -0.\overline{1}$
On a number line, $-0.8$ is farther to the left than $-0.\overline{1}$.
So, $-0.8 < -\frac{1}{9}$.
The positive numbers are: $\sqrt{5}$ and 3.5.
$2 < \sqrt{5} < 3$.
Since the value of $\sqrt{5}$ is less than 3, 3.5 must be greater than $\sqrt{5}$.
From least to greatest, the order is: $-0.8$, $-\frac{1}{9}$, $\sqrt{5}$, 3.5.

Lesson 4
Coached Example

$\pi = 3.14$
Possible work:

1.57
2/3.14
- 2
---
11
- 10
---
14
- 14
---
0

The value of $\frac{\pi}{2}$ is approximately 1.57 and is represented on the number line above.

Lesson 5
Coached Example
To raise a power to a power, you must multiply the exponents.
$(10^3)^3 = 10^{2 \cdot 3} = 10^6$
Use 10 as a factor 6 times.
$10 \times 10 \times 10 \times 10 \times 10 \times 10 = 1,000,000$
$(10^3)^3 = 1,000,000$
Lesson 6
Coached Example
The area of the garden above is 121 square yards.
To find the length of one side, take the square root of that area.
On the lines below, try squaring numbers until you find one that results in 121.
Possible work:
\[10^2 = 10 \times 10 = 100 \text{ Too low}\]
\[11^2 = 11 \times 11 = 121 \checkmark\]
The length of each side, \(s\), of the garden is 11 yards.

Lesson 7
Coached Example
Since the exponent is positive, this is a number greater than 1.
The exponent of the second factor is 7.
The exponent tells you to move the decimal point in 9.4 seven places to the right.
The number 9.4 \(10^7\) in standard form is 94,000,000.
About 94,000,000 passengers passed through the Hartsfield-Jackson Atlanta International Airport in 2013.

Lesson 8
Coached Example
The decimal point was moved 6 places to the right.
The original number is less than 1, so the exponent will be negative.
\[0.000007 = 7 \times 10^{-6}\]
Multiply to convert that number of square meters to square millimeters: \((7 \times 10^{-6})(1 \times 10^6)\)
Multiply the first factors: \(7 \times 1 = 7\)
Multiply the power-of-10 factors:
\[10^{-6} \times 10^6 = 10^{-6+6} = 10^0 = 1\]
The area is \(7 \times 10^0\) or 7 square millimeters.
It is better to measure the area in square millimeters because it is better to measure a small area using a smaller unit.

Lesson 9
Coached Example
\[n + 2 = \left(\frac{1}{3} \cdot \frac{32}{1}\right) + \left(\frac{1}{3} \cdot \frac{6}{1}\right)\]
\[n + 2 = n + 2\]
\[n + 2 - 2 = n + 2 - 2\]
\[n = n\]
Is the equation above always true, never true, or sometimes true?
The equation is always true, so any value of \(n\) makes the equation true.
Since any value of \(n\) makes the equation true, the equation has infinitely many solution(s).

Lesson 10
Coached Example
One DVD at the regular price of \(d\) dollars
Second DVD at half the regular price of \(d\) dollars
$33 in all
\[d + \frac{d}{2} = 33\]
Rewrite the equation and solve for \(d\).
\[d + \frac{d}{2} = 33\]
\[1d + \frac{1}{2}d = 33\]
\[\frac{2}{2}d + \frac{1}{2}d = 33\]
\[\frac{3}{2}d = 33\]
\[\frac{3}{2}d \cdot \frac{2}{3} = 33 \cdot \frac{2}{3}\]
\[d = 22\]
The regular price of a DVD at the store is $22.

Lesson 11
Coached Example
Let \((x_1, y_1) = (2, 8)\).
Let \((x_2, y_2) = (6, 20)\).
\[m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{20 - 8}{6 - 2} = \frac{12}{4} = 3\]
The \(y\)-axis shows the total cost in dollars.
The \(x\)-axis shows the time in minutes.
So, the slope shows a rate of change of 3 dollars to 1 minute, or $3 per minute.
Does the slope represent the connection fee or the rate per minute for the call? **rate per minute**

The slope is 3. It shows that Joanie must pay $3 per minute for the calls she makes.

**Lesson 12**

**Coached Example**

The $y$-intercept is the point at which the graph crosses the $y$-axis.

The $y$-intercept of this graph is (0, −6). So, $b = −6$.

Choose two points on the graph to find the slope, $m$.

Use the $y$-intercept, (0, −6), and the point (3, 0).

To move from the $y$-intercept to (3, 0), count 6 units up and 3 units to the right.

$$m = \frac{\text{rise}}{\text{run}} = \frac{6}{3} = 2$$

Since the line slants up from left to right, the slope is positive.

Substitute those values of $m$ and $b$ into $y = mx + b$.

$$y = 2x − 6$$

The equation of the line is $y = 2x − 6$

**Lesson 13**

**Coached Example**

$m = \frac{700}{22}$

$700 \cdot 22 = 4 \cdot x$

$\frac{15,400}{4} = 4x$

$\frac{3,850}{x}$

If he continues to drive at the same rate, Mr. Cipriati will drive 3,850 miles in 22 days.

**Lesson 14**

**Coached Example**

For (1, 350): $\frac{y}{x} = \frac{350}{1} = 350$

For (2, 700): $\frac{y}{x} = \frac{700}{2} = 350$

For (3, 1050): $\frac{y}{x} = \frac{1050}{3} = 350$

For (4, 1400): $\frac{y}{x} = \frac{1400}{4} = 350$

The ratio $\frac{y}{x}$ is constant for all ordered pairs, so the table shows a direct proportion.

**Lesson 15**

**Coached Example**

$m = \frac{5}{3}$

Find the slope of the line that passes through (−10, −4) and (−5, −1).

$$m = \frac{-1 - (-4)}{-5 - (-10)} = \frac{-3}{10}$$

$m = \frac{3}{5}$

The slopes are the same. So, the lines are not intersecting lines.

Use the point-slope form to find the $y$-intercept of the other line.

The slope, $m$, is $\frac{3}{5}$. Let $(x_1, y_1) = (−5, 1)$.

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{3}{5}(x - (-5))$$

$$y + 1 = \frac{3}{5}x + 3$$

$$y + 1 - 1 = \frac{3}{5}x + 3 - 1$$

$$y = \frac{3}{5}x + 2$$

Since $b = 2$, the $y$-intercept of that line is (0, 2).

Is that different or the same as the $y$-intercept of the first line? **same**

The lines have the same slope and $y$-intercept, so they do not intersect.

They are coincident lines.

**Lesson 16**

**Coached Example**

Find the slope of the line that passes through (−10, −4) and (−5, −1).

$$m = \frac{-1 - (-4)}{-5 - (-10)} = \frac{-3}{10}$$

$m = \frac{3}{5}$

The ratios are the same. So, the lines are not intersecting lines.

Use the point-slope form to find the $y$-intercept of the other line.

The slope, $m$, is $\frac{3}{5}$. Let $(x_1, y_1) = (−5, 1)$.

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \frac{3}{5}(x - (-5))$$

$$y + 1 = \frac{3}{5}x + 3$$

$$y + 1 - 1 = \frac{3}{5}x + 3 - 1$$

$$y = \frac{3}{5}x + 2$$

Since $b = 2$, the $y$-intercept of that line is (0, 2).

Is that different or the same as the $y$-intercept of the first line? **same**

The lines have the same slope and $y$-intercept, so they do not intersect.

They are coincident lines.
This line has a slope of \( \frac{1}{4} \) and a \( y \)-intercept of \((0, -2)\).

\[
x - x - 4y = 8 - x
\]

\[
-4y = -x + 8
\]

\[
-4y = \frac{-x + 8}{-4}
\]

\[
y = \frac{1}{4}x - 2
\]

This line has a slope of \( \frac{1}{4} \) and a \( y \)-intercept of \((0, -2)\).

The second line I graphed lies on top of the first line.

So, are the two lines intersecting, parallel, or coincident? coincident

Does the system have one solution, no solution, or infinitely many solutions? infinitely many solutions

The lines share all points in common, so the system of equations has infinitely many solution(s).

**Lesson 17**

Coached Example

So, substitute \( 8 - x \) for \( y \) in the first equation and solve for \( x \).

\[
3x - 2(8 - x) = -16
\]

\[
3x - 16 + 2x = -16
\]

Apply the distributive property.

\[
5x - 16 = -16
\]

Combine like terms.

\[
5x = 0
\]

Add 16 to both sides.

\[
x = 0
\]

Divide both sides by 5.

Substitute that value for \( x \) into the second equation.

\[
y = 8 - x = 8 - 0 = 8
\]

\[
3x - 2y = -16
\]

\[
y = 8 - x
\]

\[
3(0) - 2(8) \leq -16
\]

\[
8 \leq 8 - 0
\]

\[
-16 = -16 \checkmark
\]

\[
y = 8 \checkmark
\]

The solution for the system of linear equations is \((0, 8)\).

**Lesson 18**

Coached Example

The number of horses and people combined is 9. \( x + y = 9 \)

There were 30 legs in total. \( 4x + 2y = 30 \)

Solve the first equation you wrote for \( y \).

\[
x + y = 9
\]

\[
y = 9 - x
\]

Substitute the equation you just rewrote for \( y \) into the second equation and solve for \( x \).

\[
4x + 2(9 - x) = 30
\]

\[
4x + 18 - 2x = 30
\]

Apply the distributive property.

\[
2x + 18 = 30
\]

Combine like terms.

\[
2x = 12
\]

Subtract 18 from both sides.

\[
x = 6
\]

Divide both sides by 2.

Substitute that value for \( x \) into the first equation,

\[
x + y = 9
\]

\[
6 + y = 9
\]

\[
y = 3
\]

Subtract 6 from both sides.

The solution for the system of linear equations is \((6, 3)\).

It shows that there are 6 horses and 3 people in the stables.

**Lesson 19**

Coached Example

The data in the table correspond to the points \((1, -1), (2, -2), (2, -4), (3, -4), (4, -5), \) and \((6, -6)\).

Draw a vertical line through \((1, -1)\). Does it pass through more than one point on the graph? no

Draw a vertical line through \((2, -1)\). Does it pass through more than one point on the graph? yes

When \( x = 2 \), the relation corresponds to 2 \( y \)-value(s).

So, the relation is not a function.

**Lesson 20**

Coached Example

The rate of change for the function is equal to its slope, \( m \).

Let \((x_1, y_1) = (0, 2)\).

Let \((x_2, y_2) = (2, 5)\).

\[
m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{5 - 2}{2 - 0} = \frac{3}{2}
\]
The $y$-intercept of the graph is $(0, 2)$. That is the initial value. So, $b = 2$.

You already know that $m = \frac{3}{2}$.

Substitute those values into the slope-intercept form, $y = mx + b$.

$$y = \frac{3}{2}x + 2$$

The rate of change for the linear function is $\frac{3}{2}$, and its equation is $y = \frac{3}{2}x + 2$.

**Lesson 21**

Coached Example

<table>
<thead>
<tr>
<th>value of 630 points</th>
<th>each meal, 5 points are deducted</th>
<th>total amount left on card</th>
</tr>
</thead>
<tbody>
<tr>
<td>630</td>
<td>$5x$</td>
<td>$y$</td>
</tr>
</tbody>
</table>

$$630 - 5x = 570$$

$$630 - 630 - 5x = 570 - 630$$ Subtract 630 from both sides.

$$-5x = -60$$

$$x = 12$$ Divide both sides by $-5$.

If Tyeisha has 570 points left on her card, she has eaten 12 meals at the dining hall.

**Lesson 22**

Coached Example

The line segment from $(0, 13)$ to $(30, 1)$ is steeper than the line segment from $(30, 1)$ to $(50, 0)$.

Since a person travels faster on a bus than on foot, the line segment from $(0, 13)$ to $(30, 1)$ shows that Mr. Kowalski is taking the bus.

$30 - 0 = 30$, so that segment represents 30 minute(s) of his commute.

$13 - 1 = 12$, so it represents a distance of 12 mile(s) traveled.

Does this line segment show Mr. Kowalski taking the bus or walking? **walking**

$50 - 30 = 20$, so that segment represents 20 minute(s) of his commute.

$1 - 0 = 1$, so it represents a distance of 1 mile(s) traveled.

The graph shows that during his commute, Mr. Kowalski rode the bus for the first 30 minute(s) and traveled a distance of 12 mile(s). He then walked for the next 20 minute(s) and traveled a distance of 1 mile(s).

**Lesson 23**

Coached Example

Use the points $(0, -6)$ and $(4, 0)$.

$$m = \frac{0 - (-6)}{4 - 0} = \frac{6}{4} = \frac{3}{2}$$

Use the points $(0, 2)$ and $(2, 5)$.

$$m = \frac{5 - 2}{2 - 0} = \frac{3}{2}$$

Which function has a greater rate of change, or are they the same? **same**

Function 1 has a rate of change of $\frac{3}{2}$ and Function 2 has a rate of change of $\frac{3}{2}$, so both functions have the same rate of change.

**Lesson 24**

Coached Example

Trapezoid $Q'R'S'T'$ is a rotation of trapezoid $QRST$.

So, the figures are the same shape and the same size, or **congruent**.

How many units long is $QR$? **3**

How many units long is $Q'R'$? **3**

How many units long is $ST$? **2**

How many units long is $S'T'$? **2**

How many units long is $TQ$? **4**

How many units long is $T'Q'$? **4**

The corresponding sides of congruent figures are congruent.

$QR$ corresponds to $Q'R'$, so $QR \cong Q'R'$.

$RS$ corresponds to $R'S'$, so $RS \cong R'S'$.

$ST$ corresponds to $S'T'$, so $ST \cong S'T'$.

$TQ$ corresponds to $T'Q'$, so $TQ \cong T'Q'$.

Which sides of trapezoid $QRST$ are parallel? $QR$ and $ST$

So, which sides of trapezoid $Q'R'S'T'$ are parallel? $Q'R'$ and $S'T'$
In trapezoid $QRST$, $QR$ is parallel to $ST$.

In trapezoid $Q'R'S'T'$, $Q'R'$ is parallel to $S'T'$.

**Lesson 25**

Coached Example

Multiply the coordinates of each vertex of $\triangle XYZ$ by the scale factor, $\frac{1}{2}$, to find the vertices of the dilated image, $\triangle X'Y'Z'$.

- $X(2, 4) \rightarrow (2 \cdot \frac{1}{2}, 4 \cdot \frac{1}{2}) \rightarrow X'(1, 2)$
- $Y(6, 10) \rightarrow (6 \cdot \frac{1}{2}, 10 \cdot \frac{1}{2}) \rightarrow Y'(3, 5)$
- $Z(8, 6) \rightarrow (8 \cdot \frac{1}{2}, 6 \cdot \frac{1}{2}) \rightarrow Z'(4, 3)$

The dilated image, with vertices $X'(1, 2)$, $Y'(3, 5)$, and $Z'(4, 3)$, is graphed above.

**Lesson 26**

Coached Example

The sum of the measures of the angles in a triangle is $180^\circ$.

$$180 - (50 + 80) = 180 - 130 = 50^\circ$$

So, $m\angle T = 50^\circ$.

Angle $E$ corresponds to $\angle O$. Are they congruent? **yes**

Angle $T$ corresponds to $\angle S$. Are they congruent? **yes**

Since the triangles have two pairs of corresponding angles that are congruent, they must be **similar** triangles.

**Lesson 27**

Coached Example

An interior angle and the corresponding exterior angle of a triangle are **supplementary**.

Supplementary angles add up to $180^\circ$.

Write an equation that shows the sum of the measures of the given interior and exterior angles of this triangle equals $180^\circ$.

$$3x + 1 + 5x - 13 = 180$$

Solve the equation for $x$.

$$8x - 12 = 180$$

$$8x = 192$$

$$x = 24$$

The value of $x$ is 24.

To find the measure of the interior angle, substitute the value of $x$ into $3x + 1$.

Solve.

$$3x + 1 = 3 \times 24 + 1 = 72 + 1 = 73$$

To find the measure of the exterior angle, substitute the value of $x$ into $5x - 13$.

Solve.

$$5x - 13 = 5 \times 24 - 13 = 120 - 13 = 107$$

Check that the sum of the angle measures is $180^\circ$.

$$73^\circ + 107^\circ = 180^\circ$$

The value of $x$ is 24. The interior angle measures $73^\circ$.

The corresponding exterior angle measures $107^\circ$.

**Lesson 28**

Coached Example

Is $\angle 6$ an interior angle or an exterior angle?

**exterior angle**

$\angle 6$ and $\angle 2$ are **alternate exterior** angles.

So, $\angle 6$ is **congruent** to $\angle 2$.

The measure of $\angle 6$ is $130^\circ$, so the measure of $\angle 2$ is $130^\circ$.

$\angle 2$ and $\angle 3$ are **supplementary** angles.

The sum of the measures of $\angle 2$ and $\angle 3$ is $180^\circ$.

Find the measure of $\angle 3$.

$$180 - 130 = 50$$

The measure of $\angle 3$ is $50^\circ$.

**Lesson 29**

Coached Example

The shorter sides are 7 in. and 24 in. long. Let $a = 7$ and $b = 24$.

The longest side is 25 in. long. Let $c = 25$. 

170
Substitute those values into the formula.

\[ 7^2 + (24)^2 \neq (25)^2 \]

\[ 49 + 576 \neq 625 \]

\[ 625 = 625 \]

Do those values make the equation true? **yes**

Triangle \( XYZ \) is a right triangle.

**Lesson 30**

Coached Example

Find the horizontal distance between points \( J \) and \( K \) by subtracting their \( x \)-coordinates and then taking the absolute value.

Let point \( J(3, 4) \) be \((x_1, y_1)\).

Let point \( K(8, 4) \) be \((x_2, y_2)\).

Horizontal distance = \(|x_2 - x_1| = |8 - 3| = |5| = 5\)

The distance between points \( J \) and \( K \) is **5** units.

Let point \( J(3, 4) \) be \((x_1, y_1)\).

Let point \( L(6, 8) \) be \((x_2, y_2)\).

\[ d = \sqrt{(6 - 3)^2 + (8 - 4)^2} \]

\[ d = \sqrt{3^2 + 4^2} \]

\[ d = \sqrt{9 + 16} \]

\[ d = \sqrt{25} \]

\[ d = 5 \]

The distance between points \( J \) and \( L \) is **5** units.

Which is closer to point \( J \): point \( K \), point \( L \), or is each the same distance from point \( J \)? **same**

Points \( K \) and \( L \) are each the same distance (5 units) from point \( J \).

**Lesson 31**

Coached Example

The ladder is the hypotenuse of the triangle, so let \( c = 20 \).

The distance from the base of the ladder to the house is a leg, so let \( a = 5 \).

\[ (5)^2 + b^2 = (20)^2 \]

\[ 25 + b^2 = 400 \]

\[ b^2 = 375 \]

\[ b = \sqrt{375} \]

\[ b \approx 19.4 \]

The ladder reaches a height of about **19.4** feet.

**Lesson 32**

Coached Example

The diameter, \( d \), is **6** cm. Calculate the radius, \( r \).

\[ r = \frac{d}{2} = \frac{6}{2} = 3 \text{ cm} \]

Substitute 9 for the height, \( h \), and 3 for the radius, \( r \), into the formula.

\[ V = 3.14 \times (3)^2 \times 9 \]

\[ V = 3.14 \times 9 \times 9 \]

\[ V \approx 254.34 \]

That number rounded to the nearest tenth is **254.3**.

The volume of the can is approximately **254.3** cubic centimeters.

**Lesson 33**

Coached Example

Do most of the data points resemble a straight line? **no**

Do most of the data points resemble a curve? **no**

Do the data points appear to be randomly scattered? **yes**

Since the data points appear randomly scattered, this scatter plot shows that there is no association between students’ heights and their test scores.

**Lesson 34**

Coached Example

\[ y = 3.14 \times (3)^2 \times 9 \]

\[ y = 3.14 \times 9 \times 9 \]

\[ y \approx 254.34 \]

That number rounded to the nearest tenth is **254.3**.

The volume of the can is approximately **254.3** cubic centimeters.

What temperature corresponds to 350 feet? **69°F**

I predict the temperature will be about **69°F** at an elevation of 350 feet.
Lesson 35
Coached Example
Find the slope. Use the points (5, 3) and (10, 6).

\[ m = \frac{6 - 3}{10 - 5} \]
\[ m = \frac{3}{5} \]

So, her model shows that for each additional 5 minutes she plays, she tends to score 3 additional points. However, this model cannot predict exactly how many points Raquel will score if she plays for \( x \) minutes in any one game.

Lesson 36
Coached Example
The probability that a girl will prefer yoga is 0.60 (or 60%), while the probability that a boy will prefer yoga is 0.05 (or 5%). So, yoga is more popular among the girls.

Similarly, boys are more likely to prefer flag football.

The probability that a boy will prefer Frisbee is 0.30 (or 30%), and the probability that a girl will prefer Frisbee is 0.35 (or 35%).

Of all the electives, ultimate Frisbee seems to be similarly popular among both boys and girls.