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In this chapter, we are going to look at some basic ideas of algebra. Algebra is about discovering patterns and finding unknowns. It makes you think about number relationships and equations, such as what is equal to what. You will also focus on your thoughts that lead to your best guess when solving problems.
Martha and Amy went to the bookstore to buy calendars. Martha spent twice as much as Amy. Together they spent $27. How much did each spend?

**Keywords: twice, much, together**

**1. Read and Think**

What question are you being asked to find?

- How much did each girl spend?

What are the keywords?

- twice, much, together

What are the facts?

- They spent $27.
- Martha spent twice as much as Amy.

**2. Select a Strategy**

We will use **Guess and Test** to solve the problem. We will also **Make a Table** to organize the information gathered as we Guess and Test.

**3. Solve**

We will pick pairs of numbers. One number must be twice the other number. Their sum must equal 27.

<table>
<thead>
<tr>
<th>Martha</th>
<th>Amy</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>5</td>
<td>15</td>
</tr>
<tr>
<td>12</td>
<td>6</td>
<td>18</td>
</tr>
<tr>
<td>14</td>
<td>7</td>
<td>21</td>
</tr>
<tr>
<td>16</td>
<td>8</td>
<td>24</td>
</tr>
<tr>
<td>18</td>
<td>9</td>
<td>27</td>
</tr>
</tbody>
</table>

Martha spent $18 and Amy spent $9.

**4. Write**

We had to find two addends that have a sum of $27. We needed to find one addend that was double the other. We used the **Guess and Test** strategy to find the numbers that matched these facts. The answers were Martha spent $18 and Amy spent $9.
5. Reflect

Let’s look at our work and answer.

- Did we show that we knew what the problem asked for? Yes. We answered the question that was asked. We found how much each girl spent.
- Did we know what the keywords were? Yes.
- Did we show that we knew what facts were given? Yes.
- Did we name and use the correct strategy? Yes.
- Was our mathematics correct? Yes. Our sum is equal to $27 and one addend is double the other.
- Did we label our work? Yes.
- Was our answer correct? Yes.
- Were all of our steps included? Yes.
- Did we explain why we chose the strategy and how we used it? Yes.
- Did we write a good, clear explanation of our work? Yes.

Score

This solution would earn a perfect 4 on our rubric. The question asked, keywords, and the facts were all given. Good strategies were used and the math was correct and labeled. The answer is explained well.

For each guided open-ended math problem, there are three examples. The first lets you solve the problem. The second lets you use a rubric to score someone else’s work, and then correct it to make it a perfect score of 4. The third is an example of the work of someone who got a 4 using a strategy that may or may not differ from your own.
Greg received 3 gifts. Uncle Pete’s gift was $2.12 more than Aunt Sally’s gift. Uncle Pete’s gift cost $3.14 less than Uncle Ray’s gift. Aunt Sally’s gift cost $8.12. How much did the gifts cost all together?

Keywords: ? ?

1. Try It Yourself.
Answer the questions below to get a score of 4.
What question are you being asked to find?

What are the keywords?

What are the facts you need to solve the problem?

What strategy can you use to solve the problem?

Solve the problem.

Explain what you did to solve the problem.
2. Look at How Rick Solved the Problem.

Question: How much did the gifts cost all together?

Keywords: more, less, much, all together

Facts: Uncle Pete’s gift cost $2.12 more than Aunt Sally’s gift.
Uncle Pete’s gift cost $3.14 less than Uncle Ray’s gift.
Aunt Sally’s gift cost $8.12.

Strategy: Divide and Conquer

Solve: Let $p = \text{the amount of money Uncle Pete spent for his gift.}$
Let $r = \text{the amount of money Uncle Ray spent for his gift.}$
Let $s = \text{the amount of money Aunt Sally spent for her gift.}$

$s = 8.12$
$p = s - 2.12, p = 6$
$r = p + 3.14, r = 9.14$

$8.12 + 6.00 + 9.14 = 23.26$

The gifts cost $23.26 altogether.

Write: I used Divide and Conquer by writing number sentences. I determined what each relative spent. Then I added the amounts.

Score the Answer: According to the rubric, from 1 to 3, what score would you give Rick? Explain why you gave that score.

Make it a 4! Rewrite.

Use the rubric on page 13 to score this work.
3. There’s More Than One Way.

Remember that there is often more than one way to solve a problem. Here is how Maureen solved this problem.

Maureen’s Paper

Question: How much did the gifts cost all together?
Keywords: more, less, all together, much
Facts: Uncle Pete’s gift was $2.12 more than Aunt Sally’s gift.
Uncle Pete’s gift was $3.14 less than Uncle Ray’s gift.
Aunt Sally’s gift cost $8.12.
Strategy: Work Backward

Solve:

Let \( r \) = the amount of money Uncle Ray spent for his gift
Let \( s \) = the amount of money Aunt Sally spent for his gift
Let \( p \) = the amount of money Uncle Pete spent for his gift.

\[
\begin{align*}
  s &= \$8.12 \\
  p &= s + \$2.12 = \$8.12 + \$2.12 = \$10.24 \\
  r &= p + \$3.14 = \$10.24 + \$3.14 = \$13.38 \\
  \$8.12 + \$10.24 + \$13.38 &= \$31.74
\end{align*}
\]

Write: I used the Work Backward strategy. I used the information that I knew, which was the cost of Aunt Sally’s gift, to find the cost of the other 2 gifts. Then I added to find the total amount spent.

Score: Maureen’s paper would receive a 4 on the test. Maureen identified the question that was asked, the keywords, and the facts. She picked a good strategy. Then she clearly explained the steps she took to solve the problem. She was perfect!
4. Answers

Guided Problem #1

Greg received 3 gifts. Uncle Pete’s gift was $2.12 more than Aunt Sally’s gift. Uncle Pete’s gift cost $3.14 less than Uncle Ray’s gift. Aunt Sally’s gift cost $8.12. How much did the gifts cost all together?

Keywords: less, more, all together

Try It Yourself (page 49)

Question: What is the cost of the 3 gifts all together?

Facts: Uncle Pete’s gift was $2.12 more than Aunt Sally’s gift.

Uncle Pete’s gift cost $3.14 less than Uncle Ray’s gift.

Aunt Sally’s gift cost $8.12.

Strategies: Work Backward, Write a Number Sentence

Solve: Aunt Sally’s gift cost $8.12.

Uncle Pete’s gift cost $2.12 more than Aunt Sally’s gift: $8.12 + $2.12 = $10.24

Uncle Pete’s gift cost $3.14 less than Uncle Ray’s gift: $10.24 + $3.14 = $13.38

$8.12 + $10.24 + $13.38 = $31.74

Explanation: I used the Work Backward and Write a Number Sentence strategies. I used the cost of Aunt Sally’s gift to find the costs of the other 2 gifts. Then I added the cost of each gift to find the total amount spent.

Rick’s Paper (page 50)

Score the Answer: I would give Rick a 2 on his paper. He knew what question was asked, gave the keywords, and the facts. He used good strategies, but he did not know what to do with his information. His number sentences did not follow the question. His explanation was not detailed.

Make it a 4!

\[
s = 8.12 \\
p = s + 2.12, 8.12 + 2.12 = 10.24 \\
r = p + 3.14, 10.24 + 3.14 = 13.38 \\
8.12 + 10.24 + 13.38 = 31.74
\]

I used the Work Backward strategy. I used the cost of Aunt Sally’s gift to find the cost of the other 2 gifts. Then I added to find the total amount spent on the gifts.

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Guided Problem #2

Theo was born 7 days before Isabelle. Theo gave Isabelle the following number puzzle.

I am thinking of 2 numbers. When you add them together you get 42.

Their difference is 8. What are my 2 numbers?

Keywords: □ □

1. Try It Yourself.

Answer the questions below to get a score of 4.

What question are you being asked to find?

What strategy can you use to solve the problem?

Solve the problem.

What are the facts you need to solve the problem?

What strategy can you use to solve the problem?

Solve the problem.

Explain what you did to solve the problem.

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2. Look at How Helene Solved the Problem.

**Helene’s Paper**

**Question:** What are the 2 numbers?

**Keywords:** add, difference

**Facts:** 2 numbers add to 42.
They have a difference of 8.

**Strategy:** Write a Number Sentence

<table>
<thead>
<tr>
<th>Solve:</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Try 42 and 1</td>
<td>$42 \times 1 = 42$</td>
<td>$42 - 1 = 41$</td>
</tr>
<tr>
<td>Try 21 and 2</td>
<td>$21 \times 2 = 42$</td>
<td>$21 - 2 = 19$</td>
</tr>
<tr>
<td>Try 14 and 3</td>
<td>$14 \times 3 = 42$</td>
<td>$14 - 3 = 11$</td>
</tr>
<tr>
<td>Try 7 and 6</td>
<td>$7 \times 6 = 42$</td>
<td>$7 - 6 = 1$</td>
</tr>
</tbody>
</table>

There is no answer to the puzzle.

**Write:** I wrote a pair of number sentences. I have tried all the possible numbers. No other pair of factors multiply to 42. There were only 4 pairs of numbers.

---

**Score the Answer:** According to the rubric, from 1 to 3, what score would you give Helene? Explain why you gave that score.

---

**Make it a 4! Rewrite.**

---

3. There’s More Than One Way.

Remember there is often more than one way to solve a problem. Here is how Ian solved this problem.
Question: What are 2 numbers that have a sum of 42 and a difference of 8?
Keywords: add, difference
Facts: 2 numbers add to 42.
The difference of the same 2 numbers is 8.
Strategy: I used Logical Thinking.
Solve: 42 ÷ 2 = 21
Since the difference is 8, add 4 to one side and subtract 4 from the other:
21 + 4 = 25
21 – 4 = 17
25 + 17 = 42
25 – 17 = 8
Write: I used Logical Thinking. I found half of 42, which is 21. Since the difference is 8, I added half of 8 to 21 and subtracted half of 8 from 21. I got the numbers 25 and 17, which has a sum of 42 and a difference of 8.
Score: Ian would receive a perfect 4. He gave the question, the keywords, the facts, and answered the question. He used a good strategy and explained his work clearly.

Guided Problem #2
Theo was born 7 days before Isabelle. Theo gave Isabelle the following number puzzle.
I am thinking of 2 numbers. When you add them together you get 42.
Their difference is 8. What are my 2 numbers?
Keywords: add, difference

Try It Yourself (page 53)
Question: What are the 2 numbers that have a sum of 42 and a difference of 8?
Facts: 2 numbers add to 42.
The same 2 numbers have a difference of 8.
Strategy: Guess and Test
Solve:
| Try 24 and 18 | 24 + 18 = 42 | 24 – 18 = 6 |
| Try 25 and 17 | 25 + 17 = 42 | 25 – 17 = 8 |
**6. Algebra**

**Explanation:** I used **Guess and Test**. I picked pairs of numbers that add to 42 until finding a pair that has a difference of 8. Those numbers are 25 and 17.

**Helene's Paper (page 54)**

**Score the Answer:** I would give Helene a 2. She knew the question, gave the keywords, used only the facts that were needed to solve the problem, and listed the facts. She picked a good strategy, but she gave up on the problem. She confused multiplication with addition. Her answer was incorrect.

**Make it a 4!**

<table>
<thead>
<tr>
<th>Try 24 and 18</th>
<th>24 + 18 = 42</th>
<th>24 - 18 = 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Try 25 and 17</td>
<td>25 + 17 = 42</td>
<td>25 - 17 = 8</td>
</tr>
</tbody>
</table>

I used **Guess and Test**. I picked pairs of numbers that add to 42 until finding a pair that has a difference of 8. Those numbers are 25 and 17. I ignored that Theo was born 7 days before Isabelle because it did not help solve the problem.

**Guided Problem #3**

Sandra is working with this function table.

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>11</td>
</tr>
<tr>
<td>7</td>
<td>15</td>
</tr>
<tr>
<td>9</td>
<td>?</td>
</tr>
</tbody>
</table>

What is the missing number?

**Keywords:**

**1. Try It Yourself.**

Answer the questions below to get a score of 4.

What **question** are you being asked to **find**?

What are the **keywords**?

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What are the **facts** you need to solve the problem?

_________________________________________________________

_________________________________________________________

_________________________________________________________

What **strategy** can you use to solve the problem?

_________________________________________________________

_________________________________________________________

_________________________________________________________

**Solve** the problem.

_________________________________________________________

_________________________________________________________

_________________________________________________________

**Explain** what you did to solve the problem.

_________________________________________________________

_________________________________________________________

_________________________________________________________

**2. Look at How Lars Solved the Problem.**

**Lars’s Paper**

**Hint**

Answers include **Look for a Pattern** and **Write a Number Sentence**.

**Question**: What is the Output when the Input is 9?

**Keywords**: function, missing

**Facts**: 2 becomes 5, 5 becomes 11, 7 becomes 15.

**Strategy**: Look for a Pattern.

**Solve**: The difference in the Output between when 2 and 5 were Inputs was 6, so 15 + 6 = 21. The missing number is 21.

**Write**: I used the Look for a Pattern strategy. I determined that the difference between the Outputs for Inputs 2 and 5 was 6. So I added 6 to the Output of 15 to get 21.

**Score the Answer**: According to the rubric, from 1 to 3, what score would you give Lars? Explain why you gave that score.

_________________________________________________________

_________________________________________________________

_________________________________________________________
3. There’s More Than One Way.

Remember, there is often more than one way to solve a problem. Here is how Patrick solved this problem.

Patrick’s Paper

Question: What is the Output when the Input is 9?
Keywords: function, missing
Facts: 2 becomes 5, 5 becomes 11, and 7 becomes 15.
Strategy: I wrote an equation.
Solve: The equation Output = (2 x Input) + 1 is the rule of the function table.
(2 x 9) + 1 = 19
The missing number is 19.
Write: I wrote the equation Output = (2 x Input) + 1 as the rule of the table. I then substituted 9 for the Input to get (2 x 9) + 1 = 19. The missing number is 19.
Score: Patrick’s paper would earn a 4 on our test. He showed that he understood the question that was asked. He identified the keywords and the facts. He picked a good strategy. Then he clearly explained the steps he took to solve the problem. It is perfect!
4. Answers

Guided Problem #3

Sandra is working with this function table.

<table>
<thead>
<tr>
<th>Input</th>
<th>Output</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>11</td>
</tr>
<tr>
<td>7</td>
<td>15</td>
</tr>
<tr>
<td>9</td>
<td>?</td>
</tr>
</tbody>
</table>

What is the missing number?

Keywords: function, missing

Question: What is the Output when the Input is 9?

Facts: 2 becomes 5, 5 becomes 11, 7 becomes 15

Strategy: Look for a Pattern

Solve: I Found a Pattern. The Output is always 1 more than double the Input.

\[ 9 \times 2 = 18, \quad 18 + 1 = 19. \] The missing number is 19.

Explanation: I found a pattern of the Output being 1 more than 2 times the Input. I doubled 9, which is 18, and then I added 1 to get 19. The missing number is 19.

Lars’s Paper

I would give Lars a 3. He knew the question, gave the keywords, the facts, and picked a good strategy. He made a mistake in his pattern, which caused him to get an incorrect answer. His explanation of what he did was good.

Make it a 4!

The pattern is the Output is 1 more than 2 times the Input. Multiply \( 9 \times 2 = 18 \) and then add, \( 18 + 1 = 19 \) to get the missing number.
6. Algebra

Quiz Problems

Here are some problems for you to try. Keep your rubric handy while you solve the problem. Let’s see if you can score a 4.

1. Lorraine bought 2 notebooks and a new book bag. She spent $24. The 2 notebooks were the same price. The book bag cost twice as much as 1 of the notebooks. How much did she spend for each item?

2. Carlos sells worms to fishermen. On the first day he sold 12 worms. On the second day he sold 10 worms. On the third day he sold 15 worms. On the fourth day he sold 13 worms. On the fifth day he sold 18 worms. If the pattern continues, how many worms did Carlos sell on the seventh day?

3. In a hot-dog-eating contest, Manny ate 3 times as many hot dogs as Ollie. Stan ate 5 hot dogs fewer than Manny. Ollie ate 12 hot dogs. How many hot dogs did they eat in all?

4. Here is the puzzle that Joanie is working on.

There are 2 numbers. When you add them you get 18. When you find their difference, it is 4. What are the 2 numbers?
5. In this money function machine, you put money in and you get money out. You always get more money out than you put in.

<table>
<thead>
<tr>
<th>Money In</th>
<th>Money Out</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1</td>
<td>$12</td>
</tr>
<tr>
<td>$2</td>
<td>$14</td>
</tr>
<tr>
<td>$3</td>
<td>$16</td>
</tr>
<tr>
<td>$4</td>
<td>$16</td>
</tr>
</tbody>
</table>

Find out how much money you get from this machine when $10 is put in.

6. Becky’s dad sponsored her in a charity bike ride. He paid $0.01 for the first $\frac{1}{2}$ mile, $0.02$ for the second $\frac{1}{2}$ mile, $0.04$ for the third $\frac{1}{2}$ mile, $0.08$ for the next $\frac{1}{2}$ mile, and the pattern continues for the distance she biked. Becky rode 7 miles. How much did her dad have to pay for the last $\frac{1}{2}$ mile?

7. How many dots are in the sixth part of this diagram?

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