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Measurement

Measurement is part of many open-ended math problems. Any problem that asks about a size (such as a length, capacity, or weight), a temperature, a period of time, or even an amount of money is a measurement problem. Problems that include rates such as gallons per minute are also measurement problems. In the sixth grade, tests have lots of problems that involve measurements such as perimeter, area, and volume.
Let's look at a problem that involves area. Let's see how to solve it. Remember, we want to get a perfect score of 4 on our scoring rubric.

**Model Problem**

A fish tank has the shape of a **rectangular prism**. It has a length of 6 feet, a width of 3 feet and a height of 5 feet. What is the **volume** of the fish tank?

**Keywords:** rectangular prism, volume

---

**1. Read and Think**

What **question** are you being asked?
- What is the volume of a fish tank that is 6 feet by 3 feet by 5 feet?

What are the **keywords**?
- rectangular prism, volume

What **facts** were we given?
- The fish tank is shaped like a rectangular prism.
- The dimensions are 6 feet by 3 feet by 5 feet.

**2. Select a Strategy**

To solve this problem, you need to find the volume of the fish tank. We will use the **Make a Model** strategy.
3. Solve

Let each counter represent 1 cubic foot. We can make 6 rows and 3 columns of counters to show the first layer. There are 18 counters in the layer. We can make 5 layers, or we can multiply $18 \times 5 = 90$ to find the number of counters that the entire fish tank would comprise.

The volume of the fish tank is 90 cubic feet.

4. Write/Explain

We used the Make a Model strategy to solve the problem. We designed a model to show the bottom layer of the fish tank and then multiplied by the number of layers to find the volume.

5. Reflect

- Did we show that we knew what the problem asked for? Yes.
- Did we show that we knew what facts were given? Yes.
- Did we know what the keywords were? Yes.
- Did we name and use the correct strategy? Yes.
- Was our mathematics correct? Yes. We checked it.
- Did we label our work? Yes.
- Was our answer correct? Yes.
- Were all of our steps included? Yes.
- Did we write a good, clear explanation of our work? Yes.
Let’s look at some problems and see how students solved them.

For each guided problem there are four sections. The first lets you solve the problem. The second lets you use a rubric to score someone else’s work, and then correct that work to a perfect score of 4. The third is an example of work by someone who got a perfect score of 4 using a strategy that may differ from your own. The fourth has sample answers for sections one and two.

Guided Problem #1

Marianne’s room is shaped like a trapezoid. The width of the room is 12 feet. One of the other sides of the room is 18 feet long. If the room has an area of 192 ft², what is the length of the fourth side of the room?

Keywords: ■ ■ ■ ■ ■

1. Try It Yourself.

Answer the questions below to get a score of 4

What question are you being asked?

What are the keywords?

What are the facts you need to solve the problem?

What strategy can you use to solve the problem?

Solve the problem.

Write/Explain what you did to solve the problem.

Reflect. Use the rubric to help you.

Possible answers include: Draw a Picture, Write an Equation, and Make It Simpler.
2. Look How Caroline Solved the Problem.

Caroline’s Paper

Question: What is the length of the fourth side of a trapezoid that has an area of 192 ft² and sides of 12 ft, 12 ft, and 18 ft?

Keywords: trapezoid, area

Facts: The room is shaped like a trapezoid.
The width is 12 feet.
One of the side’s length is 18 ft.
The room has an area of 192 ft².

Strategy: I Wrote an Equation.

Solve:
Use the formula for the area of a trapezoid:
\[ A = \frac{1}{2} \times (a + b) \times h \]

\[ 192 = \frac{1}{2} \times (18 + b) \times 12 \]
\[ 192 = 9 + b \times 12 \]
\[ 192 = 108 + b \]
\[ 192 - 108 = 108 - 108 + b \]
\[ 84 = b \]

The other side of the trapezoid is 84 feet.

Write/Explain: I used the Write an Equation strategy. I substituted each of the known dimensions into the formula for the area of a trapezoid. I simplified the side of the equation with the variables and then solved the equation for the other side of the trapezoid. I found that the fourth side of the trapezoid was 84 feet.

Score the Answer.

According to the rubric, from 1 to 3 what score would you give Caroline? Explain why you gave that score.

________________________________________

Make It a 4! Rewrite.

________________________________________

________________________________________

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3. There’s More Than One Way.

Remember there is often more than one way to solve a problem. Here is how Roy solved this problem.

**Roy’s Paper**

**Question:** What is the length of the fourth side of a trapezoid that has an area of 192 ft² parallel to the side that is 18 feet long?

**Keywords:** trapezoid, area

**Facts:** The room is shaped like a trapezoid.
- The width is 12 feet.
- One of the other sides is 18 ft.
- The room has an area of 192 ft².

**Strategy:** I Made It Simpler.

**Solve:** I will Make It Simpler. I know that the width of the trapezoid is 12 feet, so the “height” of the trapezoid in the formula is 12 feet. I know that the area of a parallelogram is A = bh, so I can use that equation to find what the average of the two bases should be.

\[ A = bh \]

\[ 192 = b \times 12 \]

\[ 192 \div 12 = b \times 12 \div 12 \]

\[ 16 = b \]

So, the bases should have an average of 16 feet. I can find the fourth side of the trapezoid.

\[ 16 = (18 + b) \div 2 \]

\[ 16 \times 2 = (18 + b) \div 2 \times 2 \]

\[ 32 = 18 + b \]

\[ 32 - 18 = 18 - 18 + b \]

\[ 14 = b \]

The other side of the room is 14 feet.

**Write/Explain:** I used the Make It Simpler strategy. I used the formula for the area of a parallelogram to find what the average of the two bases of a trapezoid should be. I found that the two bases should average 16 feet. I knew that one of the bases was 18 feet, so I wrote an equation to find what the other base should be. I solved my equation and found that the fourth side of the trapezoid was 14 feet.

**Score:** Roy’s solution would earn a 4 on a test. Roy identified the question, the facts, and the keywords. He picked a good strategy and used it correctly. He gave a clear explanation of the steps he took to solve the problem. He labeled his answer. Roy’s solution is perfect!
4. Answers

Guided Problem #1

Marianne’s room is shaped like a trapezoid. The width of the room is 12 feet. One of the other sides of the room is 18 feet long. If the room has an area of 192 ft$^2$, what is the length of the fourth side of the room?

Try It Yourself (page 85)

Question: What is the length of the fourth side of a trapezoid that has an area of 192 ft$^2$ and one length of 18 feet and a width of 12 feet?

Keywords: trapezoid, area

Facts: The room is shaped like a trapezoid.

The width is 12 feet.

One of the parallel sides is 18 ft.

The room has an area of 192 ft$^2$.

Strategy: Write an Equation

Solve:

Area of a trapezoid: $A = \frac{1}{2} \times (a + b) \times h$

192 = $\frac{1}{2} \times (18 + b) \times 12$
192 $\div$ 12 = $\frac{1}{2} \times (18 + b) \times 12 \div 12$
16 = $\frac{1}{2} \times (18 + b)$
16 $\times$ 2 = $\frac{1}{2} \times 2 \times (18 + b)$
32 = 18 + b
32 $-$ 18 = 18 $-$ 18 + b
14 = b

Write/Explain: I used the Write an Equation strategy. I substituted the known values of the trapezoid, which included the area, one of the bases, and the height into the formula for the area of a trapezoid. Then I solved the equation to find the length of the fourth side of the trapezoid is 14 feet.

Caroline’s Paper (page 86)

Score the Answer: I would give Caroline a 3. She knew what the question asked, listed the facts and gave all of the keywords. She labeled her answer. Caroline did not know how to solve her equation for the base.

Make It a 4!

Area of a trapezoid: $A = \frac{1}{2} \times (a + b) \times h$

192 = $\frac{1}{2} \times (18 + b) \times 12$
192 $\div$ 12 = $\frac{1}{2} \times (18 + b) \times 12 \div 12$
I used the **Write an Equation** strategy. I substituted the known values of the trapezoid, which included the area, one of the bases, and the height into the formula for the area of a trapezoid. Then I solved the equation to find the length of the fourth side of the trapezoid is 14 feet.

16 = \(\frac{1}{2} \times (18 + b)\)
16 \times 2 = \(\frac{1}{2} \times 2 \times (18 + b)\)
32 = 18 + b
32 - 18 = 18 - 18 + b
14 = b

**Guided Problem #2**

A circular fountain is in the middle of a square park. The fountain has a diameter of 30 meters. The rest of the park consists of a grass field. If the park has sides of 120 meters, what is the area that the maintenance people have to mow?

**Keywords:** ?

**1. Try It Yourself.**

Answer the questions below to get a score of 4.

**What question** are you being asked?

**What are the keywords?**

**What are the facts** you need to solve the problem?

**What strategy** can you use to solve the problem?

**Solve** the problem.

**Write/Explain** what you did to solve the problem.

**Hint**

Possible answers include: **Divide and Conquer, Draw a Picture, and Write an Equation.**

**Reflect.** Use the rubric to help you.
2. Look How Jake Solved the Problem.

**Jake’s Paper**

**Question:** Not counting the fountain, what is the area of the park?

**Keywords:** circular, square, diameter, area

**Facts:** The park has a fountain with a diameter of 30 meters. Each side of the park is 120 meters.

**Strategy:** I used Divide and Conquer and Logical Thinking.

**Solve:** A square that has sides of 120 meters has an area of $120 \times 120 = 14,400$ m$^2$.

The fountain has a diameter of 30 meters, which is $\frac{1}{4}$ the length of the square.

Multiply the area of the square by $\frac{1}{4}$ to find the area of the fountain: $\frac{1}{4} \times 14,400 = 3,600$ m$^2$.

Subtract the area of the fountain from the area of the square: $14,400 - 3,600 = 10,800$ m$^2$.

The maintenance people have to mow 10,800 m$^2$ of grass.

**Write/Explain:** I used the Divide and Conquer strategy. First, I found the area of the park by using the formula for the area of a square. Secondly, I used logical thinking to find the area of the fountain. Since the diameter of the fountain was $\frac{1}{4}$ the length of the square, I multiplied the area of the fountain by $\frac{1}{4}$. Then I subtracted the area of the fountain from the area of the park.

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**Score the Answer.**

According to the rubric, from 1 to 3 what score would you give Jake? Explain why you gave that score.

---

**Make It a 4! Rewrite.**

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**Use the rubric on page 13 to score this work.**

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3. There's More Than One Way.

Remember there is often more than one way to solve a problem. Here is how Jordan solved this problem.

**Jordan's Paper**

**Question:** What is the area of the park, not counting the fountain?

**Keywords:** circular, square, diameter, area

**Facts:** The park has a fountain with a diameter of 30 m. Each side of the park is 120 m.

**Strategies:** I Drew a Picture and used Divide and Conquer.

First I found the area of the square park: \( A = s^2 \)

\[
A = 120 \text{ m} \times 120 \text{ m} \\
A = 14,400 \text{ m}^2
\]

Next, I found the area of the circular fountain: \( A = \pi r^2 \)

\[
A \approx 3.14 \times 15 \text{ m} \times 15 \text{ m} \\
A \approx 706.5 \text{ m}^2
\]

Last, I subtracted the area of the fountain from the area of the park:

\[
14,400.0 \text{ m}^2 \\
-\quad 706.5 \text{ m}^2 \\
13,693.5 \text{ m}^2
\]

The maintenance people have to mow 13,693.5 m\(^2\).

**Write/Explain:** I Drew a Picture to see how the fountain related to the park. Then I wrote an equation to find the area of the park by using the formula for a square. Then I found the area of the fountain by using the formula for a circle. Finally, I subtracted the area of the fountain from the area of the park. This left 13,693.5 m\(^2\) for the maintenance people to mow.

**Score:** Jordan’s solution would earn a 4 on a test. She identified the question that was asked, the facts, and the keywords, and picked a good strategy. Jordan used the strategy she chose correctly. She clearly explained the steps taken to solve the problem. She labeled her answer. Jordan’s paper is perfect.
Guided Problem #2

A circular fountain is in the middle of a square park. The fountain has a diameter of 30 meters. The rest of the park consists of a grass field. If the park has sides of 120 meters, what is the area that the maintenance people have to mow?

Keywords: ? ?

Try It Yourself (page 89)

Question: What is the area of the grass in the park?

Keywords: circular, square, diameter, area

Facts: Each side of the park is 120 m.

Strategy: Divide and Conquer and Write an Equation

Solve: First, find the area of the park by using the formula for the area of a square.

\[ A = s^2 \]

\[ A = 120 \text{ m} \times 120 \text{ m} \]

\[ A = 14,400 \text{ m}^2 \]

Second, find the area of the fountain by using the formula for the area of a circle.

\[ A = \pi r^2 \]

\[ A \approx 3.14 \times 15 \text{ m} \times 15 \text{ m} \]

\[ A \approx 706.5 \text{ m}^2 \]

Then, subtract the area of the fountain from the area of the square.

\[ 14,400 \text{ m}^2 - 706.5 \text{ m}^2 = 13,693.5\text{ m}^2 \]

Write/Explain: I used the Divide and Conquer and Write an Equation strategies. First, I found the area of the square park. Then I found the area of the circular fountain. Finally, I subtracted the area of the circular fountain from the area of the square park to find the area that the maintenance people have to mow.

Jake’s Paper (page 90)

Score the Answer: I would give Jake a 2. He listed the question, the facts, and the keywords. His explanation was clear and his work indicates that he understood the problem. However, Jake used Logical Thinking when he should have written another equation. He incorrectly found the area of the circle by using the formula for the area of a square and multiplying by \( \frac{1}{4} \), instead of using the formula for the area of a circle. He didn’t clearly label his answer.
Make It a 4!

Find the area of the fountain by using the formula for the area of a circle.

\[ A = \pi r^2 \]
\[ A \approx 3.14 \times 15 \text{ m} \times 15 \text{ m} \]
\[ A \approx 706.5 \text{ m}^2 \]

Subtract the area of the fountain from the area of the square.

\[ 14.400 \text{ m}^2 - 706.5 \text{ m}^2 = 13,693.5 \text{ m}^2 \]

The maintenance people have to mow an area of 13,693.5 m².

First, I found the area of the square park. Then I found the area of the circular fountain. Finally, I subtracted the area of the circular fountain from the area of the square park to find the area that the maintenance people have to mow.

Guided Problem #3

The houses on Teece Street are each 20 meters long. The houses are 40 meters apart from each other. The corner houses are at least 60 meters away from the intersecting streets. Teece Street is 0.6 kilometer long and is intersected only at the ends. How many houses are on both sides of Teece Street?

1. Try It Yourself.

Answer the questions below to get a score of 4.

What question are you being asked?

What are the keywords?
What are the facts you need to solve the problem?

What strategy can you use to solve the problem?

Solve the problem.

Write/Explain what you did to solve the problem.

Reflect. Use the rubric to help you.

2. Look How Dennis Solved the Problem.

Dennis’ Paper

Question: How many houses are on Teece Street all together?

Keywords: meter, kilometer, each, all together

Facts: The houses are each 20 meters long.
       The houses are 40 meters from each other.
       The street is 0.6 kilometer long.

Strategy: I can Make It Simpler

House 1: 0 to 20 meters
House 2: 60 to 80 meters
House 3: 120 to 140 meters
House 4: 180 to 200 meters

600 ÷ 200 = 3
4 × 3 = 12

There are 12 houses on each side of the street. So, there are 24 houses all together on Teece Street.

Score the Answer.

According to the rubric, from 1 to 3 what score would you give Dennis? Explain why you gave that score.

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3. There’s More Than One Way.

Remember there is often more than one way to solve a problem. Here is how Amber solved this problem.

Amber’s Paper

**Question:** How many houses are on Teece Street all together?

**Keywords:** meter, kilometer, each, all together

**Facts:** The houses are each 20 meters long. The houses are 40 meters from each other. The corner houses are at least 60 meters from the intersecting streets. The street is 0.6 kilometer long.

**Strategy:** I can Make a Table.

<table>
<thead>
<tr>
<th>House</th>
<th>Start of House (from the beginning of the street)</th>
<th>End of House (from the beginning of the street)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>60</td>
<td>80</td>
</tr>
<tr>
<td>2</td>
<td>120</td>
<td>140</td>
</tr>
<tr>
<td>3</td>
<td>180</td>
<td>200</td>
</tr>
<tr>
<td>4</td>
<td>240</td>
<td>260</td>
</tr>
<tr>
<td>5</td>
<td>300</td>
<td>320</td>
</tr>
<tr>
<td>6</td>
<td>360</td>
<td>380</td>
</tr>
<tr>
<td>7</td>
<td>420</td>
<td>440</td>
</tr>
<tr>
<td>8</td>
<td>480</td>
<td>500</td>
</tr>
<tr>
<td>9</td>
<td>540</td>
<td>560</td>
</tr>
</tbody>
</table>

**Solve:** Not enough land for the ninth house (since there must be 60 meters after it)

There are 8 houses on each side of Teece Street, so there are 16 houses all together on Teece Street.

**Write/Explain:** I Made a Table to find the number of houses on each side of the street. I started the first house at 60 meters, since the corner houses must have at least 60 meters of land between the house and the intersecting street. I found that each side of the street can have no more than 8 houses. Since the question asks for the total number of houses, multiply 8 x 2 = 16 houses.

**Score:** Amber would earn a 4 on our rubric. She identified the question that was asked, the facts, and the Keywords. She picked a good strategy and used it correctly. She explained her thinking clearly and found the correct answers. She labeled her answer. Her paper is perfect!
4. Answers

Guided Problem #3

The houses on Teece Street are each 20 yards long. The houses are 40 meters apart from each other. The corner houses are at least 60 meters away from the intersecting streets. Teece Street is 0.6 kilometer long. How many houses are on both sides of Teece Street?

Keywords: ? ?

Try It Yourself (pages 93–94)

Question: How many houses are on Teece Street all together?

Keywords: meter, kilometer, each, all together

Facts: The houses are each 20 meters long.

The houses are 40 meters from each other.

The corner houses are at least 60 meters from the intersecting streets.

The street is 0.6 kilometer long.

Strategy: Draw a Picture

There are 16 houses on Teece Street.

Write/Explain: I used the Draw a Picture strategy. I drew one side of a street that was divided into 20-meter sections. I then placed houses in the appropriate places, following the instructions given in the question. Each corner house was at least 60 meters from the corner, each house was 20 meters long, and houses were 40 meters apart. There were 8 houses on each side of the street, so there were 16 houses all together.

Dennis' Paper (page 94–95)

Score the Answer: I would give Dennis a 1. He knew what the question was asking, and listed the keywords. He labeled his answer. However, he forgot the fact that the corner houses must be at least 60 meters from the intersecting street. His strategy would have led him to an incorrect answer even if he remembered that fact. He also failed to explain his work.

Make It a 4!

Facts: The houses are each 20 meters long.

The houses are 40 meters from each other.

The corner houses are at least 60 meters from the intersecting streets.

The street is 0.6 kilometer long.
**Strategy:** Make an Organized List

- House 1: 60 to 80 meters
- House 2: 120 to 140 meters
- House 3: 180 to 200 meters
- House 4: 240 to 260 meters
- House 5: 300 to 320 meters
- House 6: 360 to 380 meters
- House 7: 420 to 440 meters
- House 8: 480 to 500 meters

There is not enough land to have a ninth house and fit all the criteria. Since there are two sides of the street, there are $2 \times 8$ houses on Teece Street. So there are 16 houses on Teece Street.

**Write/Explain:** I used the **Make a List** strategy. I started from the beginning of the street and recorded where each house would go. I made sure to leave at least 60 meters at the beginning and end of the street. Houses were all 20 meters long and there were 40 meters between houses. I found that 8 houses could fit on each side of the street, so there could be 16 houses all together.
Quiz Problems

Here are some problems for you to try. Keep your rubric handy while you solve the problem. Let’s see if you can score a 4.

1. Dan works from 8:30 A.M. to 5:00 P.M. each day from Monday to Friday. He takes 45 minutes for lunch each day. Each week, Dan earns $426.25. How much does he earn per hour?

2. Nora is going to paint the walls and the ceiling of her bedroom. Her bedroom is 15 feet long, 12 feet wide, and 8 feet high. She has three windows that are each 5 feet by 4 feet. If Nora gives her room two coats of paint, what is the total area that she will paint?

3. From his home, Larry rides his bicycle 3 miles north, 2 miles west, 4 miles north, 3 miles east, and then 5 miles south. If he can only travel east or west and north or south, what is the least distance that Larry can travel to bicycle home?
4. Michelle wants to send blocks to her cousin Emma. The blocks are 2-inch cubes. If the box that Michelle wants to use is a rectangular prism that is 8 inches long, 6 inches wide, and 6 inches high, how many blocks can Michelle give Emma?

5. Springfield Community Pool consists of two separate pools. The diving pool is 40 feet long, 30 feet wide, and 12 feet deep. The swimming pool is 80 feet long, 50 feet wide, and 5 feet deep. Which pool has a greater volume? How much more volume does the larger pool have than the smaller?

6. Mrs. Thomas wants to place a 12-inch-by-9-inch photo in a frame that has a 2-inch-thick-border around it. What is the combined area of the frame and the photo?

7. Lorraine brought 5 quarts of apple juice to a party. Christina brought 8 pints of pineapple juice to the party. Clarence brought $1\frac{1}{2}$ gallons of grape juice to the party. If Greg mixes all of the juice with the 6 cups of grape juice that he had to make fruit punch, how many fluid ounces of fruit juice are in the punch?